

# *The relationship between size and growth: the case of Italian newborn firms*

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This paper analyses the relationship between size and growth for a group of Italian newborn firms in the instruments industry. The main finding is that Gibrat's Law of Proportionate Effect exhibits a behaviour dependent on the firm's life cycle. In particular, even if in the years immediately following start-up the law could be rejected, since smaller firms have to rush in order to survive in the market, in subsequent years growth rates seem to converge towards a Gibrat-like pattern. This result is confirmed by the separate analyses carried out for micro-firms and larger firms.

## I. INTRODUCTION

According to Gibrat's Law of Proportionate Effect, 'the probability of a given proportionate change in size during a specified period is the same for all firms in a given industry – regardless of their size at the beginning of the period' (Mansfield, 1962, p. 1031).

The purpose of this paper is to investigate whether (a) Gibrat's Law fails to hold for *new entrants* in the early stage of their life cycle, and (b) there is some convergence towards a Gibrat-like behaviour with the passage of time. Gibrat's Law will therefore be tested within a subpopulation of new entrants. While failure of the law – meaning that smaller firms have higher growth rates than their larger counterparts – has been found in most previous studies (Mansfield, 1962; Hall, 1987; Evans, 1987; Contini and Revelli, 1989; Dunne and Hughes, 1994; Hart and Oulton, 1996; Audretsch *et al.*, 1999; for a survey, see Sutton, 1997), only a few studies have carried out longitudinal investigations of new firms during their infancy (to our knowledge, Dunne *et al.*, 1989; and Mata, 1994).

This paper is organized as follows. Section II presents the dataset, while Section III discusses some methodologi-

cal issues related to estimation of Gibrat's Law and carries out the longitudinal investigation. Finally, Section IV draws some conclusions.

## II. DATA

As discussed above, the peculiarity of this study is the analysis of Gibrat's law *within* a cohort of new entrants. So far, the main drawback to the empirical analysis of the post-entry behaviour of newborn firms has been the lack of longitudinal datasets tracking the evolution of firms after their birth. In this paper a unique data set is used from the Italian National Institute for Social Security (INPS), which identifies new firms in the instruments industry (with at least one paid employee) formed in January 1987 and tracks their post-entry employment performance at yearly intervals until January 1993.<sup>1</sup>

Since all private Italian firms are compelled to transfer to INPS national security payments for their employees, when a new firm is registered as 'active' for the first time an entry can be identified, while a firm cancellation denotes exit from the market (this happens when a firm ultimately stops paying national security fees). For the purposes of

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<sup>1</sup> No information on firms with zero paid employees is obtainable from the INPS file; however, these firms usually identify self-employment and only occasionally become true entrants with positive post-entry employment growth rates.

the present paper, the original INPS file has been subjected to a control aimed at correct identification of entry and failure times and at detecting inconsistencies in individual tracks for administrative reasons, problems related to file truncation in January 1993, cancellations due to firm transfers, mergers and take-overs.

### III. EMPIRICAL MODEL AND RESULTS

The main relationship tested in this study is the original specification of Gibrat's Law:

$$\log S_{i,t} = \beta_0 + \beta_1 \log S_{i,t-1} + \varepsilon \quad (1)$$

where  $S_{i,t}$  is the size of the  $i$ th firm at time  $t$ ,  $S_{i,t-1}$  is the size of the same firm at the previous period and  $\varepsilon$  is a random variable distributed independently of  $S_{i,t-1}$ . Following Chesher (1979, p. 404), if both sides of equation (1) are exponentiated, it becomes clear that if  $\beta_1$  is equal to unity, then growth rate and initial size are independently distributed and Gibrat's law is in operation. By contrast, if  $\beta_1 < 1$  smaller firms grow at a systematically higher rate than do their larger counterparts, while the opposite occurs if  $\beta_1 > 1$ .

