*Metroeconomica* 46:2 (1995) pp. 111–126

# DIRECTED GRAPH THEORY AND THE ECONOMIC ANALYSIS OF INNOVATION

Enrico Santarelli*
University of Bologna

(March 1992: revised January 1993)

## ABSTRACT

This article applies digraph theory to the theory of technological regimes. The first part of the paper identifies a number of links between the evolutionary approach and a particular version of the neoclassical approach to the economic analysis of technological change. Both these approaches are shown to take the body of presently-available technological knowledge as a quantifiable magnitude allowing firms varying in strategy, structure and core capabilities to explore a range of feasible alternatives within the frontiers imposed by such knowledge. The second part of the paper therefore considers technological knowledge to be a cognitive empirical structure which can be better defined and calculated by using the theorems of digraph theory.

## 1. INTRODUCTION

It is the aim of this article to compare those approaches which consider technological knowledge to be a cognitive empirical structure underlying the process of economic change, and to reformulate their main results in terms of the theory of directed graph.[1]

The theory of directed graph (henceforth digraph theory) is a powerful tool for an analysis of the structural properties of any empirical

---

* Financial support from the National Research Council of Italy and the Italian Ministry of University and Scientific and Technological Research is gratefully acknowledged. I wish to express my thanks to Luigi Marengo, Roberta Piergiovanni, Nathan Rosenberg, Alessandro Sterlacchini, Nick Von Tunzelmann and two referees of this *Journal* for their useful comments. The usual disclaimers apply.

[1] An introductory treatment of directed graphs can be found in Harary–Norman–Cartwright (1965). For the use of directed graphs in the economic analysis of technological change, see Green–Shoven (1983), Vega-Redondo (1993).

system. Firstly, it enables the researcher to define the complex properties of a structure which cannot be described using everyday language. This, in the case considered here, is the current language of economics. Secondly, digraph theory provides techniques for calculating the quantifiable features of an empirical structure, such as the adjacency of and the distance between its various elements. Thirdly, it has developed numerous theorems which yield additional information on empirical structures by enabling the researcher to draw conclusions about certain properties from his/her knowledge of other properties of the same structure.

Section 2 surveys the concepts of technological regime and the natural trajectory of technology developed by evolutionary theorists. Section 3 compares these concepts to the particular versions of the production function approach previously introduced by Salter (1960, 1966) and Atkinson–Stiglitz (1969). In section 4 a digraph representation of the structure of technological knowledge is outlined. Finally, section 5 makes some concluding remarks.

## 2. THE EVOLUTIONARY APPROACH TO THE EXPLANATION OF TECHNOLOGICAL CHANGE

The set of technological knowledge and skills available in a certain period can be defined as the *cognitive structure* underlying possible developments in industrial activities. In this respect, the concepts of technology and knowledge are closely related, and technology can be represented as knowledge "contained in the minds of individuals who either know directly or know how to find out [. . .] some piece of information" (Metcalfe–Boden, 1991, p. 711). The employment of this cognitive structure is closely related to a firm's *core capabilities*, which represent the set of operations it is able to perform with confidence using its organizational skills (Nelson, 1991).

Under this assumption, the search processes that lead firms to new discoveries are oriented by the conventions which prevail at any given period in the economic system, and by the specific skills, experience, and knowledge which characterize any given firm. This feature of research activity has been empirically confirmed for all of the most technologically advanced industries (see, for the cases of petroleum-refining and computer manufacture, among others, Enos, 1962, and Katz–Phillips, 1981), where successive technological advances seem to follow a common path which, as many authors stress,[2] appears almost inevitable. This path-dependent process is led by those firms which, in the pursuit of competitive advantage, display the requisite set of core capabilities in R&D. Firms which are able to exploit the increasing returns deriving from a new prevailing technology come to dominate a market, irrespectively of their previous position within it. This empirical finding has generated a new approach to the economic analysis of technological change, which aims to represent an alternative and a challenge to mainstream economics: in the following pages this will be referred to as the "evolutionary" approach.

According to evolutionary theorists,[3] one may assume that most of the R&D carried out within an industrial sector at any time is related to a well defined *natural trajectory of technology* which in turn belongs to a technological *regime*.[4] Thus a technological regime embodies a series of *natural trajectories*, which single out the direction of technical progress in specific fields and denote the results of scientific knowledge directly exploitable at the commercial level. Natural trajectories represent the tangible results of the exploitation of basic technological knowledge in industrial activities and, as such, they are more significant than technological regimes for the economic analysis of the production process in a strict sense. In some cases, natural trajectories which are specific to different technological regimes display a certain degree of complementarity. These are particularly pervasive natural trajectories which play a pivotal role in promoting technological advance.

