R&D Incentives under Bertrand Competition: A Differential Game[1]

Roberto Cellini[§] - Luca Lambertini[#]

[§] Dipartimento di Economia e Metodi Quantitativi

Università di Catania, Corso Italia 55, 95129 Catania, Italy

phone 39-095375344, fax 39-095-370574; cellini@unict.it

[#] Dipartimento di Scienze Economiche, Università di Bologna

Strada Maggiore 45, 40125 Bologna, Italy

phone 39-051-2092600, fax 39-051-2092664; luca.lambertini@unibo.it

[#] RCEA, via Angherà 22, 47900 Rimini, Italy

[#] ENCORE, University of Amsterdam

Roetersstraat 11, 1018 WB Amsterdam, The Netherlands

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Abstract

We investigate dynamic R&D for process innovation in an oligopoly where firms invest in cost-reducing activities. We focus on the relationship between R&D intensity and market structure, proving that the industry R&D investment at equilibrium monotonically increases in the number of firms. This result contradicts the established wisdom acquired from static games on the same topic. We also prove that, if competition is sufficiently tough, any increase in product substitutability reduces R&D efforts.

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1 Introduction

We propose a dynamic analysis of the relationship between market power and R&D efforts, in order to reassess a well-known issue in the theory of industrial organization, that can be traced back to the debate between Schumpeter (1942) and Arrow (1962), about the bearings of the intensity of market competition on the pace of technical progress. The so-called Schumpeterian hypothesis maintains that there exists an inverse relationship between the intensity of competition and the pace of technical progress. That is, according to Schumpeter, monopoly is the market structure that should ensure the fastest and largest technical progress. This relies upon the idea that monopoly ensures the highest profit level and therefore the larger internal sources for funding R&D activities. Exactly the opposite view is expressed by Arrow, since he focuses upon the replacement effect, according to which a monopolist should be induced to rest on his laurels, while a firm operating in a competitive environment should strive for new technologies or new products, in order to throw her rivals out of business. While the Arrovian position measures the intensity of market competition in terms of market structure (i.e., the number of firms), the interpretation of the Schumpeterian hypothesis is a bit looser, and several versions have been alternatively investigated in the literature.[1]

In order to assess this issue, we consider an oligopoly where n firms sell differentiated products and compete in prices. Moreover, they also invest at each point in time in R&D for process innovation, i.e., reducing the marginal production cost of the good. R&D activity is characterized by positive externalities, i.e., each firm receives a positive spillover from the investments carried out by all other firms in the industry.

[1]Influential studies of the relationship between market structure and innovation are those of Flaherty (1980) and Spence (1984). For an exhaustive overview of the related literature, see Reinganum (1989) and Martin (2001).

The game is *state-redundant* or *perfect*, so that the open-loop solution is a Markovian equilibrium. We proceed in two steps. First, we characterize the individually optimal path of R&D investment for a given level of marginal production cost. Second, we obtain the steady state levels of investment and marginal cost. With respect to the steady-state level of R&D investment, the following conclusions hold. The individual effort is always decreasing in the number of firms while the opposite holds for the aggregate R&D investments. This result has an Arrovian flavour, since as the degree of competition becomes tougher, the aggregate investment becomes larger. This is in contrast with the conclusions drawn from the static version of the same model (Hinloopen, 2000) where a non-monotone relationship exists between aggregate R&D investment and market structure. Our results can also be seen as an extension of the static model of Qiu (1997) - not only because we propose a dynamic framework in continuous time, but also because we do not confine ourselves to the duopoly case, and we consider the more general case in which n firms serve the market. Under this perspective, we confirm the findings of Qiu for the duopolistic competition, but we show that some of his results are not robust in the more general case of oligopoly with n firms; in particular, some ambiguity disappears when markets are served by a number of firms larger than two. Viewed from this angle, our model highlights the value added of a properly dynamic analysis over the static approach based upon a multistage game. Then, we also evaluate the effect of product differentiation on R&D efforts. We find that (i) for any admissible level of marginal cost, the individual as well as the industry incentive to invest is increasing in the degree of product differentiation (provided that the number of firms is large enough), while (ii) the steady state R&D efforts are completely unaffected by product differentiation. Result (i) is clearly Schumpeterian in spirit, since any increase in product differentiation translates into a milder price competition on the market; hence, in such a case we may put forward a Schumpeterian argument according to which softening competition by reducing the degree of product substitutability ultimately induces firms to increase their R&D investments. This of course enhances technical progress.

The remainder of the paper is structured as follows. Section 2 contains a brief review of the related literature. Section 3 illustrates the basic setup. The solution of the open-loop game is investigated in section 4, while section 5 contains comparative statics. Concluding remarks are in section 6.

