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Relational Goods, Private Consumption and Social Poverty Traps in an Evolutionary Game

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Abstract

We investigate how the patterns of consumption and accumulation, as well as the patterns of time allocation and of social interaction, may be influenced by 'social pressure', i.e., by the choices of others. We display an evolutionary model involving several generations of interacting individuals, in which different patterns may coexist in equilibrium and in which path dependence and critical mass effects play an important role. Bomze's (1983) classification is used to study the properties of this game. We find that 'social pressure' may lead to 'low welfare traps' with overwork and overconsumption, and that policy may have a role as a coordination device.

JEL-Classification: B52, D62, E21, J22, Z13

Key-words: Social interaction, Relational goods, Coordination failure

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1 Introduction

This paper represents a further step in a line of research that investigates how negative externalities may function as a *push* factor in the processes of economic growth, that is, as an 'engine' of economic growth. The mechanism through which negative externalities can fuel economic growth is very simple: individuals who are victims of negative externalities have, in many cases, the chance of alleviating the damage by increasing their consumption of private goods; growth of private consumption, in turn, may generate an increase in negative externalities and may therefore generate further victims who resort to consumption of private goods for self-protection. Such a mechanism may therefore be self-enforcing. This line of research can be subdivided in two strands, according to whether the main focus is on negative externalities of private production on the natural environment, as in Antoci and Bartolini (1997, 1999 and 2004) and Antoci and Borghesi (2001), or on the social environment, as in Antoci, Sacco and Vanin (2005 and 2006). The result that these works have in common is that when economic growth is fuelled by negative externalities, it may be undesirable; that is, economic agents' well-being would be greater if they consumed fewer private goods and accumulated less physical capital.

This paper may be seen as a further contribution to the second strand of this line of research. It takes the view that social environment matters for individual well-being and it investigates how the patterns of social interaction may change in response to agents' economic choices and how, in turn, they may influence those same choices. In particular, it focuses on choices of time allocation between production and leisure and of goods allocation between consumption and accumulation and it argues that such choices may be strongly conditioned by 'social pressure' (that is, by the choices of others). This implies that economic dynamics may exhibit *path dependence* and *critical mass effects*, in the sense that, for instance, a given pattern of capital accumulation may be selected if the proportion of individuals initially choosing it is high enough. As a consequence such pattern may become an expected, customary and self-enforcing status of things - that is, it may emerge as a *social convention* in the sense of Lewis (1969).

We investigate how such a social convention may emerge and we take the view that this process involves several generations. One after the other, and looking at past experience but also experimenting new strategies, generations slowly adopt those patterns of consumption and accumulation, of time allocation and of social interaction that turn out to be most rewarding. This process has one difficulty, though, and this is where 'social pressure' comes into the picture, since what is more rewarding depends on what other people do. Therefore, the process of gradual adoption of more rewarding strategies may lead to overall suboptimal choices if an high enough fraction of the population behaves initially in a 'wrong' way, thereby conditioning the rest of the population to respond to such behavior, rather than choosing what would otherwise be the more rewarding strategy.

We model this process in terms of an evolutionary dynamics involving several generations of interacting individuals. We find that different equilibria may emerge depending on initial conditions and critical masses, and that in the emerging equilibria different strategies may coexist. This means that the population of our economy may behave heterogeneously in equilibrium. We then discuss the welfare properties of the equilibria and argue that in some (but not in all) cases there is a scope for policy intervention, since sub-optimality is the consequence of a coordination failure.

The remainder of the paper is organized as follows. Section 2 discusses the concept of relational goods, which captures returns to socially enjoyed leisure, and the possible substitution of private goods for relational goods. Section 3 displays and analyzes the model, its dynamic properties and its policy implications. Section 4 concludes.

2 Relational goods and self-protection

As mentioned above, we consider social environment relevant for individual well-being. While such relevance may in principle take several forms, we are going to focus on the returns to social interaction in terms of relational goods. Relational goods are goods that cannot be enjoyed alone, since they can only be produced and consumed by participating to some common activity with other agents. For concreteness, think of the return to the time spent with friends or participating to a choir, a football club, a voluntary organization, and so on, or of what distinguishes a dinner alone from the same dinner in good company. Uhlaner (1989), who introduced the concept of relational goods among economists, emphasizes the relevance of joint participation. Indeed, relational goods may be seen as a special case Cornes and Sandler's (1984) joint production model.

The mixed private-public good approach[1] of the joint production model has interesting implications in terms of crowding in and of multiple equilibria, since it may display strategic complementarity in private contribution to the joint production. This means that if everybody else contributes much, this raises my private returns from contribution and therefore leads me to contribute much myself, but if other people's contribution is low, I may have no incentive to contribute much. Uhlaner applies this general result to the case of relational goods and emphasizes that an increase in the number of participants to the common activity may increase individual utility. One consequence of these features is that, due to coordination failure, participation to those activities that generate relational goods may be inefficiently low. In the present context we are going to consider in broad terms socially enjoyed leisure as the activity that generates relational goods, arguing that, even if interaction at work may be another source of relational goods, the first engine of most work relationships is private rather than relational, and it seems to be a good first approximation to consider the returns to work in terms of private goods and the returns to

[1] Observe that relational goods are an intermediate case between private and public goods: like public goods, they display relevant aspects of non-rivalry in consumption; like private goods, their consumption depends to a substantial degree upon own contribution.

socially enjoyed leisure in terms of relational goods. In Cooper and John's (1988) terminology, we are going to display a model in which, due to the role of social interaction, average leisure has positive spillovers. When a society gets stuck in a Pareto-dominated equilibrium with inefficiently low socially enjoyed leisure, we may speak of a 'social poverty trap'.

In several cases, individuals may defend themselves from a situation of social poverty by increasing their private consumption. If I try to share some activity with other people but they work too much and do not have time, I may react in a number of ways, which include devoting more time to television, eating more as a form of affective compensation, looking for private services that substitute for the interaction opportunities that are no longer available outside the market (e.g., marriage agencies, 'singles' bars, virtual dating), and working more myself to be able to buy some luxury goods. In this paper we are going to focus on this last reaction.