Within a natural trajectory, technological change is a continuous and cumulative process which follows a path-dependent, evolutionary pattern. Conversely, the passage from one technological regime to another

---

[2] Cf., among other, Nelson–Winter (1982), Arthur (1983, 1989), David (1986, 1988), and Rosenberg (1988). According to David (1988, p. 18) a path-dependent process can be seen as "the whole class of dynamical processes in which the position and motion of the economic system—or of particular, constituent sub-systems like firms and households—turn out to be sensitive to initial conditions".

[3] Cf., for example, Rosenberg (1969, 1988), Nelson–Winter (1977), Abernathy–Utterback (1978), Dosi (1982), Freeman–Clark–Soete (1982), Sahal (1985), Freeman–Perez (1986), Andersen (1991), and others. For a review of these contributions, see Nelson–Winter (1982, Ch.X), Teece (1986), Clark–Juma (1987), and Dosi (1988).

[4] Or *technological paradigm*, as defined by Dosi. Dosi and Abernathy–Utterback analyse technological change by developing an approach which combines two important notions of the modern philosophy of science: that of *scientific paradigms* (introduced by Kuhn, 1963) and that of *scientific research programmes* (developed by Lakatos, 1970). The former is a synonym for technological paradigms, the latter for technological trajectories. Other definitions are provided by the ideas of *technology system*, developed by Freeman *et al.*, and *technological guideposts*, as defined by Sahal.

constitutes a dramatic change (a *discontinuity* in Rosenberg's terms) in the traditions of practice applied to the development of technological capability and denotes a major technological breakthrough.

The origins of this evolutionary approach can be traced back to the definition of *technological convergencies* introduced by Rosenberg (1963, 1969), who pointed out how different industries converge to a common technological pattern and how *technological imperatives* guide the evolution of a given body of technological knowledge. Since the technological regime is "a pattern of solution of selected technological problems" (Dosi, 1982, p. 152) which determines the development of natural trajectories, it represents in itself a technological imperative as it "indicates fruitful directions for technological change, defines some ideas of progress, and has a powerful exclusion effect on the imagination of engineers and organisations" (Metcalfe–Boden, 1991, p. 712). Solid-state physics, for instance, led to the discovery of semiconductors and triggered off the microelectronics revolution which, in turn, determined the development of the fast growing information systems industry after the invention of the transistor in 1947. However, different firms pursue different paths, and have different structures and core capabilities. Consequently, only firms with the most appropriate R&D capabilities are able to exploit this technological knowledge to develop new inventions, while firms which choose different technological strategies will be forced out of the contest. Within this analytical framework, this diversity of firms is a crucial feature of any industry at any particular time.

## 3. TECHNOLOGICAL KNOWLEDGE IN THE PERIPHERY OF MAINSTREAM ECONOMICS

As far as their general features are concerned, the concepts of technological regime and natural trajectory of technology do not significantly differ from some of the assumptions implicit in the versions of the production function approach developed by Salter (1960, 1966) and Atkinson–Stiglitz (1969).

These authors contend that the substantial inadequacy of the production function approach in the microeconomic analysis of technical change results a) from the fact that a firm is not interested in introducing all the techniques which are already available at a given time (Atkinson and Stiglitz), and b) from the function that informational asymmetries play in the innovative process (Salter[5]). In support of the first assumption, one may cite the fact that each firm is only able to deal with a portion of technical progress which is *localized* at a few points along the isoquant: a portion which is closely related to its structure, previous experience and capability. This implies the rejection of both the usual neoclassical consideration of technological knowledge as a free good and the subsequent assumption that all firms follow a similar pattern in the adoption of improved technologies. As in evolutionary theory, these authors, too, maintain that variety more than uniformity is the password when considering technical change at the firm level. Within this theoretical framework, firms identify and select among a range of technological options and adopt only those technologies with which they are most familiar.

As regards assumption *b*) above, informational asymmetries prevent the firm from introducing techniques which were excluded by its previous routines. This explains why, in the real economy, there are firms which *adopt* innovations a certain time after their early appearance: these are firms which decide to change their strategy by imitating technological leaders, and thus develop new core capabilites to avoid being forced out of the market.