2 Related literature

The comparative assessment of R&D incentives under price and quantity competition has been the subject of a relatively large number of contributions. Hence, a review of the main ones is in order here so as to clarify the direction along which our model departs from the existing literature. The issue at hand has been investigated by Delbono and Denicolò (1990) using a stochastic R&D race approach. They compare the optimal R&D efforts under Bertrand and Cournot competition in a symmetric and homogeneous oligopoly and show that, although the R&D investment is greater under Bertrand competition, social welfare, net of R&D costs, may indeed be higher under Cournot competition. This conflict between static and dynamic efficiency arises because of excess investment under price competition.

Bester and Petrakis (1993) take a deterministic approach to investigate how the incentives for cost reduction in a differentiated industry depend upon the degree of product substitutability, under the assumptions that (i) only one firm invests in R&D and (ii) there are no technological spillovers. When goods are imperfect substitutes, both Cournot and Bertrand competition result in underinvestment in the sense that a social planner would be willing to pay more for a given cost reduction than a profit-maximizing firm. Overinvestment may occur when the goods are sufficiently close substitutes.

Likewise, Cournot behaviour provides a stronger incentive to innovate than Bertrand behaviour if the degree of product differentiation is high, and conversely if this degree is low.

The paper that is closest to ours is Qiu's (1997). He compares Bertrand and Cournot equilibria in a differentiated duopoly with R&D for process innovation and spillovers à la d'Aspremont and Jacquemin (1988). His analysis shows that Cournot behaviour induces higher R&D efforts than Bertrand behaviour. However, the price is lower and output is larger in Bertrand than in Cournot competition. Furthermore, the Bertrand equilibrium is more efficient than the Cournot equilibrium if either R&D productivity is low, or spillovers are weak, or products are very different. If R&D productivity is high, spillovers are strong, and goods are close substitutes, then the Cournot equilibrium is socially preferable to the Bertrand equilibrium.

3 The setup

We consider an oligopoly with n single-product firms selling differentiated goods over continuous time, $t \in [0, \infty)$. As in Spence (1976) and Singh and Vives (1984), *inter alia*, the representative consumer's instantaneous utility function is quasi-linear of the form

$$U(t) = x(t) + A \sum_{i=1}^{n} q_i(t) - \frac{1}{2} \left[\sum_{i=1}^{n} (q_i(t))^2 + s \sum_{i=1}^{n} \sum_{j \neq i} q_i(t) q_j(t) \right] \quad (1)$$

where $x_i(t)$ is a composite and numeraire good whose price is normalised to one and $q_i(t)$ is the quantity supplied by firm i. $\{A, s\}$ are constant parameters; A is the reservation price, or equivalently a measure of market size and $s \in [0, 1)$ measures the degree of substitutability between any two varieties: the higher is s, the lower is differentiation.[2] The maximisation under the

[2]For a model where s is a state variable changing because of R&D for product innovation, see Cellini and Lambertini (2002, 2004).

budget constraint $Y(t) = p_i(t) q_i(t) + x(t)$, $p_i(t)$ being the market price of variety i, yields the instantaneous direct demand function for firm i:

$$q_i(t) = \frac{A}{1+s(n-1)} - \frac{(1+s(n-2))\, p_i(t)}{(1-s)[1+s(n-1)]} + \frac{s}{(1-s)[1+s(n-1)]} \sum_{j \neq i} p_j(t) \tag{2}$$

where $p_i(t)$ is the market price chosen by firm i. Each firm produces at a constant marginal cost, c_i. Accordingly, her instantaneous cost function for the production of the final good is $C_i(c_i, q_i, t) = c_i(t) q_i(t)$. The marginal cost of firm i evolves over time according to the following equation:

$$\frac{dc_i(t)}{dt} \equiv \dot{c}_i = c_i(t)[-k_i(t) - \beta K_{-i}(t) + \delta] \tag{3}$$

where $k_i(t)$ is the R&D effort exerted by firm i at time t, while $K_{-i}(t)$ is the aggregate R&D effort of all other firms and parameter $\beta \in [0,1]$ measures the positive technological spillover that firm i receives from the R&D activity of the rivals.[3] Parameter $\delta \in [0,1]$ is a constant depreciation rate measuring the instantaneous decrease in productive efficiency due to the ageing of technology. The instantaneous R&D cost is:

$$\Gamma(k_i, t) = b[k_i(t)]^2, \tag{4}$$

where b is a positive parameter. Throughout the game, firms discount future profits at the common and constant discount rate $\rho > 0$.