A number of studies, from Hirsch's (1976) classical work to more recent empirical investigations, point at the fact that the substitution process we investigate is indeed relevant in the real world. The analysis of its details and causes, though, proves quite complex and there are at least a dozen of possible explanatory factors that can be cited, including pressure on time, geographical mobility, television, generational change, increased women's labor force participation rate, expansion of the welfare state, and the list could continue. For instance, reflecting on the combination of private wealth and weak social support structure in the United States, Schiff (1992) argues[2]:

> The need to cope with the high degree of isolation caused by the higher degree of geographic labor mobility may lead to the creation of alternative institutions where people who are not as close can interact (e.g., singles' bars, dating services, nursing homes, insurance, and so on). These market activities enter into the gross national product (GNP) but do not necessarily imply higher welfare than in societies where some of these functions are carried out outside the market. (p.167-168)

The empirical analyses carried out to date indicate that four factors are particularly relevant to explain the substitution process we focus on: Putnam (2000), looking at the United States, emphasizes the role of television[3] and of generational shift (from the generation born between 1910 and 1940 characterized, in view of the particular historic conditions of the period, by a strong sense of civic commitment, to the generation born after the Second World War, much more individualistically oriented); Alesina and La Ferrara (2000) focus

[2]See also Schiff (1999) for a general equilibrium model of labor mobility in the presence of social capital and Schiff (2002) for a model that points out the difference between trade and migration as of factor mobility when social capital is considered.

[3]See also Corneo (2001), who shows clear empirical evidence that more time is spent in front of the television in those countries in which people work harder, and explains this evidence with a model based on relational goods, in which coordination failure may lead to a Pareto-dominated 'work and television' equilibrium.

prevalently on the importance of the increase of economic inequality and social heterogeneity; Costa and Kahn (2001) refer that the increased number of working women also plays a key role.

We do not go deeper into the analysis of any of these aspects, but rather take it for granted that there can be a relevant substitution process of activities that yield private goods for activities that yield relational goods. In particular, we shall simply assume that non-subsistence needs may be either satisfied by a relational good or by a luxury private good, and we shall distinguish between a 'relational strategy' and two 'private strategies': according to the first one, individuals devote much time to leisure and satisfy their non-subsistence needs with the relational good, but cannot afford the luxury good; according to the other two ones, they work more and are thus able to consume the luxury good; the difference is that in the first 'private strategy' the luxury good is consumed when young, whereas in the second one individuals save and accumulate capital when young in order to consume the luxury good when old. Both of the latter two strategies protect individuals from the negative externalities of a high average working time on social interaction, but while the first one offers a better protection in the youth, the second one does it in the old age. Their performance depends on the evolution of social participation over time, which in turn depends on the adoption dynamics of the three strategies under consideration. This interdependence generates phenomena of path dependence: economies which are identical in terms of preference and technology, but with different past histories reflected in a different initial distribution of the strategies, can come to differ radically at a social and economic level in terms of the allocation of time between the private and the social sphere, and in terms of capital accumulation.

3 The model

3.1 Preferences

We analyze the decision process of a continuum of (non-overlapping) generations of individuals parameterized by $s \in (0, +\infty)$. Each generation s is formed by a continuum of individuals that live for two periods: $t = 0, 1$. In $t = 0$ they are 'young' and in $t = 1$ they are 'old'. In each generation, individuals' well-being depends on four goods: leisure $(1 - l_t)$, a subsistence private good (P_t), a luxury private good (Q_t) and a relational good (R_t). For simplicity we assume that the relational good and the luxury good are perfect substitutes. Specifically, letting r be the intertemporal discount rate and d the marginal rate of substitution between Q and R, individual preferences are represented by the utility function

$$u = \ln(1-l_0)+\ln P_0+\ln(R_0+dQ_0)+r\left\{\ln(1-l_1)+\ln P_1+\ln(R_1+dQ_1)\right\}. \quad (1)$$

3.2 Technology

To keep the technology of this economy as simple as possible, we assume that both private goods (P and Q) are produced with labor only when individuals are young and with capital only when they are old. Moreover, capital is not passed from one generation to the next one. The relational good is produced with shared leisure and past relational good.

Given this technology, it is immediate to notice that individuals will not work when old, rather devoting their entire time to leisure, i.e., $l_1 = 0$.

3.3 Matching

The fact that the relational good is not a private good, but is rather jointly produced and consumed through social interaction, is captured by assuming that at birth individuals are randomly pairwise matched to play a one-shot game. Such game determines consumption of relational good according to whether the two interacting individuals devote much time to socially enjoyed leisure (thus foregoing some private consumption), or work hard to increase their private consumption, either when young or when old, but at the cost of a reduced leisure and of enjoying less relational good. The details of the game are specified below.

Therefore, when they are young, individuals have to decide both upon the allocation of their time between leisure and work and upon the patterns of consumption and accumulation, since the part of private production that is saved when young is used as capital to produce and consume new private goods when old. For simplicity, we assume that individuals have to choose among three pure strategies S_1, S_2, S_3.

3.4 Strategies

3.4.1 Relational strategy (S_1)

Following strategy S_1, they work 'little' when young: $l_0(S_1) = \bar{l}_L \in (0, 1)$. By working $\bar{l}_L$, they are not able to consume the luxury good, either when young or when old. Rather, they just produce enough private goods to be able to satisfy their subsistence needs, and for simplicity we assume that their subsistence consumption is the same when young and when old: $P_0(S_1) = P_1(S_1) = \overline{P}$[4]. Their consumption of relational goods is not entirely determined by their own strategy, but depends also upon the strategy of the individual j with whom they are pairwise matched: we specify later how $R_0(S_1, S_j)$ and $R_1(S_1, S_j)$ are determined.

[4]One can interpret the subsistence good as being the only one that can be accumulated as capital and used in future production. In this case, individuals following strategy S_1 would be producing only the subsistence good when young (say, the amount $\overline{Y}_L$) and saving just so much as to be able to produce and consume an equal amount of it when old (they save $\overline{K}_L = \overline{Y}_L - \overline{P}$, which yields again $\overline{P}$ at $t = 1$). Other interpretations are possible, but the point is not really relevant.

$\bar{l}_L$ and $\overline{P}$ are parameters of the model which are considered as exogenously determined. Moreover, without loss of generality we normalize $\overline{P} = 1$[5].