Under these assumptions, let the shorter curve depicted on the left side of Figure 1 denote the known portion of an isoquant, which is, by definition, smaller than the regular and continuous curve represented by

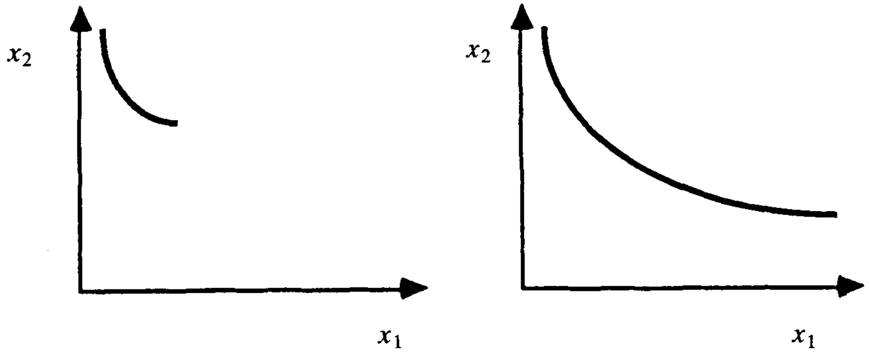


*Figure 1: Technological alternatives along the isoquant*

---

[5] In particular, in the Appendix to Chapter VI Salter (1960) provides a theoretical explanation of and empirical support for his intuition of the delay in the practical utilization of new techniques.

the whole isoquant on the right side of the same figure. In this representation the shorter curve denotes the limited set of technical alternatives which are effectively known by the firm (cf. Atkinson–Stiglitz, 1969; Rosenberg, 1975). The firm's knowledge of the other portions of the isoquant is prevented by informational asymmetries and technological idiosyncracy. This way of positing the firm's production frontier challenges neoclassical theory and the adequacy of its instruments, and may lead to a revision of the traditional approach to the economic analysis of technological change.

As we know, Salter (1960, 1966) used the concept of the production function as an analytical device to provide a general description of the pool of knowledge existing at each date, and of the range of feasible techniques that this pool makes available to industrial activities. Salter distinguished between general advances in technology, which apply to the production of many commodities (such as electricity), and others which are highly specialized and apply to only one or a few commodities (such as linotype machines). One cannot help noting that this distinction

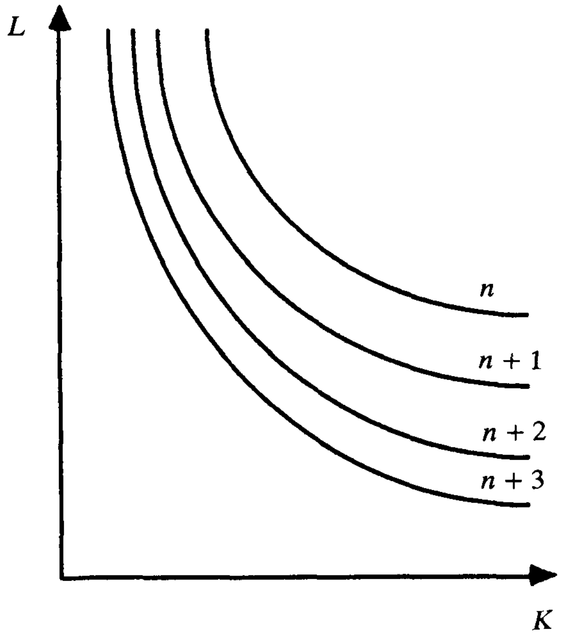


*Figure 2: Technological advances as a series of dated production functions*

resembles the definitions of technological regime and technological trajectories subsequently developed within the evolutionary approach.

The starting point of Salter's analysis is the observation that a flow of technological change determines a continuous modification in the production function for each commodity. The most radical (science-oriented) technological change modifies the whole nature of the production function; conversely, technological change of a less radical nature, for example those incremental innovations which *improve* the existing processes of production, leads to a change in "only one technique in the range of alternatives" (Salter, 1960, p. 21). Evidently, while the latter change denotes a modification in a given point of an isoquant, the former denotes a movement from one isoquant to another.

However, radical and incremental advances have a number of features in common. In particular, both of them lead to a superior production function, in which a smaller quantity of factors of production is required to produce a certain output (the input of other factors being equal). Salter summarizes this process of technological advance in a series of production functions, one for each period (Figure 2).