Firms adopt a strictly noncooperative behaviour in choosing both the price levels and the R&D efforts, each firm operating her own R&D division.[4] Firm i's instantaneous profits are $\pi_i(t) = [p_i(t) - c_i(t)] q_i(t) - b[k_i(t)]^2$.

[3] As in d'Aspremont and Jacquemin (1988).

[4] For a discussion of R&D cooperation in the same model, see Cellini and Lambertini (2009).

Consumer surplus is $CS(t) = U(t) - \sum_{i=1}^{n} p_i(t) q_i(t)$; therefore, the instantaneous social welfare function is:

$$SW(t) = CS(t) + \sum_{i=1}^{n} \pi_i(t). \tag{5}$$

The objective of firm i consists in maximizing discounted profits:

$$\Pi_i = \int_0^\infty \left\{ [p_i(t) - c_i(t)] \left[\frac{A}{1 + s(n-1)} \right.\right. \tag{6}$$
$$- \frac{(1 + s(n-2)) p_i(t)}{(1-s)[1 + s(n-1)]} +$$
$$\left.\left. \frac{s \sum_{j \neq i} p_j(t)}{(1-s)[1 + s(n-1)]} \right] - b[k_i(t)]^2 \right\} e^{-\rho t} dt$$

subject to the set of dynamic constraints (3). The corresponding Hamiltonian function is:

$$\mathcal{H}_i(\mathbf{p}, \mathbf{k}, \mathbf{c}) = e^{-\rho t} \{ [p_i(t) - c_i(t)] q_i(t) + \tag{7}$$
$$-b[k_i(t)]^2 - \lambda_{ii}(t) c_i(t) [k_i(t) + \beta K_{-i}(t) - \delta] +$$
$$- \sum_{j \neq i} \lambda_{ij}(t) c_j(t) [k_j(t) + \beta(k_i(t) + \sum_{l \neq i,j} k_l(t)) - \delta] \}$$

where $\lambda_{ij}(t) = \mu_{ij}(t) e^{\rho t}$ is the co-state variable (evaluated at time t) associated with the state variable $c_j(t)$, $q_i(t)$ is defined as in (2) and $\mathbf{p}, \mathbf{k}, \mathbf{c}$ are the vectors of control and state variables.

4 The open-loop solution

Here we characterize the Nash equilibrium under the open-loop information structure. As a first step, we prove the following result:

Lemma 1 *The open-loop Nash equilibrium is subgame (or Markov) perfect.*

Proof. We are going to show that the present setup is a *perfect game* in the sense of Leitmann and Schmitendorf (1978) and Feichtinger (1983). In summary, a differential game is *perfect* whenever the closed-loop equilibrium collapses into the open-loop one, the latter being thus strongly time consistent, i.e., subgame perfect.[5] Consider the closed-loop information structure. The relevant first order conditions (FOCs) are:[6]

$$\frac{\partial \mathcal{H}_i}{\partial p_i} = \frac{1}{(1-s)\Upsilon}\{[c_i - 2p_i][1 + s(n-2)] + A(1-s) + s\sum_{j \neq i} p_j\} = 0 \tag{8}$$

where:

$$\Upsilon \equiv 1 + s(n-1); \tag{9}$$

$$\frac{\partial \mathcal{H}_i}{\partial k_i} = -2bk_i - \lambda_{ii}c_i - \beta \sum_{j \neq i} \lambda_{ij}c_j = 0. \tag{10}$$

As a first step, observe that the system of FOCs (8) yields the instantaneous Bertrand-Nash optimal prices as a quasi-static solution at any t, as it does not contain any co-state variable. Conversely, (10) at first sight seems to imply that the R&D effort is determined at any time by the entire vectors of both states and co-states, at least for any positive spillover effect.[7] The core of the proof consists in showing that k_i is in fact independent of all c_j at any time during the game, even for positive spillover levels.

[5] The label 'perfect game' is due to Fershtman (1987), where one can find a general technique to identify any such games. Another class of games where open-loop equilibria are subgame perfect is investigated by Reinganum (1982). For further details, see Mehlmann (1988, ch. 4) and Dockner *et al.* (2000, ch. 7).

[6] Henceforth, the indication of time and exponential discountig is omitted for brevity.

[7] Intuitively, if $\beta = 0$, then the investment plans are completely independent and therefore it is apparent that no feedback effect operates.