If one does not treat growth and exit as homogeneous phenomena, that is assuming the disputable hypothesis that exit is equal to a minus one rate of growth, empirical estimates have to deal only with survived firms. Here the sample selection problem arises. Since growth can be measured exclusively for firms which have survived over the entire period examined and since slow-growing firms are more likely to exit, small fast-growing firms can be over-represented in the surviving sample and this may bias the results of the empirical research. Needless to say, this observation applies in particular to new and small firms, for which the hazard rate is generally high. As discussed in Hall (1987), Evans (1987), Mata (1994), Dunne and Hughes (1994) and Sutton (1997), the appropriate econometric method to deal with this problem is the two-step procedure suggested by Heckman (1979) (see also Amemiya, 1984). Accordingly, we added to the main equation an additional explanatory variable (the inverse Mill's ratio) obtained by a probit model (selection equation) estimating the relationship between firm's survival and firm's size at the beginning of the examined period:

$$P(f_i = 1) = F(\delta + \gamma \log S_{i,t-1} + \varphi \log S_{i,t-1}^2 + \mu) \quad (2)$$

with:  $f_i = 1$  survivor;  $f_i = 0$  exit;  $\mu$  = disturbance.

Since the relationship between size and survival can assume a non-linear feature (as in Evans, 1987; Dunne and Hughes, 1994; and Harhoff *et al.*, 1998), a squared term was introduced in the selection equation. While Equation 1 in isolation had been preliminary estimated

through OLS, the sample selection model including Equation 2 was estimated using a two-stage maximum likelihood method. As in most of previous empirical studies, tests for heteroscedasticity were carried out using the OLS estimations of Equation 1 and White's (1980) correction introduced when necessary, both in the OLS and in the sample selection model (SSM) estimations. The possible occurrence of persistence in firms' patterns of growth was tested (as in Kumar, 1985 and Dunne and Hughes, 1994) using annual growth rates: since no significant AR(1) process emerged, the aforementioned specification was not extended.<sup>2</sup>

The empirical test of Equation 1 should shed some light on the following two questions:

- (1) Is the overall inverse relationship between size and growth confirmed during the infancy of newborn firms?
- (2) Is there a process of convergence towards a Gibrat-like pattern of growth with the passage of time?

If the two hypotheses above were jointly supported by the data, Gibrat's law would exhibit a behaviour dependent on the firm's life cycle: the Law would fail to hold during the first years after entry and would become acceptable with the passage of time.

Table 1 reports the OLS and SSM results concerning the estimations of Equation 1. Each panel of the table is organized as follows. In the first two columns results from the estimations on the entire six-year period are reported, along with the usual statistical diagnoses (including the correlation between the selection and growth equation,  $\rho$ ) and with a specific  $t$ -test of the validity of Gibrat's law ( $\beta = 1$ ; question (a)); the final rows report the White's test for heteroscedasticity (when significant a consistent covariance matrix has been used) and sample sizes with and without exits. In the following columns, the same estimations are repeated for each year, in order to characterize the possible convergence path with the passage of time (question (b)).

Consider first the result for the six-year period. Both the OLS and the SSM estimates of  $\beta_1$  carried out for *all firms* are significantly less than unity (0.79 and 0.59 respectively) and this means that, in general, smaller firms grow faster than their larger counterparts (this result is consistent with those emerged from most of previous studies).

Yet, the departure from Gibrat's Law tends to become less marked year after year. In effect, the value of  $\beta_1$  in the SSM estimate is equal to 0.77 in 1988 (significantly different from 1 at the 99% level of confidence), and this suggests that, immediately after entry, smaller firms have to rush to achieve an acceptable size. Conversely, as shown by the increase of  $\beta_1$  and by the non-significant values of

<sup>2</sup> Results about persistence and those from the selection equations are available from the authors upon request.

Table 1. OLS and sample selection model (SSM) estimates of Gibrat's Law: instruments industry (Italy)

