

# Topics in Personnel Economics

Slides for Lectures

Very preliminary, incomplete and not to be quoted \*

Andrea Ichino  
(European University Institute)

October 12, 2005

## Abstract

Since its beginning labor economics has focused mainly on the functioning (or mis-functioning) of “labor markets” leaving to other disciplines the study of “employment relationships” within firms. More recently, under the label of “personnel economics”, an increasing number of labor economists have started to apply theoretical and empirical tools to the analysis of how employers and employees interact within firms. In this literature, firms are viewed as organizations to be studied from the inside, not as black boxes producing output with labor, capital and other factors. This course is intended to introduce students to this recent theoretical and empirical literature, with specific attention to the peculiarities of European industrial relations.

---

\*Address correspondence to: Andrea Ichino, Department of Economics, European University Institute, Via Piazzola 43, I-50133, Italia, e-mail: [andrea.ichino@iue.it](mailto:andrea.ichino@iue.it)  
<http://www.iue.it/Personal/Ichino/Welcome.html>

# Contents

<b>1</b>	<b>The stage and the actors</b>	<b>2</b>
1.1	The employer and the employee: typical assumptions . . . . .	4
1.2	“Non- orthodox” views on the employment relationship . . . . .	6
1.3	The phases of the employment relationship . . . . .	7
<b>2</b>	<b>The selection of workers at the beginning of an employment relationship</b>	<b>8</b>
2.1	Adverse selection, Pareto optimality and Rawlsian social justice (Mas Colell et al. (1995) ch. 13) . . . . .	10
2.2	Signaling and welfare: the problem of compulsory maternity leaves (Mas Colell et al. (1995) ch. 13) . . . . .	22
2.3	Screening through probation and upward sloping wage profiles (Lazear (1995)) . . . . .	36
2.4	Screening through “pay based on performance” (Lazear (1999)) . . . . .	46
2.5	Probation when monitoring is the only way to screen heterogeneous agents (Ichino and Muehlheusser (2004)) . . . . .	50
<b>3</b>	<b>The development of an employment relationship</b>	<b>81</b>
3.1	The canonical principal-agent model (Gibbons and Waldman (1999)) . . . . .	83
3.2	Imperfect measurement of performance (Baker (1992)) . . . . .	94
3.3	Pay based on input or output and allocation of responsibilities (Prendergast (2002)) . . . . .	109
3.4	Relative performance evaluation (Holmstrom (1979)) . . . . .	118
3.5	Relative performance and tournaments (Lazear and Rosen (1981)) . . . . .	127
3.6	The emergence of hierarchies when effort and output are not contractible (Prendergast (1993)) . . . . .	136
3.7	“Up-or-out” contracts (Kahn and Huberman (1988)) . . . . .	143
3.8	The “Peter Principle” (Lazear (2004)) . . . . .	151
3.9	Moral Hazard in teams (Holmstrom (1982)) . . . . .	164
3.10	Efficiency Wages as an incentive device (Gibbons and Waldmann (1999)) . . . . .	176
3.11	Career concerns (Holmstrom (1999)) . . . . .	183
<b>4</b>	<b>End of the employment relationship: to be completed</b>	<b>191</b>
<b>5</b>	<b>References</b>	<b>192</b>

## 1 The stage and the actors

Since its beginning, labor economics has focused mainly on the functioning (or mis-functioning) of “labor markets” leaving to other disciplines the analysis of “employment relationships” within firms.

More recently, under the label of “personnel economics”, an increasing number of labor economists have started to apply theoretical and empirical tools to the analysis of how employers and employees interact within firms.

In this literature, firms are studied from the inside as organizations, not as black boxes producing output with labor, capital and other factors.

This course is intended to introduce students to this recent theoretical literature.

## Personnel economics: normative or positive goals?

Standard “personnel economics” is sometimes seen as a normative discipline aimed at teaching employers how to solve problems in human resources management.

Here we will focus not only on the perspective of employers but also on:

- welfare considerations according to different social welfare functions;
- workers goals and attitudes;
- the market failures at the origin of some (typically “Europeans”) labor market institutions and regulations which shape employment relationships within firms.

## 1.1 The employer and the employee: typical assumptions

The employer aims at profit maximization and solves four main problems:

- Selection of employees.
- Motivation of employees.
- Assignment of employees to jobs.
- Optimal retention and turnover of employees.

Solving these problems involves issues like:

- Divergence of employers' and employees' goals.
- Imperfect and asymmetric information.
- High cost or impossibility to write complete contracts.

Many models assume perfect or Bertrand competition and therefore zero profits in equilibrium for firms. Is this an innocuous assumption?

The employee typically aims at maximizing compensation and minimizing effort.

His productivity depends on:

- A “factor” which can be acquired at some cost (before or during the employment relationship) and is observed by the employer (maybe not by others):
  - general human capital acquired before entering in the job market;
  - general human capital acquired after entering the job market;
  - firm-specific human capital acquired through on the job training.
- A non modifiable “type” which may be observed by nobody or only by the employee:
  - innate ability;
  - preferences concerning work and leisure.
- An “action” which may or may not be observable and verifiable in court:
  - effort (e.g. absenteeism).

## 1.2 “Non- orthodox” views on the employment relationship

Standard Personnel Economics textbooks put less emphasis on aspects of the employment relationship like:

- Asymmetry of roles and the “debate” on “who hires whom” .
- A psychological aspect of the relationship: domination and control.
- Inequality of bargaining powers, endowments and outside options.
- Social preference for equality of compensation levels and working conditions across workers.
- Wage and working conditions as “independent variables” .
- Unions, collective action and collective bargaining
- Strikes and conflicts resolution.
- Moral hazard on the management side.

Are these relevant issues or “dinosaurs” from the past?

### 1.3 The phases of the employment relationship

We distinguish three related phases of the employment relationship.

#### 1. The beginning:

- unobservability of workers' types and adverse selection
  - due to technological reasons;
  - due to endogenous labor market institutions;
- signaling, screening, probation;

#### 2. The development:

- motivation of employees;
- assignment of employees to jobs.

#### 3. The end:

- quits and layoffs: is there a difference?
- retirement;
- pathological and physiological terminations.

## 2 The selection of workers at the beginning of an employment relationship

A complete course should begin by what a potential employee does, before starting a job, to acquire the observable characteristics which are needed for it.

For lack of time, here we condition instead on the acquired observable characteristics and go directly to the next step of the process.

What happens when employers want to “hire the right person” for a given job but do not observe the relevant workers’ characteristics (or “type”)?

Unobservability of workers’ types may be due to “technology”.

But sometimes labor market institutions arise endogenously to prevent the observability of workers’ types.

It is interesting to study the origin of these institutions and their properties according to different social welfare functions.

## Workers' types and labor market institutions

Examples of institutions that prevent employers from observing workers' types:

- Compulsory hiring from numeric lists in employment offices.
- Restrictions on the possibility to acquire information on job applicants.
- Restrictions on the possibility to monitor during probation periods.
- Restrictions on the use of temporary contracts as extended probation periods.
- In general: non meritocratic institutions

Examples of institutions that prevent employers from using information on workers' types.

- Compensation systems independent of performance.
- Automatic promotions based on seniority not on merit.
- Seniority criteria for layoffs.

There is a continuum of workers in the  $[0, 1]$  interval, who differ by the output they can produce denoted with  $\theta \in [\underline{\theta}, \bar{\theta}]$  and distributed according to  $F(\theta)$ .

They have a reservation wage  $r(\theta)$  and accept a wage offer only if  $w > r(\theta)$ .

Firms produce the same output with a CRS technology using labor only.

Firms are price takers, the price is normalized to 1 and there is free entry.

As a benchmark, if  $\theta$  is observable the competitive equilibrium is given by

$$w_o^*(\theta) = \theta \quad (1)$$

for each different  $\theta$  and the set of workers accepting wage offers is:

$$\Theta_o^* = \{\theta : r(\theta) \leq w_o^*(\theta)\} \quad (2)$$

This is a competitive equilibrium and it is therefore Pareto optimal, but it may generate unequal ex post outcomes, which may be considered collectively undesirable if evaluated as of before the time in which nature assigns types  $\theta$ .

## Equilibrium when productivity is not observed by firms

When firms cannot observe  $\theta$ , the equilibrium is described by a wage level  $w_u^*$  and by a set of workers who accept wage offers  $\Theta_u^*$  such that

$$w_u^* = E(\theta \mid \theta \in \Theta_u^*) \quad (3)$$

$$\Theta_u^* = \{\theta : r(\theta) \leq w_u^*\} \quad (4)$$

Equation 3 ensures that the wage offered by firms is equal to the average productivity of the workers who accept job offers in equilibrium.

Equation 4 defines the set of workers who accept job offers given the equilibrium wage offered by firms.

Firms correctly anticipate the productivity of accepting workers.

This equilibrium may be preferred to the one in which  $\theta$  is observable if there is a strong social preference for equality.

## Pooling equilibria, justice and labor market institutions

If  $\theta$  is *unobservable*, the equilibrium wage cannot depend on  $\theta$ .

A wage independent of  $\theta$  would be a typical “unions’ objective” in this context. Many labor market institutions can be seen as aimed at making  $\theta$  unobservable.

This objective could be justified if social welfare is defined à la Rawls (i.e. “under the veil of ignorance”) as

$$R = \text{Min} \{w(\theta)\} \quad (5)$$

Note that  $\theta$  here is inherited, not a choice variable like effort.

Many situations of labor relations and social conflicts, particularly in Europe, can be analyzed in these terms.

Let’s examine some specific examples in our stylized framework.

**Ex. A:**  $\theta$  is observable and the reservation wage is constant

Suppose that  $r(\theta) = r$  and  $r < \underline{\theta} < \bar{\theta}$ . Since  $\theta$  is observable the equilibrium is:

- all workers are employed at the wage  $w_0^*(\theta) = \theta > r$ ;
- no one is unemployed because all wage offers are higher than  $r$ .

This equilibrium is Pareto optimal and generates full employment.

If workers' utility is equal to the wage, utilitarian social welfare is maximized (note that firms earn zero profits):

$$\Omega_o^* = E(\theta) = \int_{\underline{\theta}}^{\bar{\theta}} \theta dF(\theta) \quad (6)$$

Rawlsian social welfare is :

$$R_o^* = \underline{\theta} \quad (7)$$

This equilibrium is not egalitarian. Note again that  $\theta$  is inherited, not a choice variable like effort.

**Ex. B1:**  $\theta$  is unobservable,  $r$  is constant and  $r < \underline{\theta} < E(\theta)$

Suppose that  $r < \underline{\theta} < E(\theta)$ . The equilibrium is:

$$w_u^* = E(\theta \mid \theta \in [\underline{\theta}, \bar{\theta}]) = E(\theta) \quad (8)$$

$$\Theta_u^* = [\underline{\theta}, \bar{\theta}] \quad (9)$$

- Everybody works, and this is efficient because  $r < \underline{\theta}$ .
- Firms make losses on some workers, gains on others (viceversa for workers).
- Utilitarian social welfare is identical to the case of  $\theta$  observable with  $r < \underline{\theta}$ :

$$\Omega_u^* \mid_{r < \underline{\theta}} = E(\theta) = \Omega_o^* \quad (10)$$

- Rawlsian social welfare is higher than in the case of  $\theta$  observable with  $r < \underline{\theta}$ :

$$R_u^* \mid_{r < \underline{\theta}} = E(\theta) > R_o^* = \underline{\theta} \quad (11)$$

- The wage is completely equalized across workers. “Under the veil of ignorance” everybody is ex ante fully insured against the risk of a low  $\theta$ .

**Ex. B2:**  $\theta$  is unobservable,  $r$  is constant and  $\underline{\theta} < r < E(\theta)$

Suppose that  $\underline{\theta} < r < E(\theta)$ . The equilibrium is:

$$w_u^* = E(\theta \mid \theta \in [\underline{\theta}, \bar{\theta}]) = E(\theta) \quad (12)$$

$$\Theta_u^* = [\underline{\theta}, \bar{\theta}] \quad (13)$$

- Unproductive workers, who would not work if  $\theta$  were observable, have a job.
- Firms make losses on some workers, gains on others (viceversa for workers).
- Utilitarian welfare is lower than when  $\theta$  is observable with  $\underline{\theta} < r < E(\theta)$ :

$$\Omega_u^* \mid_{\underline{\theta} < r < E(\theta)} = E(\theta) < \Omega_o^* = rF(r) + \int_r^{\bar{\theta}} \theta dF(\theta) \quad (14)$$

- Rawlsian welfare is higher than when  $\theta$  is observable with  $\underline{\theta} < r < E(\theta)$ :

$$R_u^* \mid_{r < E(\theta)} = E(\theta) > R_o^* = r \quad (15)$$

- The wage is completely equalized across workers. “Under the veil of ignorance” everybody is ex ante insured against the risk of a low  $\theta$ .

**Ex. B3:**  $\theta$  is unobservable,  $r$  is constant and  $E(\theta) < r$

Suppose that  $r > E(\theta)$ . Under these conditions the market unravels because:

- No firm offers more than  $w_u^* = E(\theta)$ .
- But this is not enough to convince workers to accept offers.
- Thus, everybody is unemployed and  $\Theta_u^*$  is empty.
- Market production does not take place.
- Society collapses.

Thus, in this case, labor market institutions which make  $\theta$  unobservable lead to the worst outcome.

## Thoughts on the “The rise and fall of Rawlsian institutions”

- Institutions governed by Rawlsian objectives emerge as a reaction to unequal outcomes of competitive equilibria in which  $\theta$  is observable and  $r$  is low.
- Suppose that initially  $r < \underline{\theta} < E(\theta)$ . In this case making  $\theta$  not observable has no consequences in terms of production efficiency.
- Rawlsian social objectives also favor institutions aimed at increasing  $r$ .
- As soon as these combined processes lead to  $E(\theta) < r$  the market unravels and institutions which makes  $\theta$  unobservable become unsustainable.
- Rawlsian labor market institutions face a dangerous “razor-edge”.

**Ex. C:**  $\theta$  is unobservable and the reservation wage grows with  $\theta$

In the previous examples inefficiency occurs if “unproductive” workers, who should not work, are instead employed.

When the reservation wage increases with productivity, the implications for the trade-off between Pareto efficiency and social justice become even more striking.

This is the case of “adverse selection” described by Akerlof (1970), in which the market may unravel even if all workers should work.

- $r(\theta) < \theta \quad \forall \theta \in [\underline{\theta}, \bar{\theta}]$  which means that all workers should work;
- $r(\cdot)$  is strictly increasing in  $\theta$ , which means that more productive workers have better alternative opportunities.

Under these assumptions the average quality of job applicants is increasing in the wage offered by firms.

This generates the possibility of “adverse selection”: if firms lower the wage to reduce labor costs they get worse job applicants.