3.4.2 Private 'impatient' strategy (S_2)

Following strategy S_2, individuals work 'hard' when young: $l_0(S_2) = \bar{l}_H$, with $0 < \bar{l}_L < \bar{l}_H < 1$. By working $\bar{l}_H$, they are not only able to consume the quantity $\overline{P}$ of the subsistence good both when young and when old, but they are also able to increase their private consumption when young by consuming the quantity $Q_0(S_2) = \overline{Q}$ of the luxury good. Such additional consumption leaves savings (and therefore capital accumulation and subsistence consumption when old) unchanged with respect to strategy S_1. Relational goods when young and when old, $R_0(S_2, S_j)$ and $R_1(S_2, S_j)$, will be specified later.

$\bar{l}_H$ and $\overline{Q}$ are parameters of the model.

3.4.3 Private 'patient' strategy (S_3)

Finally, by following strategy S_3[6], individuals work 'hard' when they are young ($l_0(S_3) = \bar{l}_H$), but, instead of consuming the the luxury good when young, they rather save more and accumulate more capital in order to be able to increase their private consumption when old. Specifically, in their old age they consume the quantity $Q_1(S_3) = \overline{Q}$ of the luxury good[7]. Subsistence consumption is the same as in strategies S_1 and S_2. Relational goods when young and when old will now be $R_0(S_3, S_j)$ and $R_1(S_3, S_j)$.

3.5 Relational technology

We make two basic assumptions concerning the technology of joint production and consumption of relational goods: first, we assume that the amount of such goods jointly produced and consumed is the same for the two interacting individuals; second, we assume that this amount depends positively upon two factors: the quantity of time that the two interacting individuals are able to spend together (in this case, their leisure time) and the 'quality' of their interaction, which is determined by the amount of past relational goods on which

[5]Observe that also the assumption that $P_0(S_1) = P_1(S_1)$ is not restrictive, since our results do not depend upon it: these assumptions are made just to make model more easily readable.

[6]Notice that the labels 'impatient' and 'patient' are used here just to distinguish between a private strategy with low capital accumulation and one with high capital accumulation, respectively: they should not be interpreted literally, since the utility function, and in particular the discount rate r, are the same for everybody.

[7]This means that, paralleling the assumption about subsistence consumption, we are assuming that working hard allows to consume the quantity $\overline{Q}$ of luxury good, but individuals are free to choose between consuming it when young, as in strategy S_2, or when old, as in strategy S_3. It is immediate to extend all of our results, just we obvious adjustments in the inequalities (3) to (7), to the case where this equality assumption does not hold: its only role is to simplify notation.

they can count[8]. Since before birth there is no interaction, when individuals are young they cannot count on any past relational good; moreover, since no individual works when old and they all devote their entire time to leisure, the only determinant of the differences in consumption of relational goods when old is the amount of relational goods enjoyed when young.

We translate these assumption into simple functional forms in the following way. First, we write $R_1(S_i, S_j) = \delta R_0(S_i, S_j)$, where δ is a positive parameter. Second, calling $\omega > 0$ the maximum amount of relational goods that may be enjoyed by two interacting individuals when they are young, we assume that $R_0(S_i, S_j) = \omega$ if $S_i = S_j = S_1$; $R_0(S_i, S_j) = \omega - \alpha(\bar{l}_H - \bar{l}_L)$ if $S_i = S_1$ and $S_j \in \{S_2, S_3\}$; and $R_0(S_i, S_j) = \omega - 2\alpha(\bar{l}_H - \bar{l}_L)$ if $S_i, S_j \in \{S_2, S_3\}$[9].

$\alpha > 0$ is a parameter that measures the negative externality on the relational sector generated by the time spent at work over the minimum work time needed to satisfy subsistence needs (for this reason the difference $\bar{l}_H - \bar{l}_L$ enters in spite of just $\bar{l}_H$). To simplify notation, we assume throughout that $\bar{l}_H = 2\bar{l}_L$ and omit the subscript by writing $\bar{l}_L = \bar{l}$. This assumption has a possible and intuitive interpretation in terms of part-time and full-time job, but in any case is not crucial for our results.

3.6 Payoffs

We now have all the elements to fill in the 3×3 payoff matrix U, by substituting into equation (1) the amount of all kinds of goods consumed, which depends upon technology and the combination of own strategy and of the strategy followed by the individual I am matched with: $U_{ij} = u(S_i, S_j)$. This matrix is

	S_1	S_2	S_3
S_1	A	B	B
S_2	C	D	D
S_3	E	F	F

,

$$
\begin{aligned}
A &\equiv \ln(1 - \frac{\bar{l}}{2}) + \ln\omega + r\ln(\delta\omega), \\
B &\equiv \ln(1 - \frac{\bar{l}}{2}) + \ln(\omega - \alpha\frac{\bar{l}}{2}) + r\ln[\delta(\omega - \alpha\frac{\bar{l}}{2})], \\
C &\equiv \ln(1 - \bar{l}) + \ln(\omega - \alpha\frac{\bar{l}}{2} + d\overline{Q}) + r\ln[\delta(\omega - \alpha\frac{\bar{l}}{2})], \\
D &\equiv \ln(1 - \bar{l}) + \ln(\omega - \alpha\bar{l} + d\overline{Q}) + r\ln[\delta(\omega - \alpha\bar{l})], \\
E &\equiv \ln(1 - \bar{l}) + \ln(\omega - \alpha\frac{\bar{l}}{2}) + r\ln[\delta(\omega - \alpha\frac{\bar{l}}{2}) + d\overline{Q}], \\
F &\equiv \ln(1 - \bar{l}) + \ln(\omega - \alpha\bar{l}) + r\ln[\delta(\omega - \alpha\bar{l}) + d\overline{Q}].
\end{aligned}
$$

[8]Notice that the time spent in socially enjoyed leisure serves here at the same time for immediate consumption of the relational good and as a relation-specific investment whose return is given by future consumption of the relational good.

[9]Because of our assumptions, observe that the matrix $R_0(S_i, S_j)_{i,j=1,2,3}$ is symmetric.

Observe that $A > B,\ C > D,\ E > F$.