Taking the above considerations into account, the adjoint or co-state equations are:

$$-\frac{\partial \mathcal{H}_i}{\partial c_i} - \sum_{j\neq i} \frac{\partial \mathcal{H}_i}{\partial k_j}\frac{\partial k_j^*}{\partial c_i} = \frac{\partial \lambda_{ii}}{\partial t} - \rho\lambda_{ii} \tag{11}$$

yielding:

$$\begin{aligned}\frac{\partial \lambda_{ii}}{\partial t} &= q_i\left(t\right) + \lambda_{ii}(t)\left[k_i(t) + \beta K_{-i}(t) + \right. \\ &\rho - \delta] - \frac{\beta}{2b}\sum_{j\neq i}\lambda_{ji}(t)\left[\beta\lambda_{ii}(t)c_i(t) + \right. \\ &\lambda_{ij}(t)c_j(t) + \beta\sum_{l\neq i,j}\lambda_{il}(t)c_l(t)]\end{aligned} \tag{12}$$

and:

$$\begin{aligned}&-\frac{\partial \mathcal{H}_i}{\partial c_j} - \frac{\partial \mathcal{H}_i}{\partial k_i}\frac{\partial k_i^*}{\partial c_j} - \\ &\sum_{l\neq i,j}\frac{\partial \mathcal{H}_i}{\partial k_l}\frac{\partial k_l^*}{\partial c_j} = \frac{\partial \lambda_{ij}}{\partial t} - \rho\lambda_{ij}\end{aligned} \tag{13}$$

where each term

$$\frac{\partial \mathcal{H}_i}{\partial k_j}\frac{\partial k_j^*}{\partial c_i} \tag{14}$$

captures the feedback effect from j to i, and partial derivatives $\partial k_j^*/\partial c_i$ are calculated using the optimal values of investments as from FOC (10), $k_j^* = -\left(\lambda_{jj}c_j + \beta\lambda_{ji}c_i\right)/\left(2b\right)$. Now note that $\partial \mathcal{H}_i/\partial k_i = 0$ by virtue of (10). Hence, (13) yields:

$$\begin{aligned}\frac{\partial \lambda_{ij}}{\partial t} &= \lambda_{ij}\left(k_j + \beta k_i + \beta\sum_{l\neq i,j}k_l + \rho - \delta\right) - \\ &\frac{\beta}{2b}\sum_{l\neq i,j}\lambda_{lj}\left(\beta\lambda_{ii}c_i + \lambda_{il}c_l + \beta\sum_{j\neq i,l}\lambda_{ij}c_j\right)\end{aligned} \tag{15}$$

These conditions must be evaluated along with the initial conditions $\{c_i(0)\} = \{c_{0,i}\}$ and the transversality conditions

$$\lim_{t\to\infty} e^{-\rho t}\lambda_{ij}c_j = 0. \tag{16}$$

Note that, on the basis of *ex ante* symmetry across firms, $\lambda_{lj} = \lambda_{ij}$ for all l. Relatedly, observe that (15) is a differential equation in separable variables admitting the solution $\lambda_{ij} = 0$ at all t. Using this piece of information, we may rewrite the expression for the optimal investment of firm i as follows:

$$k_i^* = -\frac{\lambda_{ii} c_i}{2b}, \tag{17}$$

which entails that $\partial k_i^* / \partial c_j = 0$ for all $j \neq i$, i.e., feedback (cross-)effects are nil along the equilibrium path. Accordingly, the open-loop equilibrium is a degenerate closed-loop one, and it is strongly time consistent, or equivalently, subgame perfect. It is also worth observing that this procedure shows that FOCs are indeed unaffected by initial conditions as well. The property whereby the FOCs on controls are independent of states and initial conditions after replacing the optimal values of the co-state variables is known as *state-redundancy*, and the game itself as *state-redundant* or *perfect*. ■

On the basis of Lemma 1, we can proceed with the characterization of the open-loop solution. The FOCs on controls as well as the transversality conditions are the same as above, while the co-state equations are simplified as follows:

$$-\frac{\partial \mathcal{H}_i}{\partial c_i} = \frac{\partial \lambda_{ii}}{\partial t} - \rho \lambda_{ii} \Leftrightarrow \tag{18}$$

$$\frac{\partial \lambda_{ii}}{\partial t} = \lambda_{ii} \left[k_i + \beta K_{-i} + \rho - \delta \right] + \frac{A(1-s) - p_i \left[1 + s(n-2) \right] + s \sum_{j \neq i} p_j}{(1-s) \left[1 + s(n-1) \right]},$$

$$-\frac{\partial \mathcal{H}_i}{\partial c_j} = \frac{\partial \lambda_{ij}}{\partial t} - \rho \lambda_{ij} \Leftrightarrow \tag{19}$$

$$\frac{\partial \lambda_{ij}}{\partial t} = \lambda_{ij} \left[k_j + \beta K_{-j} + \rho - \delta \right] \tag{20}$$