## The equilibrium with adverse selection

The equilibrium is characterized by

$$w_u^* = E(\theta \mid r(\theta) \leq w_u^*) \quad (16)$$

$$\Theta_u^* = \{\theta : r(\theta) \leq w_u^*\} \quad (17)$$

Figure 13.B.1 (from Mas-Colell et al. (1995)):

- If the market does not unravel we obtain an egalitarian equilibrium which increases Rawlsian welfare but is ex post Pareto inefficient.

Figure 13.B.2 (from Mas-Colell et al. (1995)):

- However, the market may unravel if at any given wage the average quality of applicants is too low for the firm to be willing to hire anybody.

Figure 13.B.3 (from Mas-Colell et al. (1995)):

- Ranked multiple equilibria are possible but the Pareto-dominated equilibrium arises because of a “coordination failure” .

## Again the “razor edge” faced by Rawlsian institutions

Consider an initial competitive equilibrium in which  $\theta$  is observable. This equilibrium is Pareto efficient but generates unequal outcomes.

In this situation a pressure for Rawlsian institutions may prevail. By banning the observation of  $\theta$  these institutions

- can maximize the welfare of the least endowed worker;
- and can generate an outcome which is ex ante Pareto efficient “under the veil of ignorance” ,
- at the cost of reaching a Pareto inefficient ex post allocation of workers.

However, this is just one side of the edge. The other side is that by banning the observation of  $\theta$  the market may unravel.

If this happens, it means that banning the observability of  $\theta$  has led to the worse possible outcome independently of the preferred welfare function.

See Mas-Colell et al. (1995) pp. 445-450, and Holmstrom and Myerson (1983).

## How firms and workers can react to adverse selection

Whether caused by “technological” constraints or by “Rawlsian institutions”, the unobservability of workers’ types  $\theta$  damages skilled workers and firms.

This leads to the following possible reactions by the damaged parties.

- Skilled workers (the informed side of the market) may take actions that *signal* their higher quality.
- Firms (the un-informed side of the market) may offer:
  - contract features that induce only skilled workers to apply for jobs;
  - or menus of contracts that induce a separation of workers according to values of  $\theta$ , which is advantageous for firms.
- Firms (the un-informed side of the market) may allow all workers to apply and then use monitoring during probation to screen among heterogeneous job applicants.

All these reactions are typically strongly opposed by unions.

2.2 Signaling and welfare: the problem of compulsory maternity leaves (Mas Colell et al. (1995) ch. 13)

Suppose that  $\theta \in \{\theta_H, \theta_L\}$  denotes the desired number of children of a worker:

$\theta_H = \textit{hate}$  children so desire zero of them (with prob.  $\lambda$ );

$\theta_L = \textit{love}$  children so desire at least one of them (with prob.  $1 - \lambda$ ).

The worker type  $\theta$  cannot be observed directly by firms.

Consider the hiring decision of firms facing young workers without children.

$\theta_H$  workers are less likely to have children in the future and therefore can be expected to be more productive. For simplicity: productivity is equal to  $\theta$ .

The reservation wage is zero for all workers:  $r(\theta_L) = r(\theta_H) = 0 < \theta_L < \theta_H$ .

This framework can be used to discuss why maternity leaves are typically compulsory.

## The equilibrium in the absence of signaling possibilities

We know from Section 2.1 that the equilibrium in this case is

$$w_u^* = E(\theta) \quad (18)$$

$$\Theta_u^* = \{\theta_L, \theta_H\} \quad (19)$$

Everybody works and the wage is equal to average productivity.

Note that, even if  $\theta$  is non-observable, this equilibrium:

- is Pareto efficient because the  $r(\theta) < \theta$  for all  $\theta$ ;
- full employment is efficient because no one prefers the reservation wage;
- competition between firms drives ex ante expected profits to zero;
- but firms make losses on  $\theta_L$  workers and gains on  $\theta_H$  workers;
- it is an egalitarian equilibrium which maximizes Rawlsian welfare;
- however, the market would unravel if  $r$  becomes larger than  $E(\theta)$ .

## What happens when a signal for $\theta$ is possible

Suppose that, by law, workers:

- are entitled to  $E$  days of maternity leave when they have a child,
- but, if they want, they can give up  $e \in [0, E]$  of these days when they accept a job offer.

$\theta_H$  workers are interested in signaling that they desire zero children, because they are paid less than their productivity in the pooling equilibrium.

They can send this signal by giving up days of maternity leave.

However, this signal would obviously be completely uninformative if it could be given as easily by  $\theta_L$  workers.

The signal is informative, only if  $\theta_H$  workers have an incentive to send it while  $\theta_L$  workers prefer not to send it.

## Assumption on the cost of the signal

- The maternity leave in itself is irrelevant for productivity (i.e.  $E$  is small).
- On the contrary, having children reduces productivity because of induced absenteeism during the life of children.
- Increasing  $e$  is “more costly” for  $\theta_L$  workers than for  $\theta_H$  workers.
- More precisely, denoting the cost of the signal with  $C(e, \theta)$ :
  - $C(0, \theta) = 0$ : the cost is zero when no days are given up.
  - $C_e(e, \theta) > 0$ : the cost increases in the number of days for all  $\theta$ .
  - $C_{ee}(e, \theta) > 0$ : the cost is convex in the number of days for all  $\theta$ .
  - $C_\theta(e, \theta) < 0$ : the cost is lower for  $\theta_H$  workers who want no children.
  - $C_{e\theta}(e, \theta) < 0$  the cost grows less for  $\theta_H$  workers who want no children.
- The utility of accepting a wage offer  $w$  is

$$U(w, e | \theta) = w - C(e, \theta) \quad (20)$$

## Equilibrium concept and firms' strategies

Following Mas-Colell et al. (1995), we analyze this game using a Perfect Bayesian Equilibrium concept.

- The worker's strategy is optimal given the firms' strategies.
- Firms use Bayes law to form a belief  $\mu(e)$  on a worker's type given the signal.
- The firms' offers following a signal  $e$  are a Nash equilibrium of the simultaneous-move offer game in which the expected frequency of  $\theta_H$  worker is  $\mu(e)$ .

See Figure 13.C.1 for the structure of the game.

In the last stage of the game firms play "Bertrand" given the signal  $e$ .

The equilibrium of this sub-game is to offer to the worker a wage equal to her expected productivity given the belief  $\mu(e)$ :

$$w = \mu(e)\theta_H + (1 - \mu(e))\theta_L \quad (21)$$

The model can be adjusted for the situation of a single firm and a single worker.

## Workers' strategies

The preferences of the two types of workers with respect to  $w$  and  $e$  are described by the two indifference curves in figure 13.C.2 (with  $e$  on the horizontal axis).

- The indifference curve for  $\theta_L$  is steeper, indicating that a bigger wage increase must be given to the worker who loves children in order for her to give up days of maternity leave and be indifferent:

$\frac{dw}{de}|_{\bar{u}} = C_e(e, \theta) > 0$ : because the marginal cost of the signal is positive;

$\frac{dw^2}{ded\theta}|_{\bar{u}} = C_{e\theta}(e, \theta) < 0$ : because the marginal cost of the signal is decreasing in  $\theta$ .

- The two indifference curves satisfy the *single crossing property*.
- Thus *ceteris paribus*  $\theta_L$  workers are less likely to give the signal.

Different types of equilibria are possible.

## Separating equilibria

The combination of wage signals, beliefs and wage offers must be such that:

- only for the  $\theta_H$  worker it is advantageous to give up days of maternity leave in exchange for a higher wage;
- for the  $\theta_L$  worker the cost of sending the signal is too high to be compensated by the wage gain;
- thus firms can infer that a worker who gives up days of maternity leave is of type  $\theta_H$ ;

If  $e^*(\theta)$  and  $w^*(e)$  are the equilibrium choices, in any separating PBE:

- each type receives a wage equal to productivity:

$$w^*(e^*(\theta_L)) = \theta_L \quad \text{and} \quad w^*(e^*(\theta_H)) = \theta_H$$

- the worker who loves children gives up zero days while the other gives up a positive amount of days:

$$e^*(\theta_L) = 0 \quad \text{and} \quad e^*(\theta_H) > 0$$

## An example of separating equilibrium

The firm announces that:

- Workers who give up  $e > \tilde{e}$  days are expected to be of type  $\theta_H$  and therefore receive a wage  $w(e) = \theta_H$ .
- Workers who give up  $e \leq \tilde{e}$  are expected to be of type  $\theta_L$  and receive a wage  $w(e) = \theta_L$
- $\tilde{e}$  is defined as in figure 13.C.6 (left)

Given the indifference curves in figure 13.C.6 (left):

- $\theta_H$  workers maximize utility by giving up at least  $\tilde{e}$  days of leave;
- $\theta_L$  workers maximize utility when they do not give up any day of leave;

The signal is informative and beliefs are never disconfirmed in equilibrium.

## Some remarks on the features of this equilibrium

- If the threshold to discriminate the signal were set equal to  $\hat{e} < \tilde{e}$ :
  - $\theta_L$  workers would find optimal to mimic  $\theta_H$  workers;
  - all workers would give up the same number of days of leave ( $\hat{e}$ );
  - Separation would not be sustainable because signal is uninformative.
- If the threshold were set equal to  $\tilde{e} < \check{e} < e_1$  (see Fig. 13.C.7 right):
  - A separating equilibrium would be sustainable, but it would be Pareto dominated by the one in which the threshold is set at  $\tilde{e}$ ;
  - This is because  $\theta_H$  workers send a signal more costly than necessary.
  - Note that all other payoffs are unchanged.
- If the threshold were set greater than  $e_1$  (see Fig. 13.C.7 right):
  - $\theta_H$  workers would not send the signal and would mimic  $\theta_L$  workers;
  - The separating equilibrium would not be sustainable.

## Selection of equilibria and refinements

It is outside the goals of this course to discuss the problem of refinements to reduce the multiplicity of equilibria that arise in these models.

For our purposes, arguments like the *intuitive criterion* proposed by Cho and Kreps (1987) are perfectly satisfactory.

Using again Figure 13.C.7 (right), it is reasonable to argue that:

- a  $\theta_L$  worker would never give up more days than  $\tilde{e}$  because she is in any case better at  $e = 0$ ;
- thus, upon observing any signal  $e > \tilde{e}$  a firm may be sure of facing a  $\theta_H$  worker;
- so the only reasonable separating equilibrium has a threshold at  $\tilde{e}$ .
- The equilibrium selected with this criterion Pareto dominates the other separating equilibria.

## Pooling equilibria

Pooling equilibria prevail when, even if signaling is possible, it is not in the interest of anybody to give any signal (see fig. 13.C.9). Thus

$$e^*(\theta_L) = e^*(\theta_H) = e^* \quad (22)$$

Since the distribution of types is common knowledge the equilibrium wage in a pooling equilibrium is

$$W^*(e^*) = \lambda\theta_H + (1 - \lambda)\theta_L = E(\theta) \quad (23)$$

All signals in  $[0, e']$  can be supported.  $e > e'$  cannot be supported because  $\theta_L$  workers would prefer  $w = \theta_L$ .

However, the pooling equilibrium with  $e^* = 0$  Pareto dominates the others. Note that this pooling equilibrium would be observationally identical to the equilibrium prevailing when signaling is banned.

## Pareto welfare criteria and compulsory maternity leaves

Consider figure 13.C.8 (right): it shows an equilibrium in which  $\theta_H$  workers would be better off if signaling were banned.

The intuition is that if signaling is possible,  $\theta_H$  workers are forced to signal otherwise they get lumped together with  $\theta_L$  workers.

In other words they would rather get  $w = E(\theta)$  in the pooling equilibrium as opposed to giving the costly signal in order not to get  $w = \theta_L$ .

This may happen:

- when  $E(\theta)$  is too high for given slope of indifference curve (high  $\lambda$ )
- when the marginal cost of signaling is high for  $\theta_H$  workers given  $E(\theta)$

However it is unlikely that this might be the reason of compulsory maternity leaves.

## Rawls welfare criteria and compulsory maternity leaves

Consider again figure 13.C.8. The Rawlsian social welfare is given by:

$$R = \min\{w(\theta)\} = w^*(e^*(\theta_L)) \quad (24)$$

In the separating equilibrium, we have

$$R_s = \theta_L \quad (25)$$

while in the equilibrium in which signaling is banned (or equivalently in a pooling equilibrium) we have

$$R_p = E(\theta) > \theta_L \quad (26)$$

A Rawlsian objective function may induce the social planner to forbid signaling and impose compulsory maternity leaves

But see also the argument of Aghion and Hermalin(1990).

## What happens in the case of positive reservation wages

As discussed in Section 2.1, suppose the reservation wage grows so that

$$E(\theta) < r = r(\theta_L) = r(\theta_H).$$

If no signaling is allowed or if only pooling equilibria are sustainable, the market may unravel, exactly as in the standard “adverse selection” situation of Akerlof (1970).

Thus we see again the “razor edge” faced by rawlsian institutions which try to:

- maximize the welfare of the least endowed worker,
- at the cost of reaching a Pareto inefficient ex post allocation of workers.

Banning the possibility to signal may lead to a very inefficient outcome.

### 2.3 Screening through probation and upward sloping wage profiles (Lazear (1995))

We now consider a model of screening in which, since the worker's type is not observable, the firm offers a contract such that only  $\theta_H$  workers apply for a job.

The features of such a contract are:

- workers are hired on probation for an initial period of the relationship;
- only if they pass a test they remain employed in the second period;
- the wage is higher in the second period with respect to the first, even if the productivity of a given worker remains the same in both periods.

We want to see why and under what conditions these contract features make the job attractive only for  $\theta_H$  workers.

We also want to see why governments (or unions) acting according to rawlsian welfare objective functions may try to prevent the possibility of such contracts.

## The setting

Consider again the two types of workers of Section 2.2.

$\theta_H = \textit{hate}$  children so desire zero of them (with prob.  $\lambda$ );

$\theta_L = \textit{love}$  children so desire at least one of them (with prob.  $1 - \lambda$ ).

The firm faces young workers without children and is interested in avoiding  $\theta_L$  workers because they are expected to be less productive.

An important difference with respect to the setting of Section 2.2 is that the reservation wage (may) differ for the two types of workers:

- $r(\theta_L) = W_L$
- $r(\theta_H) = W_H$
- $W_H > W_L$

We can view  $W_H$  and  $W_L$  as the alternative wages that would prevail in the economy in equilibrium if all firms were able to screen the two types of workers.

A more sophisticated version of this setting is in Guash and Weiss (1981).

## The features of a screening contract

The employment relationship lasts for two periods of equal length for simplicity:

- $W_1$  is the wage offered by the firm in the first period;
- $W_2$  is the wage offered by the firm in the second period;
- in period 1 workers are tested in a way such that
  - $\theta_H$  workers pass the test with probability  $P_H$ ;
  - $\theta_L$  workers pass the test with probability  $P_L < P_H$ .
- We have also to assume that reputation prevents the possibility that firms renege their wage offer in the second period.