3.7 Strategy adoption: replicator dynamics

Up to now we have not specified how individuals choose their strategies. We are not interested in the solution to an individual maximization problem with full rationality, but rather in the analysis of plausible patterns of behavior selection at the social level. We take the view that, for a variety of reasons, individuals belonging to a given generation may end up choosing either one of the three strategies outlined above. Such reasons include the fact that it is often difficult to calculate *ex ante* all the consequences of the choices one makes when young, the fact that social pressure may sustain social norms that induce to follow a certain behavioral pattern, the fact that some decisions (especially concerning social interaction with other people) are not taken instrumentally (i.e., they are not taken on the basis of the calculation of their consequences, but rather on the basis of some 'intrinsic' value they may have), and so on. But we also take the view that more satisfactory strategies slowly spread over in society, for instance because individuals are able to observe the choices of the previous generation and to discern who fared well and who did not. Therefore, even though other factors (for instance the desire to experiment, or the idea that the world has changed) may induce some individuals to deviate from the strategy that turned out to be most rewarding for the previous generation, imitation will drive more individuals to embrace it than to leave it. According to this view, it is natural to model the process of strategy adoption by the replicator dynamics.

Let therefore $x_1(s),\ x_2(s),\ x_3(s)$ be the proportion of individuals choosing respectively strategies $S_1,\ S_2,\ S_3$ in generation $s \in (0, +\infty)$. Thus, the distribution of strategies $x = (x_1, x_2, x_3)$ belongs to the unitary simplex $K = \left\{ x \in \Re^3 :\ x_1, x_2, x_3 \geq 0 \text{ and } \sum_{i=1}^{3} x_i = 1 \right\}$. We will denote by $e_1 = (1, 0, 0),\ e_2 = (0, 1, 0),\ e_3 = (0, 0, 1)$ the vertices of K, where all agents adopt the same strategy S_1, S_2, S_3, respectively; and by K_{ij} the edge of the simplex linking the vertices e_i and e_j (obviously, $K_{ij} = K_{ji}$).

The (ex-ante) expected payoff of strategy S_i is:

$$e_i \cdot U \cdot x = \sum_{j=1}^{3} U_{ij} x_j$$

and the average payoff is:

$$x \cdot U \cdot x = \sum_{i=1}^{3} x_i e_i \cdot U \cdot x = \sum_{i=1}^{3} \sum_{j=1}^{3} U_{ij} x_i x_j$$

We assume that strategies which perform better than average are chosen with increasing frequency at the expense of the less rewarding ones by new generations. Specifically, by choosing s as time parametrization for the dynamics across generations, and following Taylor and Jonker (1978), we assume that the rate of growth of the frequency of strategy S_i is equal to the difference between its expected payoff an average payoff:

$$\frac{dx_i(s)}{ds} = x_i(s)\,[e_i \cdot P \cdot x(s) - x(s) \cdot P \cdot x(s)] \tag{2}$$

$i = 1, 2, 3$; where $dx_i(s)/ds$ is the derivative of $x_i(s)$ w.r.t. s (i.e., the derivative w.r.t. 'time').

3.8 Properties of dynamics (2)

Under dynamics (2), both the simplex K and its edges are invariant, and its vertices are fixed points (see e.g. Weibull (1995)). We use Bomze's (1983) classification to establish the main properties of dynamics (2). The details are in Appendix. We summarize here our main 'robust' results, where by 'robust' we mean those results that hold but at most under a certain equality condition on some parameters.

First, there are no attractive fixed points in the interior of the simplex, as well as there are none in the interior of the K_{23} edge. This means that the dynamics cannot converge to a situation where each of the three strategies is adopted by a positive fraction of the population, as well as it cannot converge to a situation in which a positive fraction adopts the private 'impatient' strategy S_2 and the remainder (positive) fraction adopts the private 'patient' one S_3, but nobody adopts the relational strategy S_1.

Second, while the vertices e_2 and e_3 cannot be simultaneously attractive, either e_1 and e_2 or e_1 and e_3 can be simultaneously attractive. In other words, it is possible to have two kinds of bistable dynamics, such that in the attractor everybody adopts the same strategy: in the first one, depending on initial conditions (i.e., on $x(0)$), all individuals may either end up adopting the relational strategy S_1 or the private 'patient' strategy S_3; in the second one, again depending on initial conditions, all individuals may either end up adopting the relational strategy S_1 or the private 'impatient' strategy S_2. Figure 1 illustrates the first case; figure 2 the second one.