He represents these production functions as follows

$$P = f_n(a, b, c, \ldots)$$

$$P = f_{n+1}(a, b, c, \ldots)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$P = f_{n+t}(a, b, c, \ldots)$$

where $P$ denotes output, $a$, $b$ and $c$ are production inputs, and $n$, $n + 1$, and $n + t$ are consecutive time periods. In this representation each curve refers to the same output, which in subsequent periods $n$, $\ldots$, $n + t$ is obtained by means of a different labour and capital mix, in turn determined by the level and features of technological advance. This representation is the analytical foundation of vintage models—usually employed to measure embodied technological change—which take gross fixed investments as the principal channels through which new technologies are adopted and diffused. However, apart from vintage models, these successive curves represent an advance with respect to the neoclassical tradition as well. In fact, they introduce a time perspective into the economic analysis of technological change which reflects "the way in which new technical knowledge opens up successive ranges of alternative techniques which make possible new levels of productivity" (Salter, 1960, p. 23).

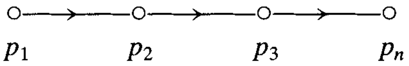
## 4. A DIGRAPH REPRESENTATION OF TECHNOLOGY AS A COGNITIVE STRUCTURE

The procedure for measuring technological knowledge introduced in this section resembles all those approaches which view technology as a cognitive empirical structure following a path-dependent dynamics. Digraph theory can be successfully employed in a reformulation, on the basis of a historically determined cognitive framework, of the *instruments* used in the economic analysis of technological change, either in the evolutionary or in the Salter–Atkinson–Stiglitz tradition.

Before applying digraph theory to the analysis of technological knowledge, I will introduce the basic concepts of this approach to the study of complex structures.

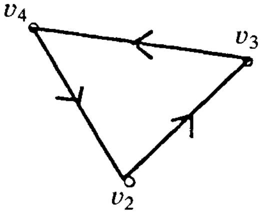
### *4.1 Some basic concepts of digraph theory*

In digraph theory a *directed path* from $p_1$ to $p_n$ consists of a sequence of points $p_1$, $p_2$, . . ., $p_n$ and lines $p_1p_2$, $p_2p_3$, . . ., $p_{n-1}p_n$, ordered as follows:



where the sequence of points and lines can be read as $p_1$, $p_1p_2$, $p_2$, $p_2p_3$, $p_3$, . . ., $p_{n-1}p_n$, $p_n$. The *length* of a path is measured by the number of lines which compose it.

By adding a line from the terminal to the initial point of a path, it is possible to construct a *cycle*. For example, if one adds the line $v_4v_2$ to the path $p_2p_3p_4$ the following cycle $v_2v_3v_4v_2$ is obtained



A collection of distinct points $p_1$, $p_2$, . . ., $p_n$ together with $n - 1$ lines is defined as a *semipath* connecting $p_1$ and $p_n$. In other words, from each pair of lines $p_1p_2$ or $p_2p_1$ there originates a semipath. Moreover, by adding a line joining the terminal point and the initial point of a semipath, one obtains a *semicycle*. In turn, the *indegree* (id) of the digraph is the number of lines from any other point to a certain point $p$, and the *outdegree* (od) is the number of lines originating from a certain point.

The presence of a semicycle denotes perfect *reachability* in the digraph, since it is possible to move—or communicate, in the case of a cognitive structure—with equal ease from $p_1$ to $p_n$ and from $p_n$ to $p_1$. A digraph is *strong* when and if any two points are mutually reachable; conversely, it is *weak* when and if any two points are joined by a semipath. A weak digraph is *unipathic* if, with $p_n$ reachable from $p_1$, there is only one path connecting $p_1$ to $p_n$. Every path in a unipathic graph is a *geodesic*, i.e. a path of minimum length connecting two given points.

### 4.2 An application to the analysis of technological knowledge as a cognitive structure

On the basis of digraph theory, it is possible to define a technological regime as a *unipathic* graph where there are no semicycles connecting two *semipaths* from one point to another. This kind of digraph is usually defined as being of the *tree from a point* type, and it is a particular version of a *weak digraph* in which one point (the start) has indegree 0 and every other point has indegree 1. In fact, in the case considered here, the starting point is not reachable from any other point and each of the subsequent points is reachable from only one point.