The firm wants to set the wage profile and the passing probabilities in a way such that only  $\theta_H$  workers find it optimal to apply.

## Participation constraints for the two types of workers

In order for  $\theta_H$  workers to be willing to apply, the contract parameters must ensure that:

$$W_1 + P_H W_2 + (1 - P_H)W_H \geq 2W_H \quad (27)$$

In order for  $\theta_L$  workers to be un-willing to apply, the contract parameters must ensure that:

$$W_1 + P_L W_2 + (1 - P_L)W_L < 2W_L \quad (28)$$

Note that there is a unique set of wages that satisfies 27 and 28 with equality given  $P_H$  and  $P_L$ .

## A wage profile that does not screen workers

Solving 27 and 28 with equality for  $W_2$ :

$$\bar{W}_2 = \frac{(1 + P_H)W_H - (1 + P_L)W_L}{P_H - P_L} \quad (29)$$

Solving 27 and 28 with equality for  $W_1$ :

$$\bar{W}_1 = \frac{-P_L(1 + P_H)W_H + P_H(1 + P_L)W_L}{P_H - P_L} \quad (30)$$

And the implied wage growth between the two periods would be:

$$\bar{W}_2 - \bar{W}_1 = \frac{(1 + P_H)(1 + P_L)(W_H - W_L)}{P_H - P_L} = X \quad (31)$$

Note however that this profile does not screen workers because both types are indifferent between applying and not applying.

To induce screening a larger spread is needed.

## A wage profile that does screen workers

Consider instead the following wage determination equation:

$$W_1^* = 2W_H - P_H W_2^* - (1 - P_H)W_H \quad (32)$$

If  $W_1$  and  $W_2$  satisfy this relationship (which is equation 27)  $\theta_H$  workers will be indifferent and (we assume) will apply for the job

Substituting 32 in 28, we find how the spread should be set to discourage  $\theta_L$  workers:

$$W_2^* - W_1^* > \frac{(1 + P_H)(1 + P_L)(W_H - W_L)}{P_H - P_L} = \bar{W}_2 - \bar{W}_1 = X \quad (33)$$

Starting from a spread equal to  $X$ , which makes both types indifferent, if the employer raises  $W_2$  and reduces  $W_1$  in a way that satisfies 32,  $\theta_H$  workers remain indifferent.

$\theta_L$  workers, instead, are no longer interested because they lose in period 1 for a gain in period 2 that they will not get with the same probability of  $\theta_H$  workers.

## Why both ingredients are needed

- Solving equation 27 with  $W_1 = W_2 = \tilde{W}$  shows that a constant wage would convince  $\theta_H$  workers to apply only if  $\tilde{W} = W_H$ .
- But equation 27 shows that obviously a constant wage  $\tilde{W} = W_H$  would convince  $\theta_L$  workers to apply independently of the passing probability.
- And with a constant wage lower than  $W_H$ , only  $\theta_L$  workers or no one would apply for the job.
- Similarly, with identical passing probabilities, either both types apply or only the bad type or no one.
- The optimal combination of wage growth and different passing probabilities is such that only for the  $\theta_H$  types it is convenient to “pay a fee” in the first period in order to gain in the second.

## Comparative statics

- Since  $\frac{\partial X}{\partial W_H - W_L} > 0$  the higher is inequality in the outside economy, the larger must be wage growth or the spread in passing probabilities to achieve screening.
- Since  $\frac{\partial X}{\partial P_H} < 0$  the higher the passing probability for  $\theta_H$  workers the lower has to be wage growth.
- On the contrary, since  $\frac{\partial X}{\partial P_L} > 0$  the higher the passing probability for  $\theta_L$  workers the higher has to be wage growth.

## Back to the real world: segregation in careers

- Suppose that in the economy there are two types of firms:
  - \* Primary market firms, interested in screening and implementing the above contract;
  - \* Secondary market firms with flat wage profiles and not interested in screening.
- Firms interested in screening ask newly hired workers to be geographically mobile at the beginning of the employment relationship.
- $\theta_L$  are less likely to pass this kind of "mobility test", for example because they are more attached to a stable partner even if young and still without children.
- In equilibrium,  $\theta_L$  workers do not even apply for primary firms. Only  $\theta_H$  workers apply for primary firms.
- Are there possible testable implications, concerning fertility and careers?
- Can this help to explain gender segregation across careers and jobs?

## Unions' attitudes and other comments

It is evident that a government or a union inspired by a Rawlsian objective function will try to

- prevent the existence of probationary period;
  - \* e.g.: opposition to Temporary Work Agencies;
- reduce the relevance of testing during probation;
  - \* e.g.: shorten the length of probation so that it becomes uninformative and the passing probability tends to 1 for all workers;
- induce flatter wage profiles;
  - \* e.g.: compress returns to seniority;

Independently of union attitudes, note that this model explains why wages may increase with seniority even when productivity is constant (in contrast with the explanations based on human capital acquisition).

## 2.4 Screening through “pay based on performance” (Lazear (1999))

“Pay based on performance” is typically considered as a way to create incentives in a situation of moral hazard, and we will get back to this later in the course.

Following Lazear (1999) I show that firms may adopt this kind of compensation system to screen workers and induce only the best to apply.

Suppose that workers are characterized by an unobservable innate productivity  $\theta$  which is distributed according to  $f(\theta)$  with support  $[\underline{\theta}, \bar{\theta}]$ .

For simplicity assume that the reservation wage is 0 for all workers.

Initially there is only one firm  $A$  offering a constant wage such that:

$$W_0^A = E(\theta) = \int_{\underline{\theta}}^{\bar{\theta}} \theta dF(\theta)$$

This is the standard egalitarian equilibrium which maximizes the Rawlsian objective function but makes skilled workers unhappy ex post.

## The introduction of pay based on performance

Suppose now that it becomes possible to observe individual productivity at a cost  $c$ .

A new firm  $B$  enters the market offering a compensation scheme based on performance. For example

$$W_0^B = \theta - c$$

Firm  $B$  cannot pay more for each  $\theta$  because it would make losses, and cannot pay less because of free entry and the zero profit constraint.

All the workers with  $\theta > E(\theta) + c$  will move to firm  $B$  (see figure).

This will however reduce the average product in firm  $A$ , which therefore has to lower the wage it pays to its remaining workers.

As a result more workers will move to firm  $B$ .

## The equilibrium when “pay based on performance” co-exist with “standard compensation”

Let  $\theta^*$  be the productivity of the worker who is indifferent between staying in firm  $A$  or moving to firm  $B$ . This indifference productivity level is defined by

$$\theta^* - c = E(\theta | \theta < \theta^*) = \frac{1}{F(\theta^*)} \int_{\underline{\theta}}^{\theta^*} \theta dF(\theta) \quad (34)$$

The equilibrium that emerges in period 1 when both firms coexist, has the following characteristics

- Utilitarian welfare may or may not increase

$$\Omega_1 = (\theta^* - c)F(\theta^*) + \int_{\theta^*}^{\bar{\theta}} (\theta - c) d\frac{F(\theta)}{1 - F(\theta^*)} \quad ? \quad E(\theta) = \Omega_0 \quad (35)$$

- Inequality certainly increases
- Rawlsian welfare certainly decreases

$$E(\theta | \theta < \theta^*) < E(\theta)$$

## Comparative statics and implications

Note that if the cost  $c$  of measuring productivity increases,

- the fraction of workers employed in firm  $B$  decreases;
- the wage in firm  $A$  increases and therefore inequality decreases and rawlsian welfare increases.

Unions may oppose the introduction of “pay based on performance” because of its sorting effects.

## 2.5 Probation when monitoring is the only way to screen heterogeneous agents (Ichino and Muehlheusser (2004))

This section is based on Ichino and Muehlheusser (2004): “How often should you open the door? Optimal monitoring to screen heterogeneous agents” .

This paper starts from the observation that the need to test the reliability of potential partners at the beginning of a project characterizes many human relationships.

This need is particularly strong whenever, once the project starts, a separation from unreliable partners becomes increasingly more difficult with time.

It is therefore not surprising that many partnerships feature, either explicitly or implicitly, an initial period of “probation” .

What is perhaps more surprising is that monitoring partners “too much” during probation periods may not be optimal, and in this paper we want to show why.

## Intuition and examples

Only by giving to a potential partner the possibility to mis-behave he might be tempted to do it, and only in this case his type could possibly be revealed when splitting would still be feasible at low cost.

- Engagement before marriage
- Probation at the beginning of labor contracts
- Trading between firms

What are the common features of these examples?

What are the necessary ingredients to conclude that “opening the door too often” at the beginning of a partnership may not be optimal?

## First ingredient: heterogeneity of agents

There must be heterogeneity of agents with respect to the cost of exerting effort.

- Nagin et al. (2002)
- Ichino and Riphahn (2003)

We distinguish between

- “good” agents who are willing to exert effort unconditionally;
- “bad” agents who instead face effort costs and therefore are potential shirkers.

Since a shirking agent yields a negative payoff to the principal, the latter wants to identify bad agents in order to stop the relationship with them.

## Second ingredient: increasing cost of splitting

Splitting from an agent must become more costly for the principal as the length of the relationship increases.

It may seem restrictive but in fact characterizes many long term relationships.

- It may result from institutional arrangement, but does not have to.
- Splitting may become increasingly difficult because of sunk costs or accumulation of match specific capital.

As a consequence of any of these reasons:

- The principal is interested in identifying bad agents as soon as possible.
- Bad agents have an extremely strong incentive to mimic good agents.

## Third ingredient: incomplete screening via contracts

We consider only situations in which it is either impossible or too costly to design menus of contracts capable to “screen” the two types of workers.

This is not particularly restrictive since in any realistic setup, the number of different contracts which can be offered by a principal is “finite” .

Thus, there might still exist some degree of heterogeneity among agents who choose a particular contract.

- extreme case: fixed wage contract.

The fact that probation periods exist in many long term relationships of different nature indicates that it is not so easy to use menus of contracts to screen agents.

## Fourth ingredient: monitoring must be possible

Monitoring of agents must be possible and thus constitutes a feasible method to identify bad agents.

Upon monitoring, the effort choice of an agent becomes “observable” although it remains “non-verifiable” in court.

During probation, splitting has low cost precisely in the sense that there is no need to verify misbehavior in court in order to end the relationship.

## The trade off faced by the principal

Whenever these four conditions are met, the principal faces a trade off concerning the optimal monitoring policy during the probation period.

- Some monitoring is beneficial for the principal because with zero monitoring she would have no chance of detecting a bad agent.
- Too much monitoring is bad because it induces more bad types to exert effort during probation, preventing their identification.

Note that even if monitoring is costless, the principal has an incentive to choose a relatively small monitoring frequency.

## Related literature

Wang and Weiss (1998): “Probation layoffs, and wage tenure: a sorting explanation”

- A commitment to “excessive monitoring” during probation allows firms to deter applications of low productivity workers.
- Our story continues to apply also in this case whenever some residual heterogeneity is left after hiring.

Dubey and Wu (2001): “More monitoring induces less effort”

- In a tournament, if the level of monitoring is high the weakest agent will think to have no chance and will exert less effort. By a “domino effect” all other agents will exert less effort.
- No need of strategic interaction between players in our paper.
- Our claim is that less monitoring induces more shirking which in turn allows for better screening, even with a single agent.

## Related literature

Cowen and Glazer 1996: “Competitive Prizes: When less scrutiny induce more effort”

- When output is stochastic and there is only one chance to pass a threshold, agents exert more effort than if there were more chances.
- A principal has an incentive to “get a less accurate picture” if this spurs effort.
- As in our paper, this effect is at work even if monitoring is costless.
- Contrary to our paper, less monitoring induces more effort.
- In our paper, less monitoring serves the purpose of improving the selection of applicants: it induces more shirking and therefore allows to identify and fire bad agents.

## The model

- One principal and  $N \geq 1$  agents.
- Production takes place in two periods  $i = 1, 2$ : probation and after.
- In each period, agents can choose an action from  $\{E, S\}$   
 $E$  denotes “exerting effort”;  
 $S$  denotes “shirking”.
- A shirking agent produces 0.
- An agent exerting effort yields an output  $v_i$  in period  $i$ .
- The output is negligible for all agents during probation:  $v_1 = 0$ .
- Period 2 can be of any length:  $v_2 > 0$  is NPV of output in period 2.

## Heterogeneity of agents

- Agents differ with respect to the (privately known) cost of exerting effort. Their type is denoted by  $\theta \in \{G, B\}$ :
  - “bad types  $B$ ” have effort costs  $c_1 = c$  and  $c_2 = c \cdot k$  with  $k > 0$ .
  - $k$  is a factor that adjusts effort costs w.r.t the length of period 2.
  - $c$  is drawn from  $H(c) \in C^2$  with support  $[0,1]$  before the game is played.
  - “Good types  $G$ ” do not face any costs of exerting effort.
- The share of good types  $0 < \alpha < 1$ .
- Each agent privately learns his type at the beginning of the game.
- $\alpha$  and  $H(\cdot)$  are common knowledge.

## Monitoring during probation

During probation the principal can monitor each agent at no cost. His choice variable is a probability of monitoring  $q \in [0, 1]$ .

The outcome of monitoring is  $M \in \{E, S\}$  which reveals shirking perfectly.

After observing the outcome and updating his beliefs, the principal makes a firing decision  $F \in \{0, 1\}$ , where  $F = 1$  means firing.

Firing cost is zero during probation and prohibitively high afterwards.

The population out of which the  $N$  agents are drawn is large. Upon monitoring one agent no inference can be made on the remaining  $N - 1$ .

# Payments

During probation the agent receives  $t_1$ , which can be set equal to 0.

If the worker is hired at the end of probation, he is entitled to a transfer  $t_2 > t_1$  independent of performance and such that:

**Assumption 1.**  $v_2 > t_2 > 1$

- In period 2 the payoff for the principal from an agent exerting effort is positive ( $v_2 > t_2$ ).
- $t_2 > 1$  is needed otherwise, exerting effort would not be privately optimal for all  $c$  in period P, if the agent is monitored with certainty.

The reason and nature of these assumptions will become clearer in the sequel.

## Firing decision

The principal wishes to continue with an agent when his belief after monitoring is greater or equal to the prior  $\alpha$ .

This implies that the following assumption must hold:

**Assumption 2.**  $\alpha \cdot (v_2 - t_2) + (1 - \alpha) \cdot (-t_2) > 0$

## The stages of the game

- At stage 0, each agent's type is determined by a nature's move and only known to the agent.
- At stage 1, the principal sets and commits to a monitoring probability  $q$  for the probation period.
- At stage 2, each agent independently decides on whether or not to exert effort. After the effort choice is made, each agent is monitored with probability  $q$ .
- At stage 3, given the outcome of the monitoring procedure, the principal decides on which agents to fire. After the firing decision period 1 ends.
- At stage 4, in period 2, all remaining agents again decide on whether or not to exert effort. Then the game ends.