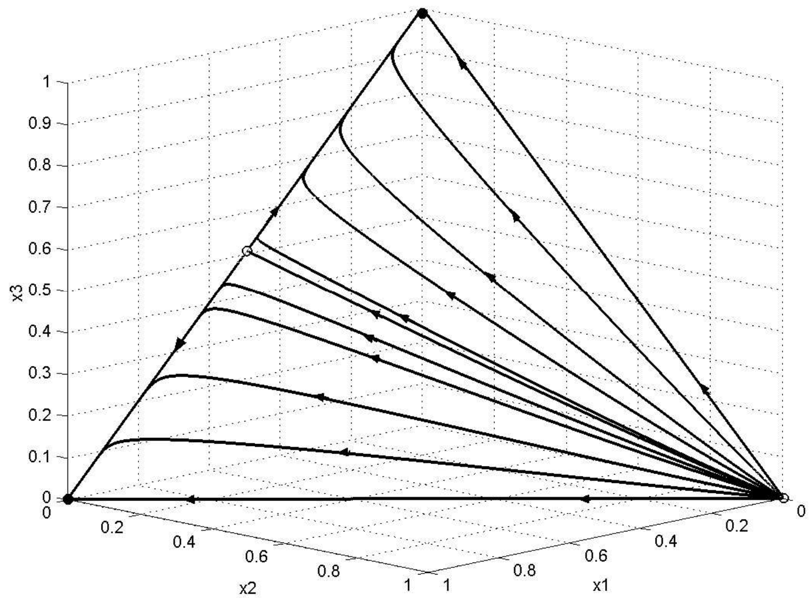

Figure 1

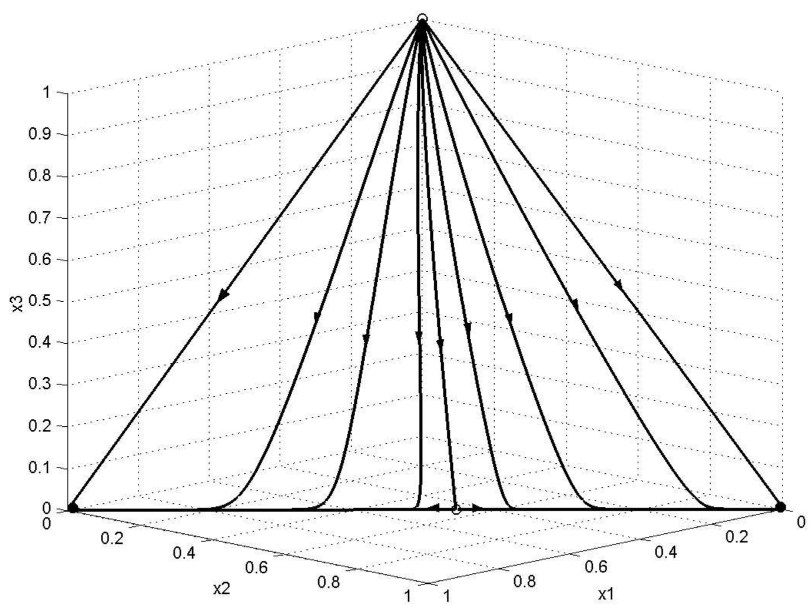

Figure 2

Figures 1 and 2 have been obtained with the following parameter values:

in both of them $\overline{Q} = \overline{P} = 200$[10], $\bar{l} = 6$, $\alpha = 10$, $\omega = 1000$; in figure 1 $r = 0.8$, $d = 2.5$, $\delta = 0.6$; in figure 2 $r = 0.5$, $d = 2.955$, $\delta = 1.12$. The change in the kind of bistable dynamics generated by this parameter change has an intuitive interpretation: a higher impatience (a lower r) renders strategy S_3 (the private strategy with high capital accumulation) less satisfactory and therefore decreases its chances of being selected under dynamics (2). Therefore, higher impatience, accompanied in this case by the simultaneous increases in the preference for luxury good over relational good (d) and in the dependence of future relational good upon the 'quality' of existing relations (δ), lets the attractor different from e_1 pass from being e_3 to being e_2, that is, we pass from a situation in which everybody may end up adopting the private strategy with high capital accumulation to one in which everybody may end up adopting the private strategy with low capital accumulation.

Our third result is that there can be attractors (even global attractors) on the edges K_{12} and K_{13}, but not contemporaneously on both. This means that the dynamics may lead to an equilibrium in which different strategies coexist, that is, a positive fraction of the population adopts the relational strategy S_1 and the remainder (positive) fraction adopts either one (but the same for all) of the two private strategies. Figure 3 illustrates the possible coexistence in the attractor of S_1 with the private 'patient' strategy S_3 and figure 4 its possible coexistence with the private 'impatient' strategy S_2.

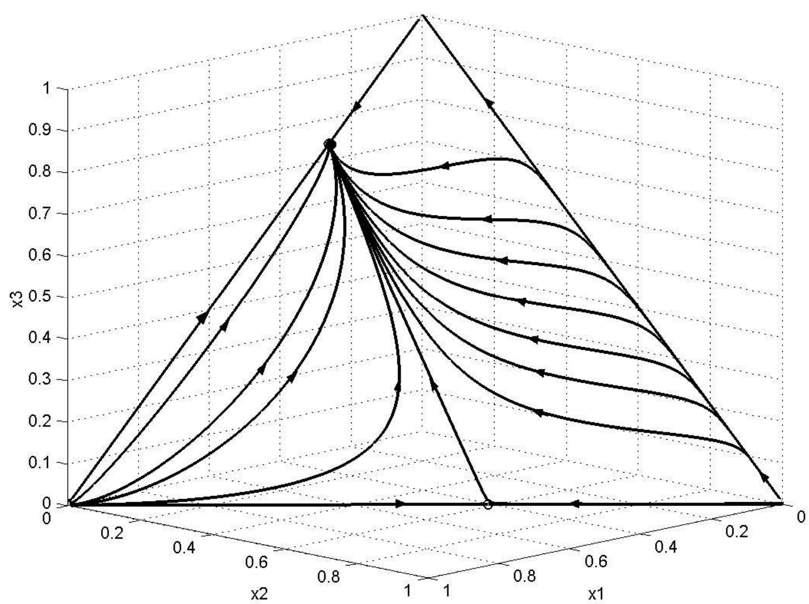

Figure 3

[10]Notice that the assumption $\overline{P} = 1$ adopted in the text was just a normalization, so that the numerical specification is not in contrast with it.