From FOCs (8-10) we have, respectively:

$$p_i^* = \frac{A(1-s) + c_i \left[1 + s(n-2) \right] + s \sum_{j \neq i} p_j}{2 \left[1 + s(n-2) \right]}, \tag{21}$$

$$k_i = -\frac{\lambda_{ii} c_i}{2b}\,, \tag{22}$$

since $\lambda_{ij} = 0$ for all $j \neq i$, at any $t \in [0, \infty)$. While (21) has the usual appearance of a standard Bertrand best reply function, the optimal R&D effort in (22) depends upon i's co-state variable. Such expression can be differentiated w.r.t. time to get the dynamic equation of $k_i(t)$:

$$\frac{dk_i}{dt} \equiv \dot{k}_i = -\frac{1}{2b}\left[c_i \dot{\lambda}_{ii} + \lambda_{ii} \dot{c}_i\right] \tag{23}$$

with $\dot{\lambda}_{ii}$ obtaining from (18). Then, (23) can be further simplified by using $\lambda_{ii} = -2bk_i/c_i$ which results from (10), and the Bertrand-Nash equilibrium price which results from (21) after imposing the obvious symmetry condition $c_j(t) = c_i(t)$, $k_j(t) = k_i(t)$ and $p_j(t) = p_i(t)$ for all j:[8]

$$p^N = \frac{A(1-s) + c\left[1 + s(n-2)\right]}{2 + s(n-3)}\,. \tag{24}$$

Using (24) we may simplify the dynamics of the R&D effort of any single firm as follows:

$$\dot{k} = -\frac{c(A-c)\left[1 + s(n-2)\right] - 2b\rho k \Upsilon \Xi}{2b\Upsilon\Xi} \tag{25}$$

where Υ is defined as in (9) and $\Xi \equiv 2 + s(n-3)$.

Imposing the stationarity condition $\dot{k} = 0$, we obtain:

$$k_B^N = \frac{c(A-c)\left[1 + s(n-2)\right]}{2b\rho\left[1 + s(n-1)\right]\left[2 + s(n-3)\right]}\,, \tag{26}$$

with $k_B^N \geq 0 \; \forall \; c \in (0, A)$ and subscript B standing for *Bertrand*. The steady state level of marginal cost c can be found by solving:

$$\dot{c} = -c\left[k^N\left(1 + \beta(n-1)\right) - \delta\right] = 0 \tag{27}$$

[8]Note that $p^N = c$ if $s = 1$.

which yields $c = 0$ and

$$c = \frac{A\Omega \pm \sqrt{\Omega\left(A^2\Omega - \Phi\Upsilon\Xi\right)}}{2\Omega} \tag{28}$$

where $\Omega \equiv [1 + \beta(n-1)][1 + s(n-2)]$ and $\Phi \equiv 8b\delta\rho$. All solutions in (28) are real if and only if $A^2 \geq \Phi\Upsilon\Xi/\Omega$. If so, they also satisfy the requirement $c \in [0,\ A]$. By checking the stability conditions, we may prove the following:

Proposition 1 *Provided that* $A^2 \geq \Phi\Upsilon\Xi/\Omega$, *the steady state point*

$$\begin{aligned} c_B^{ss} &= \frac{A\Omega - \sqrt{\Omega\left(A^2\Omega - \Phi\Upsilon\Xi\right)}}{2\Omega} \\ k_B^{ss} &= \frac{\delta}{1 + \beta(n-1)} \end{aligned}$$

is the unique saddle point equilibrium.

Proof. Under symmetry, the dynamic equations of control and state variables are written as in (25) and (27). Accordingly, the relevant Jacobian matrix is:

$$\mathfrak{J} = \begin{bmatrix} \dfrac{\partial \dot{c}}{\partial c} & \dfrac{\partial \dot{c}}{\partial k} \\ \dfrac{\partial \dot{k}}{\partial c} & \dfrac{\partial \dot{k}}{\partial k} \end{bmatrix} \tag{29}$$

whose trace and determinant are:

$$T(\mathfrak{J}) = \delta + \rho - k[1 + \beta(n-1)] \tag{30}$$

$$\Delta(\mathfrak{J}) = \rho[\delta - k(1 + \beta(n-1))] - \frac{c(A - 2c)\Omega}{2b\Upsilon\Xi}. \tag{31}$$

Then, it can be easily checked that the pair $(c^{ss},\ k^{ss})$ is the only solution yielding $\Delta(\mathfrak{J}) < 0$ always, while the other two steady state points are both unstable. ■