## The behaviour of agents in stage 4

Denote with  $a_i^\theta \in \{E, S\}$  the action chosen by type  $\theta \in \{B, G\}$  in period  $i = 1, 2$ .

A good type has no effort costs and is indifferent between exerting effort and shirking in period 2 because he gets  $t_2$  in both cases:

- will always exert effort:  $a_2^{G*} = E$ .

A bad type gets  $(t_2 - c)$  from choosing  $E$  and  $t_2$  from choosing  $S$  so:

- will always shirk:  $a_2^{B*}(c) \equiv S \forall c$ .

## The firing decision of the principal in stage 3

Denote by  $\beta \in [0, 1]$  the belief of facing a good type:

$$\beta := Pr(\theta = G \mid M) \quad (36)$$

to be consistently derived using Bayes' rule from stage 2.

The principal's expected payoff from an agent for period 2 given  $\beta$  is

$$\beta(v_2 - t_2) + (1 - \beta)(-t_2) \quad (37)$$

which may be positive or negative.

The principal will fire an agent, whenever monitoring “delivers” a belief that this agent is good which is low enough to make the expected payoff negative:

$$F^*(\beta) = \begin{cases} 1 & \text{if } \beta < \frac{t_2}{v_2} \\ 0 & \text{otherwise} \end{cases} \quad (38)$$

## The optimal effort decision of the agents in stage 2

**Lemma 1.** *At stage 2, for all  $q < \bar{q} := \frac{1}{t_2}$ , there exists a unique equilibrium continuation in which*

- a) each good type chooses  $a_1^{G*} = E$  independent of  $q$ ;*
- b) each bad type shirks whenever his realization of  $c$  is sufficiently high.*

*This happens with probability  $(1 - e(q)) > 0$ ;*

- c) the detection of shirking leads to the following beliefs*

$$\beta^* = Pr(\theta = G \mid M = E) = \frac{\alpha}{\alpha + (1 - \alpha)e(q)} > \alpha \quad (39)$$

$$\beta^* = Pr(\theta = G \mid M = S) = 0 \quad (40)$$

*and the principal optimally fires all agents for which  $M = S$  holds and thus keeps all other agents (including those who have not been monitored).*

## Intuition for Lemma 1: good types in stage 2

On the equilibrium path, when choosing  $E$ , he gets  $t_1$  in period 1, and  $t_2$  in period 2.

- If he is monitored,
  - $M = E$  and principal holds the belief  $\beta^* = \frac{\alpha}{\alpha + (1-\alpha)e(q)} > \alpha$
  - by Assumption 2,  $F = 0$  is optimal, and he is not fired.
- If he is not monitored,
  - the principal holds belief  $\beta^* = \alpha$  and he is not fired either.

On the equilibrium path when choosing  $S$  he is monitored with probability  $q$  detected of shirking and, given belief  $\beta^* = Pr(\theta = G \mid M = S) = 0$ , fired.

So, in case of shirking he gets  $t_1$  in period 1 and only  $(1 - q)t_2$  in period 2.

Therefore, it is never profitable for a good type to shirk.

## Intuition for Lemma 1: bad types in stage 2

When choosing  $E$ , he gets  $(t_1 - c)$  in period 1.

- When monitoring occurs, he is taken to be a good type.
- He gets  $t_2$  in period 2 and will then shirk in order not to pay effort costs.

When choosing  $S$ , he gets  $t_1$  in period 1 (thus saving on effort costs  $c$ ), but

- with probability  $q$  he is detected of shirking and fired,
- so his expected payoff for period 2 is only  $(1 - q)t_2$ .

It follows that  $S$  is preferred iff

$$t_1 - c + t_2 < t_1 + (1 - q)t_2 \Leftrightarrow c > qt_2 \quad (41)$$

so that the optimal decision of a bad type is:

$$a_1^{B*} = \begin{cases} S & \text{if } c > qt_2 \\ E & \text{otherwise} \end{cases} \quad (42)$$

i.e. shirking occurs whenever the cost of effort is sufficiently high.

## Uniqueness of equilibrium continuation

For  $q \geq \bar{q} := \frac{1}{t_2}$ , all bad types choose  $E$  independently of their cost  $c$ :

- Shirking would no longer occur on the equilibrium path.
- No information transmission would take place and  $\beta^* = Pr(\theta = G \mid M = E) = \alpha$  would hold allowing for many pooling equilibria.

However, we will show that the principal chooses some  $q < \bar{q}$  so that both actions,  $E$  and  $S$ , occur with positive probability on the equilibrium path.

Therefore, there is no leeway in forming off-equilibrium beliefs and so this equilibrium continuation is indeed unique.

The principal's belief conditional on  $M$  is consistent with the equilibrium strategies of both types.

## The Principal's Optimal Choice of $q$

Good types are irrelevant for the choice of  $q$ .

- In period 1, none of them shirks and they yield a payoff  $(-t_1) < 0$ .
- In period 2, none of them shirks and they yield a payoff  $(v_2 - t_2) > 0$ .

For bad types there is a trade off in setting  $q$ .

- A bad type will shirk in period 1 whenever  $c > qt_2$ .
- The probabilities of shirking and exerting effort are

$$s(q) := Pr(c > qt_2) = \max(0, 1 - H(qt_2)) \quad (43)$$

$$e(q) := Pr(c \leq qt_2) = \min(H(qt_2), 1) \quad (44)$$

- Hence  $\frac{ds}{dq} \leq 0$  and  $\frac{de}{dq} \geq 0$  and  $s'(q) = -e'(q)$ .

## The terms of the trade off in setting $q$

In period 1 the expected payoff generated by bad types is independent of  $q$ :

$$\begin{aligned}\pi_1(q) &:= (1 - \alpha) \cdot N \cdot (e(q) \cdot (-t_1) + s(q) \cdot (-t_1)) \\ &= (1 - \alpha) \cdot N \cdot (-t_1)\end{aligned}\tag{45}$$

The principal monitors in period 1 not to increase output in this period but to detect as many bad types as possible.

Precisely for this reason  $q$  must be set in a way such that:

- it induces some shirking in period 1, otherwise no bad type would be detectable;
- it does not induce too many bad types to mimic good types.

The optimal  $q$  is the one that minimizes the loss due to bad types in period 2, which depends on how many of them are detected:

$$\pi_2(q) := (1 - \alpha) \cdot N \cdot ((1 - q) \cdot s(q) + e(q)) \cdot (-t_2)\tag{46}$$

## The optimal monitoring probability $q^*$

**Proposition 1.** *Given equilibrium continuation 1, the optimal monitoring frequency for the principal induces shirking on the equilibrium path, i.e.  $q^* < \bar{q} := \frac{1}{t_2} < 1$ .*

Intuitively, although monitoring is costless,

- There is an incentive not to set the monitoring frequency too high as this would induce fewer bad types to shirk in period 1.
- Thus, it would be harder to filter out bad types.
- Given  $q^* < \bar{q}$ , the behavior determined in the equilibrium continuation is also optimal and the equilibrium is unique.

This result is robust to generalizations. For example:

- A positive value of production in 1:  $v_1 > 0$ .
- An incentive scheme to elicit effort from bad types in 2.

## Proof

Since the objective function of the principal is continuous in  $[0, \bar{q})$ ,

1. show that the expected payoff of the principal is strictly increasing at  $q = 0$  and strictly decreasing as  $q \rightarrow \bar{q}$ .
2. show that the absolute expected profit level is also higher at  $q = q^*$  than at  $q = 0$  and when  $q$  approaches  $\bar{q}$  (the payoff function of the principal is flat for all  $q \geq \bar{q}$ ).

To do this, define

$$\begin{aligned} Z(q) &:= [(1 - q)s(q) + e(q)](-t_2) = [(1 - q)(1 - e(q)) + e(q)](-t_2) \\ &= [1 - q + qe(q)](-t_2) \end{aligned} \tag{47}$$

where

$$Z'(q) = [-1 + e(q) + qe'(q)](-t_2). \tag{48}$$

## Proof (continued)

Recall that the expected payoff from the good agents and from the bad agents in period 1, respectively, is independent of  $q$ .

Furthermore, from (46),  $\pi_2(q) = (1 - \alpha) \cdot N \cdot Z(q)$  so that  $q^*$  is uniquely determined by  $Z(q)$ .

For step 1 we need to show that  $Z'(q = 0) > 0$  and  $Z'(q \rightarrow \frac{1}{t_2}) < 0$ :

$$\begin{aligned} Z'(q = 0) &= t_2 > 0 \\ Z'(q \rightarrow \frac{1}{t_2}) &= (-t_2) \cdot [-1 + \frac{1}{t_2} \cdot e'(\frac{1}{t_2}) + 1] = -e'(\frac{1}{t_2}) < 0 \end{aligned}$$

For step 2 note that we have

$$\begin{aligned} Z(0) &= Z(\frac{1}{t_2}) = -t_2 \\ Z(q^*) &= (1 - q^* + q^* \cdot e(q^*))(-t_2) \end{aligned}$$

## Proof (continued)

and thus

$$\begin{aligned} Z(q^*) - Z(0) > 0 &\Leftrightarrow \\ (1 - q^* + q^* \cdot e(q^*))(-t_2) > (-t_2) &\Leftrightarrow \\ e(q^*) < 1 \end{aligned}$$

which is true for all  $q^* < \bar{q}$ .

So, the principal's payoff is strictly higher when an interior level of  $q$  is chosen.

## Potential magnitude of the gain from setting $q$ optimally

We want the % gain in profits from monitoring with probability  $q = q^*$  compared to monitoring excessively ( $q \rightarrow \bar{q}$ ) or refraining from monitoring at all ( $q = 0$ ).

Denote by  $d(q)$  the probability that a bad type remains in period 2:

$$d(q) := (1 - q) \cdot s(q) + e(q) = 1 - q \cdot s(q) \quad (49)$$

which reflects the fact that a bad agent can only be fired if he shirks in period 1 and is detected through monitoring.

Note that  $d(q = 0) = 1 = d(q \geq \bar{q})$  which implies that *all* bad agents remain in period 2 and the principal's payoff in either of these two cases is

$$\pi_2(q = 0) = \pi_2(q \rightarrow \bar{q}) = (1 - \alpha) \cdot N \cdot (-t_2). \quad (50)$$

## Potential magnitude of the gain (continued)

When  $q^*$  is chosen, instead, the probability that a bad type remains in period 2 is strictly less than 1, i.e.  $d(q^*) = 1 - q^* \cdot s(q^*) < 1$ .

Thus the principal is able to filter out some bad types and her payoff is

$$\pi_2(q = q^*) = (1 - \alpha) \cdot N \cdot (1 - q^* \cdot s(q^*)) \cdot (-t_2). \quad (51)$$

By taking the difference between (51) and (50), her absolute gain from choosing an “interior” monitoring frequency  $q^*$  is:

$$\begin{aligned} \Delta\pi &:= \pi_2(q = q^*) - \pi_2(q = 0) \\ &= (1 - \alpha) \cdot N \cdot (-q^* \cdot s(q^*)) \cdot (-t_2) > 0 \end{aligned} \quad (52)$$

Relative to the payoff obtained in the “corner” solutions given by Eqn. (50), the percentage increase in profits can thus be calculated as

$$\frac{\Delta\pi}{|\pi_2(q = 0)|} = q^* \cdot s(q^*). \quad (53)$$

## Potential magnitude of the gain (continued)

Consider the case where  $c$  is uniformly distributed, i.e.  $H(c) = c$ .

Using (44), we then have  $e(q) = \min(qt_2, 1)$  and maximizing  $\pi_2(q)$  in Eqn. (46) with respect to  $q$  yields  $q^* = \frac{1}{2t_2}$  and thus  $e(q^*) = s(q^*) = \frac{1}{2}$ .

Therefore, from Eqn. (53) the percentage gain is given by  $\frac{1}{4t_2} = \frac{1}{4}\bar{q}$ .

Not only is it never optimal to monitor too much during probation, but:

- When  $\bar{q}$  is low because the value of employment is high, probation is not a very effective device to filter out bad agents.
- When  $\bar{q}$  is high because the cost of effort in period 1 is high compared to the benefit from employment in period 2, probation is effective.
- Even in this second case, however, it would never be optimal to set the monitoring probability  $q^*$  above  $\bar{q}$ .

## Conclusions and extensions

Monitoring too much a partner in the initial phase of a relationship may not be optimal if the goal is to determine her loyalty to the match and if the cost of terminating the relationship increases over time.

If too much monitoring induces the partner to behave well even if her inclination in the absence of monitoring would be to mis-behave, the principal does not learn what needs to be learned at the beginning of a relationship.

This general intuition applies to many long term social relationships independently of monitoring costs.

Relationships in which too much monitoring takes place at the beginning should perform worse at later stages of their development.

We are searching for empirical evidence on this prediction of our model.

### 3 The development of an employment relationship

So far we have discussed how the existence of asymmetric information on the type of workers complicates the beginning of an employment relationship.

We now consider how the employment relationship is configured and develops assuming that it has somehow started.

The employment relationship is characterized by two crucial parameters

- the compensation offered by the employers in exchange for
- the effort exerted by the employee.

It is a simplification, but it nevertheless allows to capture many important aspects of real employment relationships.

Asymmetric information still complicates matters because effort is a choice of the worker and may or may not be observed by the employer.

## Conflicting interests in the employment relationship

We know from Ichino and Muehelheusser that it is unlikely that the employer might have hired only “good” types (i.e. workers with zero cost of effort).

Inasmuch as effort is costly for some workers there is a potential conflict of interests between the two partners of the relationship.

In this section, we discuss what employers can do to induce workers to exert effort, and to what extent the adopted solutions are socially optimal.

The instruments at the employers' disposal fall in the following categories:

- compensation based on performance
- tournaments and promotions
- efficiency wages
- career concerns
- relational contracts

### 3.1 The canonical principal-agent model (Gibbons and Waldman (1999))

The classic framework to study the relation between compensation and incentives is the agency model developed by Mirrlees (1974,1976), Holmstrom (1979) and Shavell (1979).

The agent chooses an effort level  $e$  but incurs an increasing and convex cost of effort  $c(e)$ , with  $c' > 0$  and  $c'' > 0$ .

There is noise in the way effort generates output  $Y$  for the principal:

$$Y = e + \varepsilon \tag{54}$$

Thus the principal can only observe output but not effort.

The noise component  $\varepsilon$  is distributed normally with mean 0 and variance  $\sigma^2$ .