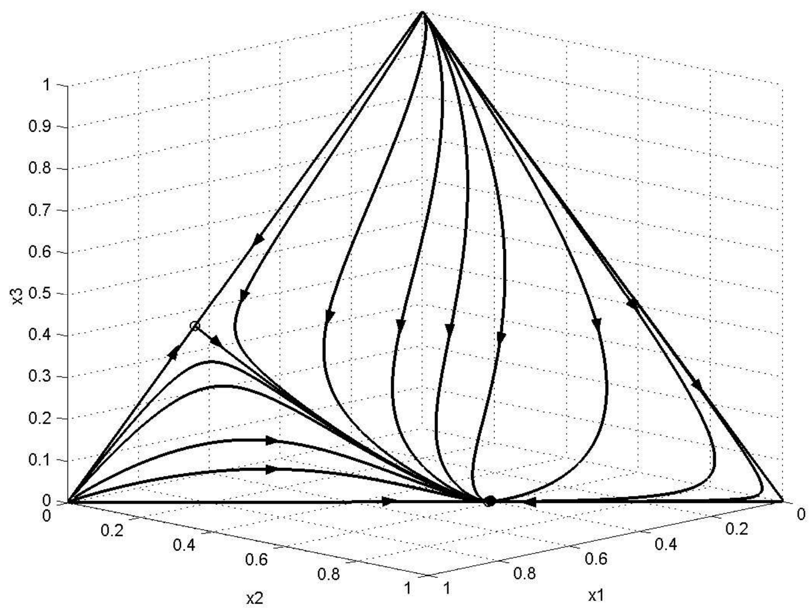

Figure 4

Figures 3 and 4 have been obtained with the following parameter values: as in figures 1 and 2, in both of them $\overline{Q} = \overline{P} = 200$, $\bar{l} = 6$, $\alpha = 10$, but now in both of them $\omega = 125$, $r = 0.8$, $d = 2.5$, whereas in figure 3 $\delta = 0.3$ and in figure 4 $\delta = 0.6$. Therefore, with respect to figure 1, figure 3 is obtained by changing only one parameter, namely by reducing the maximum amount of relational good obtainable ω. This obviously renders S_1 less satisfactory and therefore it is not surprising that e_1 stops being a sink, but what is interesting is to notice that also e_3 is not a sink anymore, that is, even if the initial frequency of private strategies is very high, under these parameters the relational strategy will have a higher than average return and so will spread over, until a certain balance between S_1 and S_3 is reached; at the same time, when the frequency of S_3 is low, it also performs better than average and spreads over, so that in the attractive fixed point we find strategies S_1 and S_3 coexisting. Something similar happens in figure 4 with strategies S_1 and S_2, and the only difference with respect to figure 3 is that we now have a lower δ, i.e., a lower 'productivity' of present relational good in terms of future relational good.

What should be clear at this point is that which strategies perform better than average, and therefore which ones spread over, is not a simple function of either the parameters or the distribution of strategies, but rather depends upon both. The two bistable dynamics show the possibility that structurally equal economies end up with very different patterns of consumption and accumulation just because they differ in the initial distribution of strategies. The two dynamics with coexistence show that even when there is a unique attractor of the dynamics, the population behavior need not be homogeneous, in the sense that we can observe very different patterns of consumption and accumulation across the population. The next question is therefore whether the fixed point to which the economy converges in each case is Pareto-efficient or whether it is dominated by another fixed point.

Our fourth result concerns welfare comparisons among fixed points of the dynamics. In both cases of bistable dynamics (figures 1 and 2), the fixed point e_1 Pareto-dominates the other attractor. This means that everybody would be better off under the universal adoption of the relational strategy than under the universal adoption of the private strategy that may be selected by the dynamics. Therefore, if the initial frequency of the attractive private strategy is sufficiently high, individuals end up in a 'poverty trap' characterized by an excess of work and private consumption and by poor relational interactions. If, as it seems reasonable in many contexts, we assume that private goods enter in the measured GDP but relational goods do not, then GDP in this case is not a good measure of welfare: while welfare is higher when everybody follows the relational strategy, GDP is higher when everybody follows one of the two private strategies. Such 'poverty trap' is due to coordination failure: to escape it, it would be sufficient that a high enough fraction of the population were able to coordinate on the relational strategy, since then the rest of the population would follow. Without coordination, what may happen is that individuals in fact try to defend themselves from social poverty, i.e., from the fact that too few other individuals follow the relational strategy, by devoting more time to work in order to consume the luxury good which is a substitute of the relational good. Notice that, while this behavior, although leading to a 'low welfare trap', has clear implications in terms of labor offer, and therefore private production and income, its implications in terms of accumulation patterns may be of two kinds: if the dynamics leads to e_2, accumulation in equilibrium is low; if it leads to e_3, it is high.

In the two cases illustrated in figures 3 and 4, in which there is only one attractive fixed point, where the relational strategy coexists with one of the private strategies, we still find that the (now repulsive) fixed point e_1 Pareto-dominates the attractor, but in this case the coordination problem is much more severe than in the case of a bistable dynamics, since now, in order to reach e_1, the entire population should be able to coordinate on S_1 at the same time, and any deviation by a whatever small but positive fraction would lead again to the Pareto-dominated coexistence equilibrium.

3.9 Policy implications

The coordination problem just outlined provides a clear rationale for policy intervention. For instance, in the cases illustrated by figures 1 and 2, consider an economy starting with such a distribution of strategies $x(0)$ that it would spontaneously end up in a 'poverty trap' with overwork: a policy of temporary (i.e., for a generation) working time regulation that were able to induce a significant but less than one fraction of the population (let's say the category of workers to which the regulation applies) to work less and therefore embrace the relational strategy would be able to take the economy out of the trap, since the rest of the population would then spontaneously follow. By contrast, the same policy would turn out to be ineffective in the cases illustrated by figures 3 and 4, since the temporary change would be reverted by next generations. Therefore, while our model points at a clear scope for policy as a coordination device, it also prevents from the temptation to reach simple conclusions that do not take adequately into account the complexity of socio-economic interaction.

4 Conclusion

The model presented shows how the patterns of consumption and accumulation, as well as the patterns of time allocation between private production and leisure, and hence those of social interaction made possible by shared leisure, may be the result of a process in which individuals try to find their personal life strategy by imitating the strategies that turned out to be most satisfactory for previous generations and by experimenting some new ones. In such a process 'social pressure' plays a key role, since which strategy is most satisfactory depends upon the distribution of strategies in the population. Because of this reason, and since the process may converge to different equilibria, we find path dependence in the form of important critical mass effects in the initial distribution of strategies. Moreover, we find that the process may both converge to equilibria in which the entire population adopts the same strategy, and to equilibria in which two different strategies coexist. We show several cases in which the dynamics leads to 'low welfare traps' where individuals work and consume private goods at a higher than efficient rate, at the expenses of socially enjoyed leisure, and we observe that in such cases GDP may not serve as a good welfare indicator, since it does not take into account the welfare loss implied by a lower consumption of relational goods. Finally, we argue that in some cases policy intervention may take the economy out of a 'low welfare trap' if it succeeds to coordinate the actions of a critical mass of individuals, but in other cases it cannot.

Let us conclude by considering two possible extensions of our setting. The first one is to consider the fact that the private consumption that substitutes for the relational good, and that we here denoted for simplicity as 'luxury' consumption, might have negative external effects on the relational good, for instance through a reduction in the opportunity to meet other people at equal levels of leisure. Preliminary simulations that take this effect into account show that in this case the dynamics may converge to an interior fixed point, where all of the three strategies considered here are present.