From each point in Figure 3 a pair of totally disconnected semipaths can branch out. These represent groups of products and/or production processes embodying the same type of knowledge and developed by firms which possess the appropriate core capabilities.[6] This digraph is useful because it represents technological change as an irreversible process of bifurcation (the branching of a tree), the outcome of which can be achieved only through some particular dynamic sequence of intervening events (David, 1988). In the digraph depicted in Figure 3, let $p_1$, $p_2$, ..., $p_{15}$, i.e. each point, denote different subsets of knowledge, skills and artefacts from which a series of natural trajectories—i.e. lines $p_1p_2$, $p_1p_3$, ..., $p_7p_{15}$ originating from the same set of

---

[6] Citing again the case of electronics, a natural trajectory may represent the transistor, another trajectory the integrated circuit, and so on.

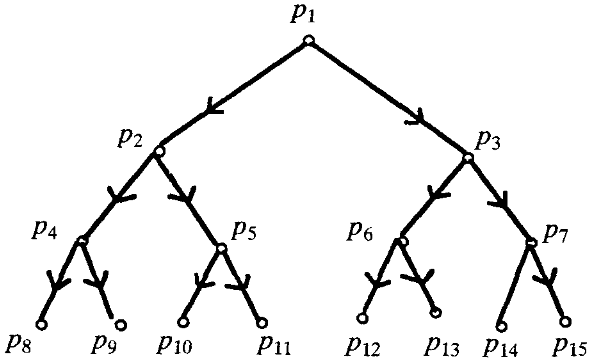


*Figure 3: A technological regime as a unipathic graph*

knowledge $p_1$—occur. Also assume that, whenever points $p_2$ and $p_3$ are reachable from $p_1$, there is exactly one path connecting $p_1$ to $p_2$ and one path connecting $p_1$ to $p_3$. The same applies to the relation between $p_4$ and $p_5$ with $p_2$, and so on, up to the relation between $p_{14}$ and $p_{15}$ with $p_7$. Each path in the digraph reflects a sequence of intervening events, such as the R&D efforts of the companies which develop successful innovations by exploiting the subset of knowledge represented by the previous point. The new point thus reached is a more complex subset of knowledge, and any departure from it towards a new hypothetical point is only possible for firms which have the appropriate structure and core capabilities and travel along a particular path.

The length of the unipathic graph considered here is a proxy for the pervasiveness of the technological regime.[7] In fact, a unipathic graph consists of at least $q = 2(p - 1)$ and up to $\infty$ lines—with $q$ denoting the length and $p$ the number of points. Accordingly, the length of the technological regime is $2(p - 1) \leqslant q < \infty$, and its pervasiveness is a direct function of $q$, since it increases together with the number of lines, viz. the number of natural trajectories originating from it.

Further information on the technological regime can be obtained by constructing the *adjacency matrix* of the digraph, which is determined by the particular order of the various points in the digraph. The information thus obtained is particularly useful since, as Nelson and Winter

---

[7] Here, the pervasiveness of a technological regime denotes either an innovation which displays a general economic effect, by means, for instance, of a reduction of input costs for all industries (an example being the discovery of new energy sources), or an innovation which originates in a given technological area but proves useful in many other technological areas as well.

(1982, p. 259) point out, there are frequently significant complementarities among the various trajectories composing a technological regime.

In the case considered here, using $q$ to denote the different lines composing the digraph enables us to find the number of points (subsets of knowledge) to which different lines $q_i$ and $q_j$ stand adjacent. The adjacency matrix $Q(D) = [p_{ij}]$ is a square matrix, with one row and one column for each point of the digraph, in which the entry $p_{ij} = 1$ if line $q_iq_j$ is in $Q$ and $p_{ij} = 0$ if $q_iq_j$ is not in $Q$.

In the matrix $Q(D)$, the sums of the row and columns indicate respectively the number of lines originating and terminating at each point of the digraph.

$$Q(D) = \begin{array}{c|ccccccccccccccc|}
 & p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 & p_8 & p_9 & p_{10} & p_{11} & p_{12} & p_{13} & p_{14} & p_{15} \\
p_1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
p_2 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
p_3 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
p_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
p_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
p_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
p_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
p_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
p_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
p_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
p_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
p_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
p_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
p_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
p_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

The problem of identifying lines adjacent to each other can be solved by using a theorem which explains how to obtain the number $s_{ij}$, which corresponds to the points to which both $q_i$ and $q_j$ are adjacent. The theorem gives $s_{ij} = \text{od}(q_i)$, which is the outdegree of $q_i$. In the case of Figure 3—as the matrix Q(D) clearly shows—no more than one line terminates at each point of the digraph, whereas two lines originate from points 1 to 7. Point 1, for instance, is adjacent to points 2 and 3, which in turn are respectively reached by lines $q_2$ and $q_3$ originating from point 1.