## The trade off between insurance and incentives

The contract between the principal and the agent specifies a linear relationship between the wage and output

$$W(Y) = s + bY \quad (55)$$

Intuitively:

- To provide first best incentives:
  - \* the principal can sell the firm to the agent for a fixed fee but in this way she would offer no insurance to the agent.
- To provide full insurance:
  - \* the principal can pay a fixed wage to the agent but in this way she would offer no incentive to the agent.

The second best must lie somewhere in between, balancing the goals of full insurance and first best incentives.

Risk aversion plays the role of “Rawlsian” concerns in Section 2.1.

## Steps for the solution of the problem

The model is solved in the standard way.

The principal maximizes profits subject to two constraints determined by the behaviour of the agent:

- the participation constraint: the agent must be willing to sign the contract;
- the incentive constraint: the agent chooses optimally his action given incentives.

Hence the solution proceeds in three steps:

1. for given parameters of the compensation schedule, solve the agent's problem;
2. derive the participation and incentive constraints;
3. solve the principal's problem deriving the optimal parameters of the compensation schedule.

## The problem of the agent

Suppose that the utility of the agent is

$$U(W, e) = -\exp\{-r(W - c(e))\} \quad (56)$$

where  $r \geq 0$  is the agent's coefficient of absolute risk aversion.

Let  $U_o = -1$  be the reservation utility of the agent, determined by outside options. The normalization to  $-1$  is innocuous.

The combination of an exponential (CARA) utility and a normally distributed random shock is called "Normal-Exponential model" and simplifies the analysis (Holmstrom and Milgrom (1991)).

## The incentive and the participation constraint

Substituting 55 and 54 in the utility function the maximization problem of the agent consists in maximizing expected utility with respect to  $e$ :

$$\text{Max} \int_{\varepsilon} -\exp\{-r(s + b(e + \varepsilon) - c(e))\} dF(\varepsilon) \quad (57)$$

and the optimal effort is  $e^*(b)$  which solves the f.o.c.

$$b = c'(e) \quad (58)$$

This is also the *incentive constraint* which says that the optimal amount of effort equates the marginal cost to the bonus parameter of the wage schedule (i.e. the marginal benefit of effort).

In addition to choosing effort optimally, the agent will not accept a contract unless it provides a utility level higher than the reservation.

Thus the *participation constraint* is

$$E(U) \geq U_0 = -1 \quad (59)$$

## The problem of the principal

The principal chooses the parameters of the compensation schedule to maximize expected profit given:

- the optimal choice of effort of the agent;
- the participation constraint of the agent.

Profit is defined as:

$$\Pi = Y - s - bY = (1 - b)(e + \varepsilon) - s \quad (60)$$

Since  $E(\varepsilon) = 0$ , the principal solves the following problem:

$$\text{Max } E(\Pi) = (1 - b)e - s \quad (61)$$

with respect to  $b$  and  $s$  subject to

$$e = e^*(b) \quad (62)$$

$$E(U) \geq U_0 = -1 \quad (63)$$

## A useful result for the “Exponential-Normal” model

The following integral

$$\int -e^{-ax} f(x) dx \quad (64)$$

where

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \quad (65)$$

can be computed as

$$\int -\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{2\sigma^2 ax + x^2}{2\sigma^2}} dx = \quad (66)$$

Dividing and multiplying appropriately, the exponent of  $e$  can be squared:

$$-e^{-\frac{(\sigma^2 a)^2}{2\sigma^2}} \int \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{2\sigma^2 ax + x^2 + (\sigma^2 a)^2}{2\sigma^2}} dx = \quad (67)$$

Note that under the integral we now have the normal density, and thus:

$$= -e^{-\frac{\sigma^2 a^2}{2}} \quad (68)$$

## Using this result to solve the principal's problem

We can solve for the fixed component of the wage schedule  $s$  applying this result to the participation constraint of the worker:

$$\int_{\varepsilon} -\exp\{-r(s + b(e + \varepsilon) - c(e))\}dF(\varepsilon) = -1 \quad (69)$$

which can be written as

$$-\exp\{-r(s + be - c(e))\} \int_{\varepsilon} \exp\{-rb\varepsilon\}dF(\varepsilon) = -1 \quad (70)$$

Using result 68

$$-\exp\{-r(s + be - c(e)) + \frac{1}{2}r^2b^2\sigma^2\} = -1 \quad (71)$$

and this equality is satisfied when the argument of the exponential is equal to 0, which yields:

$$s = -be + c(e) - \frac{1}{2}rb^2\sigma^2 \quad (72)$$

## Solving for the optimal bonus parameter

Substituting 72 in 61 the principal's problem reduces to maximizing with respect to  $b$  the following expression:

$$\text{Max } E(\Pi) = e^*(b) - c(e^*(b)) - \frac{1}{2}rb^2\sigma^2 \quad (73)$$

with the following f.o.c.:

$$e^{*'} - c'e^{*'} - rb\sigma^2 = 0$$

Note that from the agent's problem  $c' = b$  and  $e^{*'} = \frac{1}{c''}$ , which allows to solve for the optimal  $b^*$ :

$$b^* = \frac{1}{1 + r\sigma^2 c''}$$

which can be substituted in 72 to obtain the optimal fixed component of the wage schedule

$$s^* = -b^*e^* + c(e^*) - \frac{1}{2}r(b^*)^2\sigma^2 \quad (74)$$

## The efficient bonus parameter

The expression for the efficient bonus parameter is intuitive:

$$b^* = \frac{1}{1 + r\sigma^2 c''} \leq 1 \quad (75)$$

- If the agent is risk neutral ( $r = 0$ ), the efficient bonus is  $b^* = 1$ :
  - the principal sells the firm to the agent for a fixed fee  $F = s_{b^*=1}^*$ ;
  - the agent has first best incentives.
- If the agent is risk averse ( $r > 0$ ), the efficient bonus is  $b^* < 1$  and decreases:
  - the higher is the variance of output noise;
  - the higher is the rate at which the marginal cost of effort increases.

Is this a good characterization of the wage contracts that we see in reality?

It can be argued that in reality we see less “pay based on performance” (i.e.  $b^*$  much lower than 1) than we would expect in the light of this model.

If this is the case (which is an open question) what could be the explanation?

Can the model explain why we do not see very often  $b^* = 1$  ?

- Can risk aversion be high enough to explain why we often see  $b^* = 0$ .
- $b^*$  could be less than 1 because profit is just a fraction of output.
- The linear contract is too simple: we see kinky wage schedules, partly dependent on output.
- Psychologists say that monetary incentives may reduce workers' motivation.

Other reasons require more structure and are examined in the next sections:

- Individual output is measured imperfectly.
- It is less costly to measure input than output.
- When more agents are involved, only joint output may be observed.
- When more agents are involved relative performance evaluation may be more effective.
- Also a wage independent of output may motivate workers: efficiency wages.
- The principal has other (non-wage-based) tools to motivate workers.

### 3.2 Imperfect measurement of performance (Baker (1992))

A first reason why the canonical Principal-agent model is “too” simplistic is that the principal objective is typically not the performance indicator on which compensation can be made consistent.

The management and economic literature is full of examples of incentive schemes that end up inducing workers to take actions which do not correspond exactly to what the principal was hoping for.

In many cases, the real objective of the principal is simply not measurable.

This intuition was first proposed by Kerr(1975) who titled his article: “On the folly of rewarding A while hoping for B” .

Only much later, it was formalized by Baker (1992), which we follow here.

## Principal's objective and measurable performance indicator

The principal's objective is defined as the value of output minus the cost of all factors of production except for the agent's compensation.

This value, denoted by  $V(e, \varepsilon)$ , is not contractible and depends on:

- the agents action  $e$ ;
- a random shock  $\varepsilon$  that characterizes the state of the world.

A contract can instead be written on a performance measure  $P(e, \varepsilon)$  which also depends on the agents effort and on the state of the world.

The Principal can only offer a (linear) compensation schedule like:

$$W = s + bP(e, \varepsilon) \quad (76)$$

The imperfect correspondence between  $V$  and  $P$  generates incentive schemes that may be too “strong”, too “weak” or in any case distorted, even with risk neutral agents.

## Timing and information

- Neither the Principal nor the agent know  $\varepsilon$  before signing the contract.
- After signing the contract and before choosing an action, the Agent sees  $\varepsilon$  while the Principal does not.
- Therefore, in this sense the agent has superior information.
- Both parties cannot renege after signing
- Both marginal products  $V_e(e, \varepsilon)$  and  $P_e(e, \varepsilon)$  change with the state of the world, and, as effort, are therefore random variables for the Principal.
- The standard deviation of  $V_e(e, \varepsilon)$  is denoted by  $\sigma_{v_e}$  and measures the amount of valuable information that the agent has:
  - if  $\sigma_{v_e}$  is low, the agent cannot affect  $V$  very much;
  - note that marginal productivity can even be high, but if  $\sigma_{v_e}$  is low, how the agent reacts to the shock does not matter much.
- Similarly for  $\sigma_{p_e}$ .

## A real life example to think about this problem

The principal is the manager of an hospital who wants to design an incentive schemes for doctors in the maternity division.

$V$  is the satisfaction of families who come to the hospital for delivery. A possible imperfect measure of performance,  $P$ , is the number of healthy babies delivered in the hospital.

Cesarean sections require less effort from doctors, but may be necessary in cases of child stress. Nevertheless, families are unhappy when a non-necessary C-section is performed.

Child stress is the unpredictable state of the world  $\varepsilon$  (for example measured by the child heart beat).

The doctor, but not the principal, sees the heart beat in the delivery room when decisions have to be taken.

## Some additional assumptions

With no loss of generality we normalize the performance measure so that

$$E(P_e(e, \varepsilon)) = E(V_e(e, \varepsilon)) \quad (77)$$

This is needed if we want to compare the bonus parameter in this setting with the one of the canonical Principal agent model.

Effort is not observable and has a cost  $c(e)$  with  $c' > 0$  and  $c'' > 0$ .

The Agent is risk neutral, has reservation utility  $U_0$  and expected utility

$$E(U(e)) = E(s + bP(e, \varepsilon) - c(e)) \quad (78)$$

The assumption of risk neutrality is to show that deviations from the first best  $b^* = 1$  are possible even in the absence of risk aversion.

Also the Principal is risk neutral and has expected profit

$$E(\Pi(s, b)) = E(V(e, \varepsilon) - s - bP(e, \varepsilon)) \quad (79)$$

## The Agent's problem: incentive and participation constraints

The agent maximizes with respect to  $e$ :

$$\text{Max } E(U(W, e)) = E(s + bP(e, \varepsilon) - c(e)) \quad (80)$$

and the optimal effort is  $e^*(b)$  which solves the f.o.c.

$$bP_e(e, \varepsilon) = c'(e) \quad (81)$$

This is also the *incentive constraint* which says that the optimal amount of effort equates the marginal cost to the bonus parameter of the wage schedule (i.e. the marginal benefit of effort).

The *participation constraint* is instead

$$E(U(e)) = E(s + bP(e, \varepsilon) - c(e)) \geq U_0 \quad (82)$$

## The Principal's problem

The principal maximizes expected profit with respect to the  $s$  and  $b$

$$\text{Max } E(\Pi(s, b) = E(V(e, \varepsilon) - s - bP(e, \varepsilon)) \quad (83)$$

subject to the incentive and participation constraints 81 and 82.

Substituting the two constraints in the maximand yields:

$$\text{Max } E(V(e^*, \varepsilon) - U_0 + bP(e^*, \varepsilon) - c(e^*) - bP(e^*, \varepsilon)) \quad (84)$$

which can be maximized with respect to  $b$  only, keeping in mind that  $e^* = e^*(b)$ .

The f.o.c. for this problem is

$$E(V_e e_b^*) - E(c' e_b^*) = 0 \quad (85)$$

where  $V_e$  and  $e_b^*$  are partial derivatives.

Substituting  $c'$  from the incentive constraint  $c'(e) = bP_e(e, \varepsilon)$  we get  $b^*$

$$b^* = \frac{E(V_e e_b^*)}{E(P_e e_b^*)}$$

## Comparison with the canonical result with risk neutrality

$$b^* = \frac{E(V_e e_b^*)}{E(P_e e_b^*)} \quad (86)$$

Note first that if the agent chooses effort without knowing the state of the world  $\varepsilon$ , then  $e^*(b)$  is independent of  $\varepsilon$  and simplifies out of 86.

Thus, in this case the optimal bonus parameter would be  $b^* = 1$  as in the canonical case when there is no risk aversion.

Hence, what matters is not only that  $V \neq P$ , but also the superior information of the agents who knows the state of the world before choosing on effort.

This captures the idea that if measurement of performance is imperfect the agent can “game” the incentive scheme.

If the agent cannot react to the state of the world it means that it cannot “game” the system.

## Steps to interpret the result

Differentiation with respect to  $b$  of the incentive constraints allows to solve for  $e_b^*$ .

$$P_e + bP_{ee}e_b^* = c''e_b^*$$

and rearranging

$$e_b^* = \frac{P_e}{c'' - bP_{ee}} \quad (87)$$

Substituting 87 in the expression for the optimal bonus 86:

$$b^* = \frac{E\left(V_e \frac{P_e}{c'' - bP_{ee}}\right)}{E\left(P_e \frac{P_e}{c'' - bP_{ee}}\right)} \quad (88)$$

Assuming constant second derivatives (or taking a second order Taylor expansion for  $C$  and  $P$ ) we obtain:

$$b^* = \frac{E(P_e V_e)}{E(P_e^2)} \quad (89)$$

## Further steps to interpret the result

Normalize  $E(P_e) = E(V_e) = 1$  at  $e^*$  and recall that  $E(Y \cdot X) = E(Y) \cdot E(X) + \text{cov}(Y, X)$  the optimal bonus can be written as:

$$b^* = \frac{1 + \text{cov}(P_e, V_e)}{1 + \text{var}(P_e)} \quad (90)$$

Using the following notation:

- $\sigma_{v_e}^2 = \text{var}(V_e)$
- $\sigma_{p_e}^2 = \text{var}(P_e)$
- $\sigma_{v_e p_e} = \text{cov}(V_e, P_e)$
- $\rho = \frac{\sigma_{v_e p_e}}{\sigma_{v_e} \sigma_{p_e}}$

we obtain

$$b^* = \frac{1 + \rho \sigma_{v_e} \sigma_{p_e}}{1 + \sigma_{p_e}^2}$$

which is the coefficient of the regression of  $V_e$  on  $P_e$  without intercept.

## Comments on the result

$$b^* = \frac{1 + \rho\sigma_{v_e}\sigma_{p_e}}{1 + \sigma_{p_e}^2} \quad (91)$$

- If  $\sigma_{v_e} = \sigma_{p_e}$  and  $\rho = 1$  than  $b^* = 1$  which is the canonical result with risk neutrality.
- Note that  $V$  and  $P$  can still differ, but what matters is that  $V_e$  and  $P_e$  are perfectly correlated.
- When this happens the agent cannot game the incentive scheme by exploiting the difference between  $V$  and  $P$ . What she does for  $P$  is good for  $V$ .
- All else equal, the higher the correlation between  $V_e$  and  $P_e$  the higher is the bonus parameter
- It seems counter intuitive, but the principal wants  $b^* > 0$  even if  $\rho < 0$ . This as long as the marginal product of effort is positive on average.