A second extension would be to consider the possibility to leave bequests or to transfer capital from one generation to the next one. This would allow to fully study the growth implications of our model, whereas, by assuming that individuals are endowed with no capital at birth, we are able to properly discuss accumulation patterns within one generation, but not growth patterns through generations. It is clear that the two aspects are interconnected, and it is to expect that the analogous to our 'low welfare traps' with overwork would be something like the possibility of dynamic inefficiency; but at the moment this is just a supposition. It may be also interesting to enrich the model by taking into account that relational activities may imply a short run cost in terms of output and growth, but a long run benefit, as the literature on social capital seems to indicate; but this remains for future work.

Appendix

Proposition 1 ***Stability of*** e_1
The fixed point e_1 has one eigenvalue positively proportional to the expression $C - A$ (in direction of the vector $e_1 - e_2$, that is, $e_1 - e_2$ is an eigenvector corresponding to such eigenvalue) and one eigenvalue positively proportional to the expression $E - A$ (in direction of the vector $e_1 - e_3$).

$C - A < 0$ and $E - A < 0$ if and only if, respectively,

$$d\overline{Q} < \frac{\left(1-\frac{\bar{l}}{2}\right)\omega^{1+r}}{(1-\bar{l})\left(\omega-\alpha\frac{\bar{l}}{2}\right)^r} + \alpha\frac{\bar{l}}{2} - \omega, \quad \text{and} \tag{3}$$

$$d\overline{Q} < \delta\left\{\left[\frac{\left(1-\frac{\bar{l}}{2}\right)\omega^{1+r}}{(1-\bar{l})\left(\omega-\alpha\frac{\bar{l}}{2}\right)}\right]^{\frac{1}{r}} + \alpha\frac{\bar{l}}{2} - \omega\right\}. \tag{4}$$

Proof The result is an application of Bomze (1983), Proposition 1.

Proposition 2 ***Stability of*** e_2
The fixed point e_1 has one eigenvalue positively proportional to the expression $B - D$ (in direction of the vector $e_2 - e_1$) and one eigenvalue positively proportional to the expression $F - D$ (in direction of the vector $e_2 - e_3$).

$B - D < 0$ and $F - D < 0$ if and only if, respectively,

$$d\overline{Q} > \frac{\left(1-\frac{\bar{l}}{2}\right)\left(\omega-\alpha\frac{\bar{l}}{2}\right)^{1+r}}{(1-\bar{l})(\omega-\alpha\bar{l})^r} + \alpha\bar{l} - \omega, \quad \text{and} \tag{5}$$

$$1 + \frac{d\overline{Q}}{\omega-\alpha\bar{l}} > \left[1 + \frac{d\overline{Q}}{\delta(\omega-\alpha\bar{l})}\right]^r. \tag{6}$$

Proof The result is an application of Bomze (1983), Proposition 1.

Remark 1 *Inequality (6) is always satisfied if $\delta \geq 1$; it is not satisfied if δ is small enough.*

Proposition 3 ***Stability of*** e_3
The fixed point e_3 has one eigenvalue positively proportional to the expression

$B-F$ (in direction of the vector $e_3 - e_1$) and one eigenvalue positively proportional to the expression $D-F$ (in direction of the vector $e_3 - e_2$).

$B-F<0$ if and only if

$$d\overline{Q} > \delta \left\{ \left[\frac{\left(1-\frac{\bar{l}}{2}\right)\left(\omega - \alpha\frac{\bar{l}}{2}\right)^{1+r}}{(1-\bar{l})(\omega-\alpha\bar{l})} \right]^{\frac{1}{r}} + \alpha\bar{l} - \omega \right\}. \tag{7}$$

$D-F<0$ if and only if inequality (6) holds with reversed sign.

Proof The result is an application of Bomze (1983), Proposition 1.

Recall that a fixed point is locally attractive (a sink) if has two strictly negative eigenvalues; it is repulsive (a source) if it has two strictly positive eigenvalues; it is a saddle if it has a strictly positive eigenvalue and a strictly negative one. Moreover, in the following we define 'hyperbolic' a fixed point that has both eigenvalues different from zero.

Remark 2 *The eigenvalues of e_2 and e_3 cannot be all contemporaneously strictly negative. Therefore, while e_1 and e_3, as well as e_1 and e_2, can be contemporaneously attractive, e_2 and e_3 cannot.*

Proposition 4 ***Pareto-dominance among the vertices***
If e_1 and e_2 are both hyperbolic sinks, e_1 Pareto-dominates e_2.
If e_1 and e_3 are both hyperbolic sinks, e_1 Pareto-dominates e_3.

Proof We prove here the first part of the proposition (the proof of the second part follows the same lines). e_1 and e_2 are both hyperbolic sinks if and only if $A > C, E$ and $D > B, F$. Since the payoff matrix U satisfies $A > B$, $C > D$, $E > F$, it follows that $A > C > D$, which proves the result since $u(S_1, S_1) = A > D = u(S_2, S_2)$.

Proposition 5 ***Fixed points on the edge*** K_{12}
K_{12} is pointwise fixed (that is, it is entirely constituted of fixed points) if and only if $C - A = D - B = 0$.

The interior of K_{12} contains a unique fixed point if and only if

$$(C-A)(D-B) < 0. \tag{8}$$

If neither inequality (8) nor the previous equality hold, then there are no fixed points in the interior of K_{12}.

If inequality (8) holds, then the fixed point in the interior of K_{12} has one eigenvalue positively proportional to the expression $A-C$ (in direction of K_{12}) and one eigenvalue positively proportional to the expression

$$\frac{(D-B)(E-A)-(C-A)(F-B)}{D-B}. \tag{9}$$

Proof The result is an application of Bomze (1983), Proposition 2.

Remark 3 *If inequality (8) holds, a necessary condition for the unique fixed point in the interior of* K_{12} *to be attractive is that both* e_1 *and* e_2 *have a strictly negative eigenvalue in direction of* $e_1 - e_2$ *(that is, inequalities (3) and (5) must hold with reversed sign).*

Proposition 6 ***Fixed points on the edge*** K_{13}
K_{13} *is pointwise fixed if and only if* $E - A = F - B = 0$.