All firms														
	OLS 88-93	SSM 88-93	OLS 88	SSM 88	OLS 89	SSM 89	OLS 90	SSM 90	OLS 91	SSM 91	OLS 92	SSM 92	OLS 93	SSM 93
$\beta_0$	0.98***	1.44***	0.67***	0.61***	0.23***	0.24	0.01	-0.01	0.13*	0.11	0.05	0.15	0.08	0.03
$\beta_1$	0.79***	0.59***	0.77***	0.95***	0.95***	1.02***	0.99***	0.94***	0.94***	0.94***	0.96***	0.94***	0.94***	0.95***
$\rho$	—	-0.40	—	0.66***	—	-0.15	—	0.08	—	0.12	—	-0.70***	—	0.35
$t(\beta_1 = 1)^\circ$	3.53***	6.53***	5.75***	6.73***	2.50**	1.07	1.00	0.42	2.00**	0.98	1.33	1.55	2.00**	1.15
$F$	108.20***	53.78***	526.44***	266.44***	1408.94***	721.47***	1528.35***	766.45***	1035.51***	518.73***	1013.57***	503.11***	1064.19***	542.72**
$R^2$ adj.	0.45	—	0.72	—	0.88	—	0.90	—	0.87	—	0.88	—	0.89	—
LRI	—	0.20	—	0.42	—	0.71	—	0.73	—	0.65	—	0.66	—	0.72
White§	1.17	—	6.17***	—	3.86**	—	3.31**	—	2.29	—	1.05	—	0.29	—
N. tot	214	—	214	—	200	—	183	—	168	—	155	—	141	—
N. surv.	131	—	200	—	183	—	168	—	155	—	141	—	131	—
Microfirms (1-5 employees)														
	OLS 88-93	SSM 88-93	OLS 88	SSM 88	OLS 89	SSM 89	OLS 90	SSM 90	OLS 91	SSM 91	OLS 92	SSM 92	OLS 93	SSM 93
$\beta_0$	1.23***	0.33	0.70**	0.58***	0.32***	0.34	0.04	-0.44	0.40***	-0.48	0.10	-0.66	0.18	0.06
$\beta_1$	0.61***	0.81***	0.74***	0.73***	0.84***	0.83***	1.01***	1.28***	0.61***	1.30***	0.84***	0.92**	0.85***	0.92***
$\rho$	—	0.91***	—	1.00***	—	-0.07	—	0.98***	—	0.99***	—	1.00***	—	0.68
$t(\beta_1 = 1)^\circ$	1.99**	0.67	2.60**	1.73*	1.78*	0.83	0.08	0.94	2.79***	0.66	1.60	0.19	1.36	0.47
$F$	9.81***	4.91**	44.13***	22.93***	87.93	43.65***	66.75***	33.90***	19.98***	10.72***	69.83***	35.86***	55.54***	29.37***
$R^2$ adj.	0.11	—	0.27	—	0.52	—	0.51	—	0.27	—	0.60	—	0.57	—
LRI	—	0.06	—	0.15	—	0.40	—	0.36	—	0.22	—	0.59	—	0.58
White§	1.24	—	2.46*	—	3.86**	—	2.01	—	1.42	—	0.74	—	0.59	—
N. tot	129	—	129	—	91	—	72	—	58	—	56	—	49	—
N. surv.	75	—	119	—	82	—	64	—	52	—	47	—	42	—
Larger firms (more than 5 employees)														
	OLS 88-93	SSM 88-93	OLS 88	SSM 88	OLS 89	SSM 89	OLS 90	SSM 90	OLS 91	SSM 91	OLS 92	SSM 92	OLS 93	SSM 93
$\beta_0$	0.60***	1.24***	0.48*	0.48**	0.12	-2.28	-0.14	-0.04	-0.01	-0.02	0.18	0.18	-0.04	0.16
$\beta_1$	0.80***	0.76***	0.83***	0.84***	0.99***	1.03	1.06***	1.03***	0.98***	0.99***	0.92***	0.92***	0.98***	0.91***
$\rho$	—	-0.96***	—	-0.58	—	0.98	—	-0.97***	—	0.17	—	0.00	—	0.99***
$t(\beta_1 = 1)^\circ$	1.68*	1.68*	1.70*	3.46***	0.25	—	2.00**	0.65	0.50	0.15	1.60	0.62	0.40	1.18
$F$	45.07***	23.69***	165.39***	83.96***	608.00***	310.07***	1159.48***	585.23***	681.15***	338.36***	306.03***	151.36***	405.57***	201.72**
$R^2$ adj.	0.44	—	0.67	—	0.86	—	0.92	—	0.87	—	0.77	—	0.82	—
LRI	—	0.24	—	0.46	—	0.79	—	0.94	—	0.76	—	0.56	—	0.65
White§	0.92	—	2.92*	—	0.58	—	0.19	—	0.40	—	1.58	—	0.53	—
N. tot	85	—	85	—	109	—	111	—	110	—	99	—	92	—
N. surv.	56	—	81	—	101	—	104	—	103	—	94	—	89	—

\*\*\* = significant at 99% level of confidence; \*\* = significant at 95% level of confidence; \* = significant at 90% level of confidence.