## Aversion to income uncertainty or aversion to effort uncertainty

Note that  $b^* = 1$  is not only not in the interest of the principal. It is also socially inefficient.

Why first best incentives are socially inefficient given that agents are risk neutral?

Because agents are risk neutral with respect to income variations but risk averse with respect to effort variations given the convexity of effort costs.

By setting  $b^* = 1$ , the principal would induce too much variability in effort choices for given variability of the state of the world.

Effort fluctuations are costly for the agent and it is socially efficient to reduce them inasmuch as they do not increase average output.

Effort fluctuations would increase average output only when  $\rho = 1$ .

## What happens if effort is observable

In this case the principal can offer a contract that conditions on effort, and this would be first best efficient in the canonical case.

Interestingly the optimal contract is more complicated in Baker's setting.

Even if the Principal can observe effort, she does not observe the shock.

Therefore she cannot write an optimal contract for each level of effort without knowing the value of the shock

Still effort observability enlarges the set of possible contracts.

## A contract with observable effort and superior agent's information

Consider the wage offer

$$W = s + b_1 E(P | e) + b_2 (P - E(P | e)) \quad (92)$$

This contract has two incentive components

- $b_1$  induces the agent to increase the average (over the state of the world) level of  $P$  given effort
- $b_2$  induces the agent to adjust effort to the specific realization of the state of the world, inasmuch as this differs from the average

The optimal bonus parameters are (see paper):

$$b_1^* = \frac{E(V_e)}{E(P_e)} = 1 \quad (93)$$

$$b_2^* = \rho \frac{\sigma_{v_e p_e}}{\sigma_{p_e}^2} \quad (94)$$

## Why is this contract more efficient than one that forces effort?

When  $\rho \neq 0$  this contract is superior to a “forcing” contract which conditions on effort.

A contract that conditions just on effort forces the agent to exert effort in a way such that  $c' = E(V_e)$ . This is what would happen with  $b_2 = 0$ .

But this induces the agent just to do the best on average, not to exploit her superior information.

The second component induces instead the agent to use her superior information to improve outcomes with respect to average, once she gets to know the shock.

More generally this paper suggests that:

- There are other reasons, in addition to aversion to income risk, which explain why pay based on performance is not as frequently observed.
- the canonical principal agent model is too parsimonious.

### 3.3 Pay based on input or output and allocation of responsibilities (Prendergast (2002))

A further complication of the canonical principal agent models derives from the fact that the choice of the appropriate compensation system can be independent of the allocation of responsibilities.

Consider this example from Prendergast (2002).

- A US firm involved in large-scale construction projects around the world.
- The firms hire two managers to work on projects in Canada and in Armenia.
- The company knows about Canada and can tell the manager what to do.
- The company knows nothing about Armenia and the manager has to decide.
- Should the two managers be payed as a function of input or output?

The models seen so far say that where there is more uncertainty and less precise measure of performance (Armenia) we should see more payment on input.

## The tenuous trade-off between risk and incentives

Prendergast (2002) proposes a model suggesting that the Armenian manager should be paid based on output and the Canadian one on input.

More generally he highlights the following complication of the standard problem:

- Input measurement is easy and less costly in certain environments.
- Thus, with more certainty there is no reason to pay on output because the principal can tell the manager what to do exactly and can check that it is done.
- With greater uncertainty more delegation is needed and the impossibility to measure input requires pay based on output.

These considerations suggest the existence of reasons for a positive relationship between “uncertainty” and “pay based on performance”

This intuition is supported by the evidence (see the paper).

## The structure of the model

A principal hires an agent to exert *one* of  $n$  possible tasks.

If the agent chooses action  $i$  applies effort  $e_i$ . Output is given by

$$y_i = e_i + \rho_i \quad (95)$$

where  $\rho_i$  is a shock.

Effort has a cost  $c(e_i)$  for the agent and as usual:  $c' > 0$  and  $c'' > 0$

The distributions of the random shocks  $\rho_i$  are  $\Phi_i$ , with different means  $\bar{\rho}_i$  and common variance  $\sigma^2$  for all  $i$ .

An increase in uncertainty is characterized by an increase in  $\sigma^2$ .

All individuals are risk neutral and the agents' reservation utility is normalized to 0.

## Information and actions' space

Agents know the realizations  $\rho_i$  for all  $i$  before taking actions. The principal knows only the distribution.

The technology is such that only one action can be implemented by the agent.

The principal can collect two pieces of information to design the compensation scheme:

- information on effort  $e_i$  at the cost  $m_e$ ;
- information on output  $y_i$  at the cost  $m_y$ ;
- measuring output is assumed to be more costly:  $m_y > m_e$ .

The principal can take two actions:

- determine (or put a constraint on the) tasks that the agent can undertake;
- design the compensation scheme.

## The first best outcome

Suppose that the agents are indifferent between the various tasks  $i$ .

Since monitoring effort is less costly the principal offers a compensation:

$$w(e_i) = c(e_i) \quad (96)$$

and let the agent free to choose the action.

Because of competition among agents

- they get their reservation utility  $U = w - c = 0$ ;
- they choose the activity with the highest shock  $\rho_i$  which they observe;
- they choose the optimal amount of effort  $e^*$ .

This input based compensation delivers the first best allocation of effort and activity selection, and maximizes social surplus.

An output based compensation would be more costly for the principal.

## What happens if agents have preferences over actions

Suppose that there is one action from which the agent derives a benefit  $B > 0$  while for the other  $n - 1$  actions the benefit is 0.

There is no correlation between  $B_i$  and  $\rho_i$ .

Suppose that the principal offer the input based first best compensation and leaves the agent free to choose the action.

The agent will then undertake the action she prefers, which is not necessarily the optimal one from the principal's viewpoint.

The expected social surplus, which is the principal's payoff given that the agent is squeezed against her reservation utility, is

$$\Omega_1 = \frac{\sum_{i=1}^n \bar{\rho}_i}{n} + e^* - c(e^*) + B - m_e \quad (97)$$

but the principal can do better.

## Assigned actions and input-based contracts

The principal can restrict the actions taken by the agent and offer the same input-based compensation schedule.

Suppose that  $k$  is the action with the highest expected shock  $\bar{\rho}_k$ .

The principal forces the agent to take action  $k$  and offer the compensation  $w(e_k) = c(e_k)$  and the agent accepts. The expected Social surplus is

$$\Omega_2 = \bar{\rho}_k + e^* - c(e^*) + \frac{B}{n} - m_e \quad (98)$$

Note that  $\bar{\rho}_k > \frac{\sum_{i=1}^n \bar{\rho}_i}{n}$  and thus if the agent's benefit  $B$  is small,  $\Omega_2 > \Omega_1$ .

Hence, the principal prefers to restrict the action of the agent, and once the action is restricted an input-based compensation cost less because  $m_e < m_y$ .

But there is also the alternative to pay based on output without restricting actions.

## Delegated actions and output-based contracts

The purpose of an output-based contract is not only to induce the agent to exert the right amount of effort but also to choose the right kind of action.

Under this contract the agent chooses the activity and the effort that maximizes

$$y_i - c(e_i) + B \quad (99)$$

which is again the payoff of the principal.

This may look like relabeling what effort means but the implications are crucial when uncertainty is considered.

Note that even if  $m_e < m_y$ , only a compensation scheme based on output has a chance to be optimal when actions are delegated to agents.

This because delegating actions with input based pay can be optimal only if agents are indifferent between actions, otherwise they choose what gives them the highest private benefit.

## When delegated actions and output-based pay are better?

When the agent is payed as a function of social surplus, she will choose the action which delivers the highest realization  $\rho_i^*$ . Thus social surplus is

$$\Omega_3 = \rho_i^* + e^* - c(e^*) + \frac{B}{n} - m_y \quad (100)$$

This has a chance to be higher than  $\Omega_2$  only if

$$\rho_i^* - \bar{\rho}_k > m_y - m_e \quad (101)$$

i.e. when the gain by choosing the action with the highest realized shock, as opposed to the action with the highest expected shock, overcomes the increasing cost of paying based on output instead of input.

$\rho_i^*$  is distributed as the “first order statistic” of the distribution  $\Phi_i$  and therefore the difference  $\rho_i^* - \bar{\rho}_k$  grows with  $\sigma^2$ .

If uncertainty increases it is more likely that the principal might find it optimal to delegate actions and to pay based on performance.

### 3.4 Relative performance evaluation (Holmstrom (1979))

Another reason why individual incentive schemes might be inefficient and therefore rarely seen is that shocks to individual output may have two components:

- an idiosyncratic component which is specific to the agent;
- an aggregate common component which affects also other agents.

In these instances the optimal incentive scheme should try to filter out common risk from individual compensation contracts.

The aggregate shock makes output a more noisy signal of the agent's effort.

Removing the common shock from the indicator on which compensation is based has the same effect of a reduction in uncertainty about the state of the world.

Such a reduction allows to raise the bonus parameter of the contract which generates stronger incentives without reducing the agents' welfare.

## Introducing common and idiosyncratic shocks

Consider the following example. The agent's output is given by:

$$Y_i = e_i + \varepsilon_i = e_i + \eta_i + \theta \quad (102)$$

Suppose that the Principal proposes the optimal contract:  $W_i^* = s^* + b^*Y_i$  with

$$b^* = \frac{1}{1 + r\sigma_\varepsilon^2 c''} \leq 1 \quad (103)$$

where:

- $\varepsilon_i$  is the total shock to individual output;
- $\eta_i$  is the idiosyncratic component;
- $\theta$  is the aggregate and common shock component.

and the common and idiosyncratic components are orthogonal, so that

$$\sigma_\varepsilon^2 = \sigma_\eta^2 + \sigma_\theta^2 \quad (104)$$

## Why filtering out common shocks is welfare improving

As shown in a more general framework by Holmstrom (1979), it is intuitive that if the principal observes  $\theta$ , the optimal contract must take the form:

$$\tilde{W}_i = \tilde{s} + \tilde{b}(Y_i - \theta) = \tilde{s} + \tilde{b}(e_i + \eta_i) \quad (105)$$

and in this case the uncertainty component that matters is only  $\eta_i$ , with  $\sigma_\varepsilon^2 > \sigma_\eta^2$ . So the optimal bonus parameter is

$$\tilde{b} = \frac{1}{1 + r\sigma_\eta^2 c''} \geq b^* \quad (106)$$

Note that the agent chooses effort in a way that satisfies the incentive constraint

$$b = c'(e) \quad (107)$$

and therefore, given that effort cost is convex, if:

$$\tilde{b} > b^* \quad \Rightarrow \quad \tilde{e} > e^* \quad (108)$$

The observation of  $\theta$  allows the principal to generate stronger incentives and higher effort without reducing the agent's welfare (which remains  $U_o$ ).

## How can the common shock be filtered out in practice

A natural way to obtain information on the common aggregate shock is to look at the output of other comparable agents like for example:

- other agents working in the same project, if available;
- agents working on similar tasks in other firms.

Of course this makes sense only if individual shocks have a common component.

For example, shareholders of company  $A$  may use the average performance of competitors  $B$  to estimate the common shock and design the compensation of their own manager.

Suppose that there are  $n$  identical workers, each one producing

$$Y_i = e_i + \eta_i + \theta \quad (109)$$

Under which conditions can we use the average of the  $n - 1$  other agents to filter out the common shock in the compensation scheme of agent  $i$ ?

## The optimal contract when all shocks are independent

Suppose that the shocks  $(\theta, \eta_1, \dots, \eta_n)$  are independent noise terms.

Let  $Z_{-i}$  denote the average output of the  $n - 1$  agents other than  $i$ , which is:

$$Z_{-i} = \frac{Y_1 + \dots + Y_{i-1} + Y_{i+1} + \dots + Y_n}{n - 1} = \bar{e}_{-i} + \bar{\eta}_{-i} + \theta \quad (110)$$

where

- $\bar{e}_{-i}$  is the average effort chosen by the other workers
- $\bar{\eta}_{-i}$  is the average idiosyncratic shock of the other workers

The compensation schedule

$$\tilde{W}_i = \tilde{s} + \tilde{b}(Y_i - Z_{-i}) = \tilde{s} + \tilde{b}(e_i + \eta_i - \bar{e}_{-i} - \bar{\eta}_{-i}) \quad (111)$$

is called “relative performance evaluation”.

## When is relative performance evaluation better?

A compensation scheme based on relative performance like:

$$\tilde{W}_i = \tilde{s} + \tilde{b}(Y_i - Z_{-i}) = \tilde{s} + \tilde{b}[e_i + \eta_i - (\bar{e}_{-i} + \bar{\eta}_{-i})] \quad (112)$$

filters out the common shock but subjects the agents to the risk of the average shock experienced by others.

Inasmuch as the variance of the  $(\bar{e}_{-i} + \bar{\eta}_{-i})$  is smaller than the variance of  $\theta$ , the use of relative performance instead of absolute performance subjects the agent to less risk and is a less noisy measure of the agent's effort.

Note that the larger the number  $n - 1$  of other agents on which relative performance is computed, the smaller is the variance of  $(\bar{e}_{-i} + \bar{\eta}_{-i})$ .

Holmstrom (1982) has shown that if idiosyncratic shocks  $\eta_i$  are normally distributed with zero mean, the optimal contract must condition also on the performance of others, i.e. must have the general form  $\tilde{W}_i = \tilde{W}_i(Y_i, Z_{-i})$ .

## A dangerous implication of relative performance evaluation.

When an agent is paid based on relative performance, she gains more

- if her performance increases or
- if the performance of others decreases.

Thus, relative performance evaluation may induce an agent not to put effort in her own task, but to put effort in sabotaging the work of others.

Sabotage can take many subtle and explicit forms, and works even if all outputs decrease, as long as the output of others decreases more than mine.

In general, RPE may reduce cooperation, group moral and incentives to introduce innovations that improve the productivity of all agents.

It may also distort the hiring process of new workers: I have no incentive to let in someone who is better than me.

## When RPE distorts incentives in an unproductive way

Baker (1992) clarifies the conditions under which RPE may induce sabotage.

Let  $\varepsilon$  indicate the vector of individual and idiosyncratic shocks that characterize the state of the world and define with

- $V(e_i, \varepsilon)$  the absolute performance of agent  $i$ .
- $R(e_i, \varepsilon)$  the performance of the reference group.

Note that if the partial derivative  $R_{e_i} \neq 0$ , agent's  $i$  effort affects not only her own performance but also the performance of others.