The concept of adjacency within a digraph is a useful tool in the analysis of technological change. By processing information on technological trends and perspectives, one can use the adjacency matrix of a digraph to depict the current features of technological change. One can represent, for instance, the successive stages of discovery and innovation, and mark out the direction of further development within a given technological regime. In fact, the adjacency matrix can be used to order natural trajectories in groups on the basis of their technical complementarities. It is thus possible to predict which trajectories will originate from the preceding ones. This *ex ante* knowledge of the new potential paths of technological change can be used by firms to change their strategy and structure and to improve their core capabilities.

The next step is to conduct a more thorough analysis of the content of each trajectory.

In effect, within each trajectory it is possible to single out several connected developments, or incremental innovations, introduced by different firms. A natural trajectory can thus be depicted as a digraph composed of a sequence of related techniques and developments, or incremental innovations. A *complete symmetric digraph* of this type is one which contains a line $q_1, \ldots, q_5$ whenever it contains lines $q_1q_2, \ldots, q_3q_4$, for any distinct point $q_1, \ldots, q_5$. Accordingly, a structure embodying every line of a digraph is called *line-complete*, and a digraph which has a line-complete closed structure is called *traversable*. In this case all the problems connected to limited *reachability* are overcome, because every change within a natural trajectory represents an incremental innovation and requires the exploitation of an amount of knowledge and core capabilities which does not significantly differ from that which was needed to develop the preceding incremental innovation.

The complete symmetric digraph presented in Figure 4 is *traversable*, since it is composed of a line-complete closed sequence of incremental innovations $q_1q_2q_3q_4q_5q_2q_5q_4q_3q_2q_5q_1$. Here, $q_1$ denotes the first innovation developed by a certain firm within a certain natural trajectory originating from a given technological regime, and the subsequent lines $q_1 \ldots q_5$ represent further developments within the same natural trajectory.

I will return briefly to the case of microelectronics and assume that certain incremental innovations introduced into this field by a specific firm (e.g. Intel Co.) result from a chip which is more powerful than its predecessors—for instance the 80486 32-bit chip developed by Intel in the early 1990s. The phase of development of chips can be plausibly taken to be a reversible process, since it is technologically feasible to move from the superior (i.e. the most effective) chip $q_5$ to the original

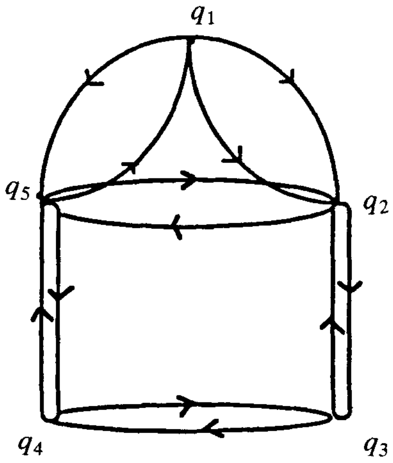


*Figure 4: A natural trajectory as a complete symmetric digraph*

microchip $q_1$ and *viceversa*. This series of incremental innovations results from the exploitation of a firm's R&D core capabilities and can be formally represented as a digraph in which for at least two points $v$, $\mathrm{id}(v) = \mathrm{od}(v)$.[8]

By developing the adjacency matrix we can compute further information about the digraph depicted in Figure 4. We may wish to find the number of points to which two given lines $v_i$ and $v_j$ are adjacent. The additional information thus obtained is particularly useful when trying to single out the distribution of incremental innovations which are even more likely than new scientific discoveries to influence any further applications of knowledge to industrial activities (cf. Rosenberg, 1985).[9]

Using the theorem introduced above to identify adjacencies, we find that incremental innovations $q_2$ and $q_5$ are adjacent to three other innovations (respectively $q_1q_3$, $q_5$ and $q_1q_2$, $q_4$), whereas all the remaining innovations are adjacent to only two others.

---

[8] The extreme case of a digraph where for every point $v$, $\mathrm{id}(v) = \mathrm{od}(v)$ can be called an isograph.