It is possible to calculate the equilibrium trajectory of k and c along the saddle path to the steady state, following the standard procedure. The relevant dynamic system $\dot{k} = F(k,c)$, $\dot{c} = G(k,c)$ is given by equations (25) and (27) respectively, so that around the steady state variables follow respectively:

$$k^N(t) = -\frac{\rho - \sqrt{\rho^2 + \Theta}}{2\left[1 + \beta\left(n-1\right)\right] c_B^{ss}}(c_0 - c_B^{ss}) \exp(\frac{\rho - \sqrt{\rho^2 + \Theta}}{2} \cdot t) + k_B^{ss}; \quad (32)$$

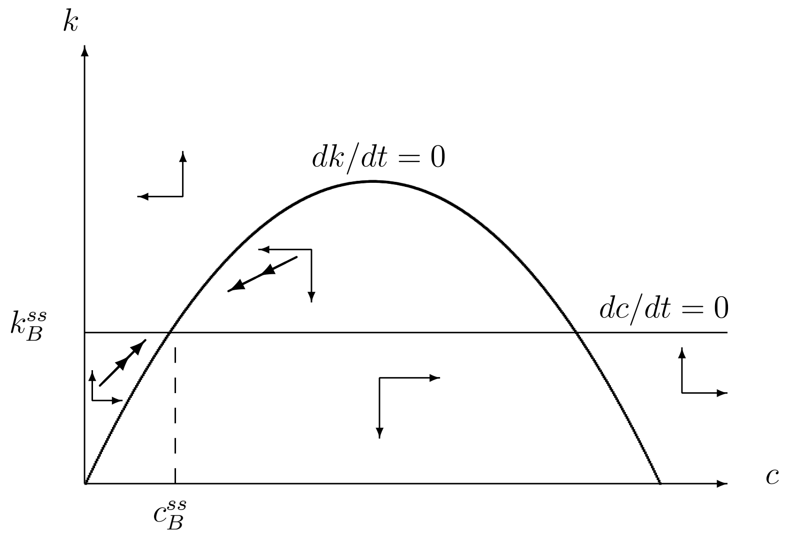
$$c^N(t) = (c_0 - c_B^{ss}) \exp(\frac{\rho - \sqrt{\rho^2 + \Theta}}{2} \cdot t) + c_B^{ss} \quad (33)$$

where $\Theta = \Omega(A - 2c_B^{ss})c_B^{ss}/(2b\Upsilon\Xi) > 0$. The equilibrium R&D efforts for a given level of the marginal cost may be expressed as

$$k^N(t) = -\frac{\rho - \sqrt{\rho^2 + \Theta}}{2\left[1 + \beta\left(n-1\right)\right] c_B^{ss}}(c(t) - c_B^{ss}) + k_B^{ss} \quad (34)$$

which can be interpreted as the feedback representation of the open-loop Nash equilibrium (see, e.g., Benchekroun and Long, 1998). The phase diagram is illustrated in Figure 1, portraying the stable and efficient steady state point (c_B^{ss}, k_B^{ss}) and the saddle path leading to it.

Figure 1: Phase diagram



5 Comparative statics

Now we focus on the effects of market structure (measured by the number of firms) and product substitutability (measured by parameter s) upon the incentive to invest in process R&D. To this aim, we examine effect of a change in n and s on individual and aggregate steady state R&D efforts.

This discussion revisits the debate between Schumpeter (1942) and Arrow (1962). Their respective views can be summarized as follows. According to the Schumpeterian hypothesis, R&D investments and technical progress are positively related to the flow of profits and therefore we should expect to observe higher R&D efforts and a faster innovation process under monopoly than any other market form. Conversely, Arrow claims that the incentive to generate technical progress is negatively affected by market power, being then maximized under perfect competition. The Arrovian position relies upon the idea that innovation is more attractive for a competitive firm than for a monopolist who, by definition, cannot improve its own degree of market power.

In order to assess this issue in the present model, we proceed as follows. The aggregate R&D investment in steady state is

$$K_B^{ss} = \frac{\delta n}{1 + \beta (n-1)} . \tag{35}$$

Taking into account the integer constraint on n, it is immediate to check that

$$\frac{\partial K_B^{ss}}{\partial n} \geq 0 \tag{36}$$

while clearly $k_B^{ss} = \delta / [1 + \beta (n-1)]$ is everywhere decreasing in n. That is, the individual firm's incentive to invest at equilibrium is negatively affected by an increase in n, while on aggregate the industry reacts in the opposite way.