Using the notation of Baker's model the measure of performance that determines the bonus accruing to agent  $i$  is

$$P(e_i, \varepsilon) = V(e_i, \varepsilon) - R(e_i, \varepsilon) \quad (113)$$

Thus, the compensation schedule is:

$$W = s + bP(e, \varepsilon) \quad (114)$$

but the principal is interested in maximizing  $V(e_i, \varepsilon)$

## Sabotage in Baker's framework

From equation 90, the optimal bonus parameter is given by

$$b^* = \frac{1 + \text{cov}(P_e, V_e)}{1 + \text{var}(P_e)} \quad (115)$$

and the  $b^* = 1$  when  $\text{cov}(P_e, V_e) = \text{var}(P_e)$ .

An obvious case when this condition is met is when

$$R_e(e_i, \varepsilon) = 0 \quad (116)$$

which means that agent  $i$  cannot affect the reference performance level. In other words agent  $i$  cannot sabotage the output of other agents.

Note this may not be enough to make RPE advantageous because we also want the variance of  $P$  to be smaller than the variance of  $V$ .

When this condition on variances is met, it may be that even a small but non-zero  $R_e$  make RPE socially advantageous.

### 3.5 Relative performance and tournaments (Lazear and Rosen (1981))

The previous analysis may suggest that the conditions under which relative performance evaluation is socially advantageous are rarely met.

As far as compensation design is concerned it is indeed observed only in some managerial contracts, where the reference performance is the output of managers in competing firms. (Gibbons and Murphy (1990)).

It is not surprising to observe it in this case because even if sabotage (of others) can take place, it goes to the advantage of the principal.

We do not see compensation based on RPE in many other blue- or white-collar contracts, but here we see quite often RPE as a way to decide on promotions.

The way to think about this is “tournament models” (Lazear and Rosen (1981)).

## Setup and notation

Consider a firm that has two workers  $j$  and  $k$  who produce, respectively,

$$Y_j = e_j + \varepsilon_j \quad (117)$$

$$Y_k = e_k + \varepsilon_k \quad (118)$$

where

- $e_j$  and  $e_k$  are effort levels of the two agents;
- $\varepsilon_j$  and  $\varepsilon_k$  are zero-mean random luck components;
- output is observable and contractible.

The workers earn a starting fixed wage  $W_0$  and faces a cost of effort  $C(e_j)$ .

In order to motivate agents to exert effort, the principal offers a prize to the agent who produces more. The winner will therefore earn  $W_1 > W_0$ .

We can call this a “promotion” but we want to abstract from changes in the job description for the agent who “wins” the wage increase.

## The decision problem of the worker

The agent  $j$  maximizes with respect to  $e_j$  the following objective

$$\text{Max } W_0(1 - P^j(e_j, e_k)) + W_1P^j(e_j, e_k) - C(e_j) \quad (119)$$

where  $P^j(e_j, e_k)$  is the (endogenous) probability of winning the prize.

The first order condition is

$$(W_1 - W_0)P_{e_j}^j - C'(e_j) = 0 \quad (120)$$

where  $P_{e_j}^j$  is a partial derivative and there is a corresponding f.o.c. for agent  $k$ .

The probability to win the prize for  $j$  is given by

$$\begin{aligned} P^j(e_j, e_k) &= \text{Prob}(e_j + \varepsilon_j > e_k + \varepsilon_k) = \text{Prob}(e_j - e_k > \varepsilon_k - \varepsilon_j) \\ &= G(e_j - e_k) \end{aligned} \quad (121)$$

where  $G$  is the symmetric cdf of the difference between the shocks  $\varepsilon_k - \varepsilon_j$ . So the f.o.c is

$$(W_1 - W_0)g(e_j - e_k) - C'(e_j) = 0 \quad (122)$$

## The effort exerted by the two workers

The two workers are identical, and solve the problem in the same way. Thus:

$$e_j^* = e_k^* \quad (123)$$

and the f.o.c. becomes

$$(W_1 - W_0)g(0) = C'(e_j^*) \quad (124)$$

which can be solved for  $e_j^*$  (and similarly for  $e_k^*$ ).

This result has two interesting implications

- The higher is the spread between the two wage levels the higher is effort.
- The higher is  $g(0)$  the higher is effort.

Note that  $g(0)$  captures the role of luck in determining the winner.

## The importance of luck

What happens when luck is not important.

- Luck is irrelevant when the two idiosyncratic shocks are identical:  $\varepsilon_j = \varepsilon_k$ .
- In this case the distribution  $G$  is degenerate in 0 and  $g(0)$  goes to infinity.
- In this case effort is crucial to win the context, and it pays to exert effort.

What happens when luck is important?

- Luck is instead very important in determining the winner when the distribution  $G$  of  $\varepsilon_k - \varepsilon_j$  has fat tails.
- Fat tails of  $G$  mean that with high probability one agent has been lucky and the other unlucky.
- In this case the distribution  $G$  is flat and  $g(0)$  is low.
- It does not pay to exert effort because who is the winner will largely depend on luck.

## The problem of the principal

Given the optimal behaviour of agents, the principal wants to determine the wage levels  $W_1$  and  $W_0$  which maximize expected profits per worker

$$\text{Max } \Pi = e - \frac{W_0 + W_1}{2} \quad (125)$$

subject to the incentive constraint

$$(W_1 - W_0)g(0) = C'(e) \quad (126)$$

and to the participation constraint

$$\frac{W_0 + W_1}{2} \geq C(e) \quad (127)$$

Note that the left hand side of the participation constraint is equal to expected earnings, given that the probability of winning the prize is  $G(0) = .5$ .

We are also implicitly assuming that the agents are risk neutral and have utility

$$U = E(W) - C \quad (128)$$

## Solving the problem of the principal

Substituting the participation constraint in the maximand we get:

$$\text{Max } \Pi = e - C(e) \quad (129)$$

and the f.o.c, respectively with respect to  $W_1$  and  $W_0$ , are:

$$[1 - C'(e)] \frac{\partial(e)}{\partial W_0} = 0 \quad (130)$$

and

$$[1 - C'(e)] \frac{\partial(e)}{\partial W_1} = 0 \quad (131)$$

These conditions are satisfied when  $C'(e) = 1$ . Using the incentive constraints this requires that the spread between the two wages is determined as

$$(W_1 - W_0) = \frac{1}{g(0)} \quad (132)$$

## Comments

- $C'(e) = 1$  implies that tournaments induce the effort that equates the marginal cost for the worker to the marginal benefit for the firm, which is socially optimal.
- But from the viewpoint of workers, this is a “Prisoners’ Dilemma” outcome.
- Such amount of effort must also satisfy the participation constraint.
- The size of the spread is inversely proportional to the importance of luck as measured by  $g(0)$ .
  - When luck is not important,  $g(0)$  is high and a small spread is enough to induce agents to exert effort.
  - When luck is very important,  $g(0)$  is low and a large spread is needed to induce agents to exert effort.
- High compensation for managers are not so much meant to motivate them, but to motivate those below them in the hierarchy.
- Tournaments are a powerful way to think about the compensation structure.

## Sabotage and tournaments

Also tournaments are subject to the risk of increasing sabotage.

Consider the model we have just seen with the following modification of the output equations(Lazear (1989)):

$$Y_j = e_j + \eta_k + \varepsilon_j \quad (133)$$

$$Y_k = e_k + \eta_j + \varepsilon_k \quad (134)$$

Where  $\eta_k$  is the damage that  $k$  can inflict on the output of  $j$  and viceversa.

When this modification is introduced the model highlights an important trade-off. Less wage compression along the firm's hierarchy:

- increases effort incentives in combinations with tournaments for promotions;
- increases sabotage between workers competing for promotions.

This captures the emphasis of the Human Resources literature on raising productivity by “maintaining internal harmony and cooperation within firms”.

### 3.6 The emergence of hierarchies when effort and output are not contractible (Prendergast (1993))

In addition to reducing cooperation among agents, tournaments are difficult to implement when not only effort but also output is not contractible.

In these cases hierarchies of jobs with wages attached to them may emerge to solve the problem of inducing agents to exert effort.

This intuition might help solve Baker et al.(1988) concern:

- Why we see an “overwhelming use of promotion based [compensation] schemes” rather than pay based on performance.
- In other words why sorting workers to tasks should not be separated from compensating workers for performance?

Note that in the simple tournament model, promotions and wage increases are not necessarily associated to changes of tasks assignment.

Prendergast (1993) shows why compensation may be linked to a hierarchy of tasks, in order to induce incentives when effort and output are not contractible.

## Non verifiable effort in a one-job model

Consider a risk neutral worker with reservation utility  $r$ , hired by a risk neutral firm for a single period, and suppose that:

- effort takes the form of acquiring a skill for a given task.
- effort and output are not verifiable in court and therefore not contractible.

At the beginning of the period the worker has to decide whether:

- to invest in the acquisition of the skill:  $s = 1$ ;
- not to invest:  $s = 0$ .

If the principal pays ex post and the acquisition takes place, absent reputational concerns, she has a strong incentive to renege.

If the principal pays ex ante, then the worker has the incentive to renege.

How can the firm induce the worker to incur the cost of an investment in an efficient but non-verifiable investment opportunity?

## Introducing a hierarchy of tasks

Suppose now that the worker can be assigned to

- either an easy job  $E$ ;
- or a difficult job  $D$ ;

The firm can commit to pay different wages for each task.

- $W_E$  for task  $E$ ;
- $W_D$  for task  $D$ ;
- and  $W_D > W_E$ .

The worker's utility is

$$U = w - sc \tag{135}$$

- $c$  is the cost of acquiring the skill, i.e. the cost of effort;
- $w$  is the wage.

## Output in the different tasks

Let  $Y_k(s)$  denote the output of task  $k \in \{E, D\}$ , with or without effort and assume that:

$$Y_D(0) < Y_E(0) < Y_E(1) < Y_D(1) \quad (136)$$

- Effort increases productivity in both tasks.
- Workers exerting effort should be efficiently assigned to job  $D$ .
- Workers not exerting effort should be efficiently assigned to job  $E$ .
- Social returns to effort are higher in job  $D$  than in job  $E$ .

We also assume that when workers exerting and not exerting effort are sorted in the above way, paying the effort cost is socially efficient, i.e.:

$$Y_D(1) - Y_E(0) > c \quad (137)$$

## Mutually credible commitments of the principal and the agent

It is in the interest of the principal to assign a worker to the  $D$  task if

$$Y_D(1) - W_D \geq Y_E(1) - W_E \quad (138)$$

i.e., if wages are set to satisfy this condition given output levels, the principal gains more by assigning workers who exert effort to  $D$ .

Thus, the firm can credibly commit to assign workers exerting effort to  $D$ .

Workers can credibly commit to exert effort if

$$W_D - W_E \geq c \quad (139)$$

Combining the two conditions, the wages attached to the tasks should satisfy:

$$Y_D(1) - Y_E(1) \geq W_D - W_E \geq c \quad (140)$$

## Conditions for the emergence of hierarchies of jobs and wages?

Note that the condition for effort to be efficient

$$Y_D(1) - Y_E(0) > c \quad (141)$$

is not sufficient to insure a situation of mutually credible commitment, which requires

$$Y_D(1) - Y_E(1) \geq c \quad (142)$$

For 141 and 142 to be contemporaneously satisfied it must happen that

$$Y_E(1) - Y_E(0) \text{ is sufficiently small} \quad (143)$$

which means that the firm must not gain much by assigning a worker who exerts effort to an easy job.

This means that what is needed is not just a hierarchy of “formal” job titles, but a hierarchy of jobs characterized by increasingly difficult different tasks.

## Comments

If we add heterogeneity of the cost of effort across workers this model resembles a model of screening.

The interesting feature is that the separating equilibrium that emerges originates a hierarchy of increasingly difficult tasks to which different wages are attached.

In the absence of such hierarchy of jobs and wages, workers would not be induced to exert effort given that both effort and output are not contractible.

The discussion of Section 2.1 on the role of Rawlsian institutions applies here as well.

But why we observe situations in which these hierarchies are combined with “up-or-out” promotion contracts?

### 3.7 “Up-or-out” contracts (Kahn and Huberman (1988))

In some organizations, if promotion does not occur within some time, workers are not retained even if still productive.

This is for example the case in US law firms where young lawyers who do not become partners are fired.

It is also the case of the US academia, where professors who are not given tenure are induced to change university.

The model of Kahn and Huberman (1988) explains these contract features as the result of a “double moral hazard” problem (similar to Prendergast (1993)):

- The firm promises a promotion to induce the worker to exert effort in acquiring new skills.
- The firm must commit to fire un-promoted workers, otherwise it could claim ex post that no one has passed the threshold for promotion.

## Setup and notation

Consider a worker whose output is a random variable taking two values:

- low output,  $H_1$ ;
- high output,  $H_2$ ;

The worker can exert effort  $e = 1$  to increase the probability of drawing  $H_2$ :

- $f(0)$  is the probability of  $H_2$  if  $e = 0$  ;
- $f(1)$  is the probability of  $H_2$  if  $e = 1$ ;
- $f(1) > f(0)$ .

The investment is assumed to be firm specific, has no value elsewhere and is taken before production starts.

The productivity of the worker elsewhere is  $r$  with  $r < H_1 < H_2$ .

## The social benefit of the worker's investment

We assume that both the firm and the worker are risk neutral.

Denote with  $C$  the cost of the investment for the worker.

It is efficient that the investment takes place if

$$Z = [f(1) - f(0)](H_2 - H_1) - C \geq 0 \quad (144)$$

The quantity  $Z$  is the net social gain yielded by the investment.

We also assume that  $U_0$  is the reservation expected utility of the worker. In no instances the worker can receive less than  $U_0$ .

## The case of contractible effort and output

Suppose that:

- the worker's investment is perfectly observed by the firm (and the worker);
- the worker's output is perfectly observed by the worker (and the firm).

It is easy to see that the optimal contract, written ex ante, will specify:

- the worker invests in the acquisition of the skill;
- the worker is retained and produces at the firm no matter what the draw of productivity may be (because  $r < H_1 < H_2$ ).
- the worker is payed  $U_0 + C$ , given that competition among workers ensures that they are squeezed against their reservation utility.

## The case of contractible effort and non-contractible output

Suppose that the information on the output draw of the worker is private information of the firm.

In other context this could lead to inefficient outcomes.

For example, in the case of bargaining on how to share total output, the firm would have an incentive to manipulate the information on output.

In this specific example, however, there is no reason why the first best contract should not emerge.

This is because in any case workers get  $U_0 + C$ .

## The case of contractible output and non-contractible effort

This is the case of the canonical principal agent model, in which:

- full insurance would kill the agent's incentive to exert effort;
- the wage must be tied to performance ;
- it is actually tied perfectly to performance, given risk neutrality;
- nevertheless, also in this case there is no reason to fire the worker independently of the results of the investment.