The interior of K_{13} *contains a unique fixed point if and only if*

$$(E - A)(F - B) < 0. \tag{10}$$

If neither inequality (10) nor the previous equality hold, then there are no fixed points in the interior of K_{13}.

If inequality (10) holds, then the fixed point in the interior of K_{13} *has one eigenvalue positively proportional to the expression* $A - E$ *(in direction of* K_{13}*) and one eigenvalue positively proportional to the expression*

$$\frac{(C - A)(F - B) - (D - B)(E - A)}{F - B}. \tag{11}$$

Proof The result is an application of Bomze (1983), Proposition 2.

Remark 4 *If inequality (10) holds, a necessary condition for the unique fixed point in the interior of* K_{13} *to be attractive is that both* e_1 *and* e_3 *have a strictly negative eigenvalue in direction of* $e_1 - e_3$ *(that is, inequalities (4) and (7) must hold with reversed sign).*

Moreover, if in the interior of both K_{12} *and* K_{13} *there exists a unique fixed point, they cannot be both hyperbolic sinks*[11].

Proposition 7 ***Pareto-dominance along the edges*** K_{12} ***and*** K_{13}
e_1 *Pareto-dominates any fixed point in the interior of the edges* K_{12} *and* K_{13}*, independently of their respective stability.*

Proof In any fixed point in the interior of the edge K_{12} (respectively, K_{13}) the expected payoff of strategy S_1 must be equal to the expected payoff of strategy S_2 (respectively, S_3). Since the expected payoff of strategy S_1 reaches its maximum value A in e_1 and is strictly lower outside of it, we have proved the result.

Proposition 8 ***Fixed points on the edge*** K_{23}
K_{23} *is pointwise fixed if and only if* $D = F$.
If $D \neq F$*, there are no fixed points in the interior of* K_{23}.

Proof The result is an application of Bomze (1983), Proposition 5.

[11] To see this, observe that (8) and $A - C < 0$ imply $D - B < 0$, that (10) and $A - E < 0$ imply $F - B < 0$, and that therefore expressions (9) and (11) have opposite sign, since they both have a negative denominator while their numerators are the opposite of one another.

Proposition 9 ***Fixed points in the interior of*** K
If $(C - A)(F - B) \neq (D - B)(E - A)$*, there are no fixed points in the interior of the simplex* K *(that is, there are no fixed points where the three strategies* S_1*,* S_2*,* S_3 *all coexist).*

If $(C - A)(F - B) = (D - B)(E - A)$*, there exist fixed points in the interior of the simplex* K*, but they are not isolated. In particular, if* $(C - A)(F - B) = (D - B)(E - A) = 0$*, there exists a segment of fixed points in the interior of* K*; if* $(C - A) = (F - B) = (D - B) = (E - A) = 0$*, the entire simplex is pointwise fixed.*

Proof The result is an application of Bomze (1983), Proposition 6.

Remark 5 *In the same way as we proved the result on the Pareto-dominance along the edges, one easily proves that* e_1 *Pareto-dominates any fixed point in the interior of* K*.*

References

[1] Alesina A., La Ferrara E. (2000), Participation in Heterogeneous Communities, The Quaterly Journal of Economics, CXV, 847-904

[2] Antoci A., Bartolini S. (1997), Externalities and Growth in an Evolutionary Game, Discussion Papers n. 9, University of Trento, Department of Economics, Trento

[3] Antoci A., Bartolini S. (1999), Negative Externalities as the Engine of Growth in an Evolutionary Context, Nota di lavoro 83/99, Fondazione Eni Enrico Mattei, Milan

[4] Antoci A., Bartolini S. (2004), Negative Externalities, Labor Input and Economic Growth in an Evolutionary Context, Environment and Development Economics, forthcoming

[5] Antoci A., Borghesi S. (2001), Working Too Much in a Polluted World: A North-South Evolutionary Model, Quaderni di Dipartimento n.334, Department Economics, University of Siena, Siena

[6] Antoci A., Sacco P.L., Vanin P. (2005), On the Possible Conflict Between Economic Growth and Social Development, in Gui B. and R. Sugden (eds.), Economics and Social Interaction: Accounting for Interpersonal Relations, Cambridge University Press

[7] Antoci A., Sacco P.L., Vanin P. (2006), Social Capital Accumulation and the Evolution of Social Participation, Journal of Socio-Economics, forthcoming

[8] Bomze I. (1983), Lotka-Volterra Equation and Replicator Dynamics: A Two-Dimensional Classification, Biological Cybernetics, 48, 201-211

[9] Corneo G. (2001), Work and Television, Institute for the Study of Labor (IZA), Bonn, Discussion Paper n.376

[10] Cornes R., Sandler T. (1984), Easy Riders, Joint Production, and Public Goods, The Economic Journal, 94(375), 580-598

[11] Cooper R., John A. (1988), Coordinating Coordination Failures in Keynesian Models, The Quarterly Journal of Economics, CIII(3), 441-463

[12] Costa D. L., Kahn M. E. (2001), Understanding the Decline in Social Capital, 1952-1998, NBER Working Paper No. 8295

[13] Hirsch F. (1976), Social Limits to Growth, Cambridge, Mass., Harvard University Press

[14] Lewis D. (1969), Convention, Harvard University Press, Cambridge, Mass.

[15] Putnam R. (2000), Bowling Alone: The Collapse and Revival of American Community, Simon and Schuster, New York

[16] Schiff M. (1992), Social Capital, Labor Mobility, and Welfare, Rationality and Society 4, 157-175

[17] Schiff M. (1999), Labor Market Integratin in the Presence of Social Capital, Development Research Group, World Bank

[18] Schiff M. (2002), Love Thy Neighbor: Trade, Migration and Social Capital, European Journal of Political Economy, 18 (1), 87-107

[19] Uhlaner C.J. (1989), Relational Goods and Participation: Incorporating Sociability into a Theory of Rational Action, Public Choice n.62, 253-285

[20] Taylor P., Jonker L. (1978), Evolutionary Stable Strategies and Game Dynamics, Mathematical Biosciences 40, 145-156

[21] Weibull J. W. (1995), Evolutionary game theory, The MIT Press, Cambridge, Ma