$^\circ$   $t$  test; null hypothesis:  $\beta_1 = 1$ ; in the only case (...) in which  $\beta_1$  was not significantly different from 0, the  $t$  test was not carried out.

§.  $F$ -statistic; null hypothesis: homoskedasticity; in case of heteroskedasticity (at least at 90% level of confidence) a consistent covariance matrix was used (White's correction).

$t(\beta_1 = 1)$  for subsequent years, once smaller firms reach a size large enough to enhance their likelihood of survival, their patterns of behaviour conform to those of larger firms.

These results on all firms capture both the dynamics within size classes and the differences between size classes. In order to control for this problem, separate estimates for micro-firms (less than five employees) and larger firms have been carried out.<sup>3</sup>

Looking at the results for micro-firms and larger firms over the entire period and year by year, one can conclude that previous results are confirmed within each of the two size classes. While in general Gibrat's Law is not con-

firmed, once attention is turned to firms' post-entry evolution a convergence – although not monotonic – toward a Gibrat-like pattern of growth can be detected.

#### IV. CONCLUSIONS

The results from the OLS and SSM estimates support the hypothesis that Gibrat's Law exhibits a behaviour dependent on the firm's life cycle. The law fails to hold during the first years after start-up, and becomes acceptable after a given threshold in terms of age and size is reached. Thus, this empirical investigation suggests that while in their very

<sup>3</sup> Given the characteristics of our database, including all firms born in January 1987, and the specific features of the instruments industry in Italy (dominated by small firms), most of the included firms show a very small start-up size and micro-firms (less than five employees) represent more than 50% of the initial sample. For this reason and in order to assure an acceptable number of observations over time, data have been split into two size classes: micro-firms (one to five employees) and larger firms (more than five employees).

early stages of life smaller newborn firms have to rush in order to reach the minimum efficient scale (thereby increasing their likelihood of survival), after a few years following start-up the growth rates of all new entrants seem to converge towards a Gibrat-like pattern.

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## REFERENCES

- Amemiya, T. (1984) Tobit models: a survey, *Journal of Econometrics*, **24**, 3–61.
- Audretsch, D.B., Santarelli, E. and Vivarelli, M. (1999) Start-up size and industrial dynamics: some evidence from Italian manufacturing, *International Journal of Industrial Organization*, **17**, 965–83.
- Chesher, A. (1979) Testing the law of proportionate effect, *Journal of Industrial Economics*, **27**, 403–11.
- Contini, B. and Revelli, R. (1989) The relationship between firm growth and labor demand, *Small Business Economics*, **1**, 309–14.
- Dunne, P. and Hughes, A. (1994) Age, size, growth and survival: UK companies in the 1980s, *Journal of Industrial Economics*, **42**, 115–40.
- Dunne, T., Roberts, M. J. and Samuelson, L. (1989) The growth and failure of US manufacturing plants, *Quarterly Journal of Economics*, **104**, 671–98.
- Evans, D. S. (1987) The relationship between firm growth, size, and age: estimates for 100 manufacturing industries, *Journal of Industrial Economics*, **35**, 567–81.
- Hall, B. (1987) The relationship between firm size and firm growth in the US manufacturing sector, *Journal of Industrial Economics*, **3**, 583–606.
- Harhoff, D., Stahl, K. and Woywode, M. (1998) Legal form, growth and exit of West German firms- empirical results for manufacturing construction, trade and service industries, *Journal of Industrial Economics*, **46**, 453–88.
- Hart, P. E. and Oulton, N. (1996) Growth and size of firms, *Economic Journal*, **106**, 1242–52.
- Heckman, J. J. (1979) Sample selection bias as a specification error, *Econometrica*, **47**, 153–61.
- Kumar, M. S. (1985) Growth, acquisition activity and firm size: evidence from the United Kingdom, *Journal of Industrial Economics*, **33**, 327–38.
- Mansfield, E. (1962) Entry, Gibrat's Law, innovation, and the growth of firms, *American Economic Review*, **52**, 1023–51.
- Mata, J. (1994) Firm growth during infancy, *Small Business Economics*, **6**, 27–40.
- Sutton, J. (1997) Gibrat's legacy, *Journal of Economic Literature*, **35**, 40–59.
- White, H. (1980) A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity, *Econometrica*, **48**, 817–38.