[9] In effect, incremental innovations represent small improvements (as defined by Usher, 1954) which, although individually are of little importance, considered as a cluster may cause a major breakthrough in knowledge directly applicable to industrial purposes. As both Usher and Rosenberg pointed out, small improvements do not break new ground for the scientific tradition, but they provide insights which can be particularly useful in re-orienting basic research.

$$A(D) = \begin{array}{c|ccccc|} & q_1 & q_2 & q_3 & q_4 & q_5 \\ q_1 & 0 & 1 & 0 & 0 & 1 \\ q_2 & 1 & 0 & 1 & 0 & 1 \\ q_3 & 0 & 1 & 0 & 1 & 0 \\ q_4 & 0 & 0 & 1 & 0 & 1 \\ q_5 & 1 & 1 & 0 & 1 & 0 \end{array}$$

This outcome tells us, first, that the development of two (or more) incremental innovations may be closely related, or that two (or more) incremental innovations have significant complementarities. Secondly, and more significantly, the matrix provides information directly related to the possibility that, according to the assumptions of Rosenberg and Usher, the cluster of small improvements represented by incremental innovations $q_1$, $q_2$, $q_3$, $q_4$, $q_5$ is likely to foster the overall process of technological change. Hence firms with the appropriate core capabilities may find it more useful to readdress their search processes within the contours of this cluster, rather than continue with the usual trial and error procedure. By imitating the strategies of earlier developers of these small improvements, new firms move in to compete on the same ground; a process which generates a swarm of imitators of the kind described by Schumpeter in his early writings (cf. Santarelli–Pesciarelli, 1990).

## 5. CONCLUSIONS

This article has sought to show that the evolutionary approach to the analysis of technical advance has some features in common with the particular version of the production function approach suggested by Salter (1960, 1966), and Atkinson–Stiglitz (1969).

The nature of the cognitive empirical structure which—according to these approaches—is typical of technology can be usefully represented using digraph theory. This enables the development of an operational structure which measures certain characteristics of the innovation process, such as the pervasiveness of technological regimes, the connectedness of natural trajectories of technology, and the distribution of small improvements. In particular, digraph theory can be used to order the natural trajectories making up a technological regime on the basis of their complementarities or diversities, and to apply the same method of classification to incremental innovations belonging to a given natural trajectory. These results can be taken to be a first step towards the development of a viable system for measuring the inner features of technological change. They also reveal that the leading-edge companies in a particular field are subject to change because different technologies are introduced which require different strategies, structures and core capabilities.

## REFERENCES

Abernathy, W. J. and Utterback, J. M. (1978), "Patterns of Industrial Innovation". *Technology Review*, 7: pp. 40–47.

Andersen, E. S. (1991), "Techno-economic Paradigms as Typical Interfaces Between Producers and Users". *Journal of Evolutionary Economics*, 1: pp. 119–144.

Arthur, W. B. (1983), "On Competing Technologies and Historical Small Events: The Dynamics of Choice Under Uncertainty". Paper presented at the Technological Innovation Program Workshop, Department of Economics, Stanford University.

Arthur, W. B. (1989), "Competing Technologies, Increasing Returns, and Lock-in by Historical Events". *Economic Journal*, 99: pp. 116–131.

Atkinson, A. B. and Stiglitz, J. B. (1969), "A New View of Technological Change". *Economic Journal*, 79: pp. 573–578.

Clark, N. and Juma, C. (1987), *Long-run Economics: An Evolutionary Approach to Economic Growth*, Pinter Publishers, London.

David, P. (1986), "Understanding the Economics of QWERTY: The Necessity of History". In: Parker, W.N. (ed), *Economic History and the Modern Economist*, Basil Blackwell, Oxford.

David, P. (1988), "Path-dependence: Putting the Past into the Future of Economics", Stanford University, Institute for Mathematical Studies in the Social Sciences, Technical Report No. 533.

Dosi, G. (1982), "Technological Paradigms and Technological Trajectories: a Suggested Interpretation of the Determinants and Directions of Technical Change". *Research Policy*, 11: pp. 147–162.

Dosi, G. (1988), "Sources, Procedures, and Microeconomic Effects of Innovation". *Journal of Economic Literature*. 26: pp. 1120–1171.