The same qualitative properties emerge if one looks at the optimal individual and aggregate industry investment for any given level of marginal cost, by assessing the effects of a change in n on the position of the locus $dk/dt = 0$ in the phase diagram. From (26) we obtain:

$$\frac{\partial k_B^{ss}}{\partial n} = -\frac{c(A-c)s\left[1+(2n-5)s+(n^2-4n+5)s^2\right]}{2b\rho\left[1+s(n-1)\right]\left[2+s(n-3)\right]^2} < 0 \qquad (37)$$

for all $n \geq 1$ and $s \in [0,1)$. With reference to Figure 1, this entails that the concave locus $dk/dt = 0$ in shifts downward in the space (c, k) as n increases. As to the industry investment, we have:

$$K_B^N = nk_B^N = \frac{c(A-c)\left[1+s(n-2)\right]n}{2b\rho\left[1+s(n-1)\right]\left[2+s(n-3)\right]}; \qquad (38)$$

and it is immediate to verify that:

$$\frac{\partial K_B^N}{\partial n} \geq 0 \qquad (39)$$

in the admissible range of parameters. So, it is clear that an increase in n drives an increase in the industry effort in process R&D. Put it differently, *all else equal*, an industry characterised by a higher concentration will invest less than an industry where the population of firms is larger. Our results can be summarised in the following

Proposition 2 *While the single firm's steady state incentive to invest reacts negatively to an increase in the intensity of competition, the equilibrium R&D investment of the whole industry is non-decreasing in the number of firms.*

That is, the industry performance exhibits a clear Arrovian flavour. The driving force behind the behaviour of individual and aggregate investments for process innovation is twofold: (i) tougher market competition reduces profits and therefore the funds available for financing R&D activity; (ii) a larger population of firms means a larger positive spillover that any firm receives from rivals. On balance, a scale effect prevails, so that the overall expenditure of the industry is monotonically increasing in n.

Hinloopen (2000) has solved a multi-stage game in which n firms may compete either *à la* Cournot or *à la* Bertrand in the market phase. From his results concerning Bertrand competition (in particular eq. A4, p. 176) one can check his finding that both aggregate and individual R&D efforts are non-monotone w.r.t. n. Under this respect, the static approach proves to fall short of appropriately accounting for the inherently dynamic nature of R&D which is not captured by multistage game modelling.

Now examine the effect of s on optimal investments. First, note that steady state levels are independent of the degree of product substitutability.[9] Second, considering the locus $dk/dt = 0$, i.e., expression k_B^N in (26), we have:

$$\frac{\partial k_B^N}{\partial s} \propto -\left[s^2(n-2)(s-3)+2s(n-3)+1\right] \tag{40}$$

and obviously $\partial K_B^N/\partial s = n\partial k_B^N/\partial s$. The partial derivative (40) is always negative, except at $n = 2$, where $\partial k_B^N/\partial s \propto 2s - 1 > 0$ for all $s \in (1/2, 1]$. Hence, we may state:

Proposition 3 *For all $n \geq 3$, the incentive to invest in R&D at equilibrium is decreasing in product substitutability, for any given c. At $n = 2$, R&D efforts are decreasing in s for $s \in (0, 1/2)$, and conversely for $s \in (1/2, 1]$.*

Any increase in substitutability, or decrease in differentiation, damages operative profits, shifting thus downward the cocave locus on Figure 1. Hence, the net effect on both k_B^N and K_B^N is the balance of two opposite tendencies: (i) the decrease in operative profits lowers the funds for R&D activity; (ii) any increase in R&D for process innovation may allow firms to recover on the cost side what is being lost on the differentiation side. Proposition 3 says that, if n is sufficiently large, the first effect dominates

[9]This is in sharp contrast with the static models on the same topic (see Delbono and Denicolò, 1990; Bester and Petrakis, 1993; Qiu, 1997; Hinloopen, 2000; and Lambertini and Mantovani, 2001).

the second because competition is too tough and the prize is not worth the effort, while the opposite holds for $n = 2$. Contrary to Proposition 2, the flavour of Proposition 3 is Schumpeterian, at least for $n \geq 3$: any increase in product differentiation amounts to a decrease in the intensity of competition, and brings about an increase in R&D efforts.

Note that our results complement the findings obtained by Qiu (1997): he proposes a multi-stage game in which *two* firms produce differentiated products, and finds that the relationship between product substitutability and R&D efforts is ambiguous. Such finding is confirmed by our analysis (for the case of duopoly). However, the ambiguity disappears if one considers the case in which the market is populated by a number of firms larger than two. In such a case - not investigated by Qiu - clearcut results emerges: product substitutability hampers R&D investment, *ceteris paribus* (this is a pro-Schumpeter result); moreover, we are also able to see that the larger the number of firms, the higher the aggregate investment in R&D efforts (a pro-Arrow result).

6 Bertrand vs Cournot

The analysis of the Cournot game with process R&D extends the contents of Cellini and Lambertini (2005), where the case of homogeneous goods was considered. Under Cournot competition, at any instant the relevant (inverse) demand function for firm i is:

$$p_i(t) = A - q_i(t) - s \sum_{j \neq i} q_j(t) . \tag{41}$$

After taking much the same steps as for the Bertrand case, we obtain the dynamics of the R&D effort:

$$\dot{k} = \rho k - \frac{c(A-c)}{2b[2+s(n-1)]} \tag{42}$$

and then, imposing stationarity, the expression of the optimal R&D effort as a function of marginal cost:[10]

$$k_C^N = \frac{c(A-c)}{2b\rho[2+s(n-1)]} \tag{43}$$

which can be evaluated against (26) to yield:

$$k_B^N - k_C^N \propto 2 + s(n-3) \geq 0 \tag{44}$$

for all $s \in [0,1)$ and all $n \geq 1$. Note that the inequality is strict for all $n \geq 2$.[11] The same conclusion applies if one compares instantaneous investment rates, i.e., (42) and (25). Therefore, we can formulate the following:

Proposition 4 *For any given level of the marginal cost, Bertrand competition yields higher R&D incentives than Cournot competition, for any industry size and any degree of product substitutability.*

The above result has a strong Arrovian flavour, clearly based on the intuition that facing a tougher competition in the market phase, Bertrand firms are more keen on reducing marginal costs than Cournot firms. Then, observe that in both cases the steady state amount of R&D has just to make up for depreciation, i.e., $k_B^{ss} = k_C^{ss} = \delta/[1+\beta(n-1)]$. However, a straightforward implication of Proposition 4 is:

Corollary 1 $c_B^{ss} < c_C^{ss}$ *for all* $s \in [0,1)$ *and all* $n \geq 1$.

[10] Also here, the steady state is a saddle point. We omit the details of the stability analysis for the sake of brevity. However, the proof is available upon request and can be found, although for the homogeneous good case only, in Cellini and Lambertini (2005).

[11] In connection with (44), it is worth observing that the phase diagram for the Cournot model would look qualitatively the same as in Figure 1, with the concave locus $dk/dt = 0$ lying everywhere below the one generated by Bertrand competition.

That is, Bertrand firms are more efficient than Cournot firms at the long run equilibrium. This fact has a relevant implication as far as the steady state welfare level is concerned. All else equal, welfare is higher at the Bertrand equilibrium than at the Cournot equilibrium (as in Singh and Vives, 1984). This is *a fortiori* true in the present case, where marginal cost is lower under price competition than under quantity competition, while individual and aggregate R&D expenditure is the same in the two cases. Under symmetry, the expression of social welfare (5) simplifies as follows:

$$SW(t) = nAq - \frac{nq^2}{2}\left[1 + s(n-1)\right] - nk. \tag{45}$$

Given that in steady state $k_B^{ss} = k_C^{ss}$, any difference in social efficiency between Bertrand and Cournot behaviour boils down to noting that the two related facts that (i) all else equal, $q_B^{ss} > q_C^{ss}$; and (ii) $p_B^{ss} < p_C^{ss}$ are enhanced by the result stated in Corollary 6 concerning the ranking of marginal costs at the two steady state equilibria. Hence, without further proof, we may state our final result:

Proposition 5 *In steady state, $SW_B^{ss} > SW_C^{ss}$ over the whole admissible range of parameters.*

7 Conclusions

We have analyzed dynamic R&D investments for cost-reducing innovation in a Bertrand oligopoly in order to evaluate the influence of market structure and product differentiation on R&D incentives.

Three features of our analysis are worth stressing. First, the game is perfect, or state-redundant, so that the open-loop solution is Markovian, or subgame perfect. Second, if we look at the effects of market structure on innovation, an Arrovian conclusion obtains, since the aggregate R&D effort at equilibrium is increasing in the number of firms, for any degree of product differentiation. This sharply differs from the ambiguous conclusions reached by the static models, where the smoothing of investment efforts over a long time horizon is ruled out by definition. Third, we have shown that the interplay between R&D incentives and product differentiation is ambiguous if $n = 2$, while individual and industry investments are monotonically decreasing in product substitutability if $n \geq 3$. This, in turn, is a Schumpeterian result. Therefore, as a final remark, we may say that, if the intensity of market competition is measured by market structure, *all else equal*, then the answer of the model is Arrovian; if instead the intensity of competition is measured by product substitutability for a given market structure, then the model points to a Schumpeterian conclusion.

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