Enos, J. (1962), "Invention and Innovation in the Petroleum Refining Industry". In: *The Rate and Direction of Inventive Activity*. Princeton University Press for NBER, Princeton.

Freeman, C., Clark, J. and Soete, L. (1982), *Unemployment and Technical Innovation*. Frances Pinter, London.

Freeman, C. and Perez, C. (1986), "The Diffusion of Technical Innovation and Changes of Techno-Economic Paradigm". Paper presented at the Conference on Innovation Diffusion, held in Venice, DAEST, March.

Green, J. R. and Shoven, J. B. (1983), "The Effects of Financing Opportunities and Bankruptcy on Entrepreneurial Risk-Bearing". In: Ronen, J. (ed), *Entrepreneurship*, Kluwer Academic Publishers, Dordrecht Boston London.

Harary, F., Norman, R. Z. and Cartwright, D. (1965), *Structural Models: An Introduction to the Theory of Directed Graph*. John Wiley & Sons Inc, New York.

Katz, B. G. and Phillips, A. (1981), "Government, Technological Opportunities and the Emergence of the Computer Industry". In: Giersch, H. (ed), *Emerging Technologies*, J.C.B. Mohr, Tubingen.

Kuhn, T. (1963), *The Structure of Scientific Revolutions*. University of Chicago Press, Chicago.

Lakatos, I. (1970), "Falsification and the Methodology of Scientific Research Programmes". In: Lakatos, I. and Musgrave, A. (eds), *Criticism and the Growth of Knowledge*. Cambridge University Press, Cambridge.

Metcalfe, J. S. and Boden, M. (1991), "Innovation Strategy and the Epistemic Connection: An Essay on the Growth of Technological Knowledge". *Journal of Scientific & Industrial Research*, 50: pp. 707–717.

Nelson, R. R. (1991), "The Role of Firm Differences in an Evolutionary Theory of Technical Advance". *Science and Public Policy*, 18: pp. 347–352.

Nelson, R. R and Winter, S. G. (1977), "In Search of Useful Theory of Innovation". *Research Policy*, 5: pp. 36–76.

Nelson, R. R. and Winter, S. G. (1982), *An Evolutionary Theory of Economic Change*. The Belknap Press of Harvard University Press, Cambridge (Mass.).

Rosenberg, N. (1963), "Technological Change in the Machine-Tool Industry 1840–1910", *Journal of Economic History*. 23: pp. 414–443.

Rosenberg, N. (1969), "The Direction of Technological Change: Inducement Mechanisms and Focusing Devices". *Economic Development and Cultural Change*, 18: pp. 1–24.

Rosenberg, N. (1975), "Problems in the Economist's Conceptualization of Technological Innovation". *History of Political Economy*, 7: pp. 456–481.

Rosenberg, N. (1985), "The Commercial Exploitation of Science by American Industry". In: Clark, K. B. and Hayes, R. (eds), *The Uneasy Alliance. Managing the Productivity—Technology Dilemma*. Harvard Business School Press, Boston.

Rosenberg, N. (1988), "Qualitative Aspects of Technological Change: Some Historical Perspectives". Department of Economics, Stanford University, unpublished manuscript.

Sahal, D. (1985), "Technological Guideposts and Innovation Avenues". *Research Policy*, 14: pp. 61–82.

Salter, W. E. G. (1960), *Productivity and Technical Change*. Cambridge University Press, Cambridge.

Salter, W. E. G. (1966), "Productivity Growth and Accumulation as Historical Processes". In Robinson, E.A.G. (ed), *Problems in Economic Development*, Macmillan, London.

Santarelli, E. and Pesciarelli, E. (1990), "The Emergence of a Vision: The Development of Schumpeter's Theory of Entrepreneurship". *History of Political Economy*, 22: pp. 677–696.

Solow, R. M. (1960), "Technical Change and the Aggregate Production Function". *Review of Economics and Statistics*, 38: pp. 312–320.

Teece, D. (1986), "Profiting From Technological Innovation: Implications for Integration, Collaboration, Licensing and Public Policy". *Research Policy*, 15: pp. 285–305.

Usher, A. P. (1954), *A History of Mechanical Inventions*. Harvard University Press, Harvard.

Vega-Redondo, F. (1993), "Technological Change, Path-Dependence and Industrial Dynamics", Working Paper *A Discussion*, WP-AD 93–4 Universidad de Alicante.

Department of Economics
University of Bologna
Strada Maggiore 45
40125 Bologna
Italy