So this situation does not generate up-or-out contracts.

## The case of non contractible output and effort

This is the case in which the double moral hazard problem arises:

- In order to ensure that the worker finds it optimal to invest, he must receive a reward when the high productivity state is realized.
- But if the state of productivity is private information of the firm, the firm has an incentive to always say that productivity has been low.

If the contract specifies that the wage changes with productivity but employment status does not there is no cost for the firm in saying that the productivity has been low.

To induce the firm to reveal the true state of productivity, it must be costly for it to declare low productivity when in fact it was high.

The way to generate this cost is to link firing of the worker to a declaration of low productivity.

## When “up-or-out” contracts are more likely?

“Up-or-out” contracts are common in situations where the promotion implies just a change of job title, but not an effective change of task.

This is for example the case of lawyers and academics who do not change their activity before and after receiving partnership or tenure.

In this case the moral hazard problem of the firm can be solved by making it costly for the firm to say that output is low.

In the case of Prendergast (1993), instead the problem is solved by making it advantageous for the firm to say that productivity is high when this is the case.

And this is possible because tasks differ with different job titles.

In firms where hierarchies involve jobs with different tasks there is less need for “up-or-out” contracts.

### 3.8 The “Peter Principle” (Lazear (2004))

“Workers are promoted to their level of incompetence” (Peter and Hull (1969)).

Outside economics it is often taken as an indication that something is wrong in the mechanism of promotions.

Is this conclusion compatible with the models we have seen in which promotions and hierarchies are meant to solve the problem of generating incentives?

Lazear (2004) argues that the “Peter’s Principle” is due to:

- mean regression of luck and/or
- strategic behavior of workers in tournaments against a standard.
- Firms anticipate both explanations but the effect is never eliminated.

According to Lazear, rather than evidence of a mistake, the “Peter Principle” is a necessary consequence of any promotion rule.

## Setup and notation

- There are two periods.
- A worker has a time invariant productivity component  $A \sim f(A)$ .
- There is also a time varying productivity component  $\varepsilon_t \sim g(\varepsilon_t)$  for  $t = 1, 2$ .
- The time varying components are identically, independently and symmetrically distributed with zero mean. Thus  $\varepsilon_t = \varepsilon \sim g(\varepsilon)$  for all  $t$ .
- Both the firm and the worker observe the sum of the two components  $A + \varepsilon_t$  but cannot disentangle them. Thus there is “symmetric ignorance”.
- There are two jobs, easy,  $E$  and difficult,  $D$ :

– In the easy job productivity is

$$Y_t^E = \alpha + \beta(A + \varepsilon_t) \quad (145)$$

– In the difficult job productivity is

$$Y_t^D = \gamma + \delta(A + \varepsilon_t) \quad (146)$$

- where  $\alpha > \gamma$  and  $\delta > \beta$  (see figure).

## Productivity and comparative advantage in the difficult job

Given this setup, within each period and absent any strategic consideration it pays to assign a worker to the difficult job only if

$$A + \varepsilon_t > X \quad (147)$$

where  $X$  is the crossing point of the two lines defined as

$$X \equiv \frac{\alpha - \gamma}{\delta - \beta} \quad (148)$$

Thus the most able workers have a comparative advantage in the difficult job.

In the first period, when no information is available, expected productivity is too low to assign anyone to the  $D$  job. This happens when  $f(A)$  is such that

$$\int \int [\alpha + \beta(A + \varepsilon_1)] dGdF > \int \int [\gamma + \delta(A + \varepsilon_1)] dGdF \quad (149)$$

The principal wants to use the information on output in period 1 to assign optimally workers to the two jobs in period 2.

## Average ability of promoted workers given a promotion rule

At the end of period 1 the principal has an observation on productivity:

$$\hat{A}_1 = A + \varepsilon_1 \quad (150)$$

Suppose that  $A^*$  is the criterion (to be derived later) that the principal uses to decide if a worker should be assigned to the difficult job.

Thus, a worker is promoted if

$$\hat{A}_1 = A + \varepsilon_1 > A^* \quad (151)$$

which implies an average transitory shock in period 1 for promoted workers:

$$E(\varepsilon_1 \mid \hat{A}_1 > A^*) = \int E(\varepsilon \mid \varepsilon > A^* - A) dF(A) > 0 \quad (152)$$

Note that this is positive because the expectation of  $\varepsilon$  conditional on it being larger than any number is positive, since  $\varepsilon$  is zero-mean noise

So, for given permanent ability, promoted workers are the “lucky” ones.

## Average ability of promoted workers after promotion

Consider now the average transitory shock of promoted workers in period 2:

$$E(\varepsilon_2 \mid \hat{A}_1 > A^*) = E(\varepsilon_2 \mid \varepsilon_1 > A^* - A) = 0 \quad (153)$$

The reason why it is zero is that  $\varepsilon_2$  is independent of  $A$  and  $\varepsilon_1$ .

Let's now compare the average productivity of promoted workers before and after promotion, conditioning on their permanent component  $A$  we have:

- before promotion:  $A + E(\varepsilon_1 \mid \varepsilon_1 > A^* - A) > A$
- after promotion:  $A + E(\varepsilon_2 \mid \varepsilon_1 > A^* - A) = A$

which says that, on average, promoted workers are more productive before than after promotion because luck helps to be promoted but it is mean reverting.

Exercise: Suppose the firm promotes only workers who get two heads on two consecutive coin tosses.

How many of the promoted workers will be able to repeat the performance?

## Optimal promotion rule if the firm anticipates mean regression

The firm maximizes profits (or workers utility) by selecting for each candidate the job with the highest expected value. So it selects  $A^*$  to maximize

$$\text{Max} \int_{-\infty}^{\infty} \int_{A^* - \varepsilon_1}^{\infty} (\gamma + \delta A) dF(A) dG(\varepsilon) + \int_{-\infty}^{\infty} \int_{-\infty}^{A^* - \varepsilon_1} (\alpha + \beta A) dF(A) dG(\varepsilon) \quad (154)$$

where note that

- the choice of  $A^*$  takes place at the beginning of period 1
- the promotion rule  $A + \varepsilon_1 > A^*$  implies  $A > A^* - \varepsilon_1$  which explains the bounds of integration
- $\varepsilon_2$  has zero mean and therefore drops out.

In general  $A^* > X$  indicating that anticipation of the “Peter Principle” induces firms to adopt a rule which is stricter than in the case of no strategic consideration.

## A numerical example

For the specific case in which

- $A$ ,  $\varepsilon_1$  and  $\varepsilon_2$  are normally distributed
- $\alpha = 1$ ,  $\beta = 0.5$ ,  $\gamma = 0$  and  $\delta = 1$ ,

It is possible to show that  $X = 2$  while  $A^* = 4.01$ .

In this case the promotion threshold is two standard deviations higher than what would prevail without strategic considerations.

If the shock  $\varepsilon_1$  had zero variance the maximization problem would reduce to

$$\text{Max} \int_{A^*}^{\infty} (\gamma + \delta A) dF(A) + \int_{-\infty}^{A^*} (\alpha + \beta A) dF(A) \quad (155)$$

and it is easy to see that the solution would be

$$A^* = X \equiv \frac{\alpha - \gamma}{\delta - \beta} \quad (156)$$

## Workers' effort in tournaments against a standard

A second interpretation of the “Peter Principle” is a simple application of the tournament's model of Lazear and Rosen (1981), to the case of competition against a standard.

Output in period 1, when the worker is assigned to the easy job  $E$ , is given by:

$$Y_E = A + \varepsilon + e_E \quad (157)$$

and the worker is promoted to the difficult job in period 2 if

$$Y_E = A + \varepsilon + e_E > A^* \quad (158)$$

The worker does not know her ability and exerts extra-effort in order to induce the principal to believe that she has high ability.

If promoted, in the absence of further incentives, her effort will drop.

## The solution of the model

Denoting with  $W_E$  and  $W_D$  the wage levels respectively in the easy and difficult jobs the worker chooses effort  $e_E$  in the easy job in period 1 to maximize

$$\text{Max} \int \{W_D \text{Pr}(A + \varepsilon + e_E > A^*) + W_E [1 - \text{Pr}(A + \varepsilon + e_E > A^*)] - C(e_E)\} dG(A) \quad (159)$$

where  $C(e_E)$  is the cost of effort.

The first order condition is

$$(W_D - W_E)g(A^* - A - e_E) = C'(e_E) \quad (160)$$

which shows that, as in the standard tournament model, the principal can induce any amount of effort by raising the spread between the two wage levels.

It is however not particularly deep to predict that effort will decrease for promoted workers after the tournament in the absence of further incentives

## More interesting implications when workers know their ability

The model becomes more interesting if workers know their ability and we consider how the incentive to distort effort in period 1 changes with ability.

Define:

- $e_1$  is effort in period 1;
- $e_2^D$  is effort in period 2 in the difficult job;
- $e_2^E$  is effort in period 2 in the easy job;
- effort in period 1 is determined before the promotion decision;
- the cost of effort is  $C(e)$ , independent of ability for simplicity;
- The wage is equal to output in both periods;
- $\varepsilon_1$  and  $\varepsilon_2$  denote noise terms in the two periods;

## Effort choices in period 2

Remembering that workers are paid based on performance, those who are not promoted choose  $e_2^E$  to maximize:

$$\text{Max } \alpha + \beta(A + e_2^E + \varepsilon_2) - C(e_2^E) \quad (161)$$

and the f.o.c. is

$$C'(e_2^E) = \beta \quad (162)$$

A similar problem is solved by workers who are promoted with respect to  $e_2^D$

$$\text{Max } \gamma + \delta(A + e_2^D + \varepsilon_2) - C(e_2^D) \quad (163)$$

and the f.o.c. is

$$C'(e_2^D) = \delta \quad (164)$$

Promoted workers exert more effort in the difficult job, because  $\delta > \beta$  implies  $e_2^{D*} > e_2^{E*}$ .

## Effort choices in period 1 given expected choices in period 2

The worker chooses effort in period 1 knowing that in period 2 she will choose  $e_2^{D*}$  or  $e_2^{E*}$  depending on the occurrence of a promotion.

$$\begin{aligned} \text{Max } & \alpha + \beta(A + e_1^E + \varepsilon_1) - C(e_1^E) & (165) \\ & + \text{Prob}(A + e_1^E + \varepsilon_1 > A^*) E[\gamma + \delta(A + e_2^{D*} + \varepsilon_2) - C(e_2^{D*})] \\ & + \text{Prob}(A + e_1^E + \varepsilon_1 \leq A^*) E[\alpha + \beta(A + e_2^{E*} + \varepsilon_2) - C(e_2^{E*})] \end{aligned}$$

The f.o.c is:

$$\begin{aligned} C'(e_1^E) = & \beta + g(A^* - e_1^E - A) \{ [\gamma + \delta(A + e_2^{D*} + \varepsilon_2) - C(e_2^{D*})] \\ & - [\alpha + \beta(A + e_2^{E*} + \varepsilon_2) - C(e_2^{E*})] \} \end{aligned} \quad (166)$$

Note that the f.o.c. for optimal effort in the absence of strategic considerations would be  $\beta = C'(e_1^E)$ .

This will occur only when the second term on the LHS of 166 is zero. However, this does not happen in general.

## How does effort distortion depend on ability

Consider figure A1 in Lazear (2004) and note that the term:

$$g(A^* - e_1^E - A) \{ [\gamma + \delta(A + e_2^{D*} + \varepsilon_2) - C(e_2^{D*})] - [\alpha + \beta(A + e_2^{E*} + \varepsilon_2) - C(e_2^{E*})] \} \quad (167)$$

- is positive for high  $A$  workers who have a comparative advantage in  $D$  jobs:
  - \* thus, they will over-produce in period 1.
- negative for low  $A$  workers who have a comparative advantage in  $E$  jobs:
  - \* thus, they will under-produce in period 1 (do not want to be promoted).
- it is near zero for workers whose ability is near the threshold of promotion:
  - \* thus, they will not be distorted because their payoff does not change much with promotion.
- it is larger or smaller in absolute value as a function of the distribution  $g$ :
  - \* for extreme values of  $A$ , the promotion decision cannot be changed by distorting effort.

### 3.9 Moral Hazard in teams (Holmstrom (1982))

Incentives are even more problematic in the presence of team production:

- A team of agents co-operate to the production of some output.
- Joint output is observable and contractible.
- Individual output is not observable.
- Individual effort is not observable.

In these situations the “free rider” problem arises (Alchian and Demsetz (1972), Holmstrom (1982)), which causes a reduction of individual effort even in the absence of uncertainty.

While the design of incentive schemes in this context is problematic, solutions may come from:

- the emergence of an hierarchy;
- the existence of repeated interactions between members of teams.

## A simple model of free riding

Consider:

- two workers  $i \in \{1, 2\}$ ;
- who contribute to a joint project with effort  $e_i \in [0, \infty)$ ;
- incurring a cost of effort  $c(e)$ , with  $c' > 0$  and  $c'' > 0$ ;
- and have utility function

$$U_i = w_i - c(e_i) \quad (168)$$

where  $w_i$  is the compensation for taking part into the project.

Joint output is observable without uncertainty and given by

$$Y = f(e_1 + e_2) \quad (169)$$

with  $f' > 0$  and  $f'' < 0$ .

Individual effort is not observable.

## The first best

The social planner would choose the effort levels  $e_1$  and  $e_2$  of the two agents to solve the following problem:

$$\text{Max } f(e_1 + e_2) - c(e_1) - c(e_2) \quad (170)$$

The f.o.c. would be:

$$f' = \frac{dc}{de_1} = \frac{dc}{de_2} \quad (171)$$

For example, if

- $C = \frac{e^2}{2}$
- $f(e_1 + e_2) = e_1 + e_2$

the first best effort exerted by the two agents is:

$$e_1^* = e_2^* = 1 \quad (172)$$

## The outcome under a fifty-fifty sharing rule

Suppose that agents decide to split the output according to the sharing rule

$$w_i = \frac{Y}{2} \quad (173)$$

This would be a situation of co-operative effort in the absence of principal.

Each agent chooses her own optimal level of effort  $e_i$  maximizing:

$$\text{Max} \frac{f(e_i + e_j)}{2} - c(e_i) \quad (174)$$

and the f.o.c. is

$$\frac{f'}{2} = \frac{dc}{de_1} = \frac{dc}{de_2} \quad (175)$$

Denoting with  $\bar{e}$  the level of effort prevailing in this case, it is easy to see that

$$\bar{e}_i < e_i^* \quad (176)$$

## The free-rider problem

Considering again the specific example in which

- $f(e_1 + e_2) = e_1 + e_2$
- $C = \frac{e^2}{2}$

we have

$$\bar{e}_i = \frac{1}{2} < e_i^* = 1 \quad (177)$$

Note also that total output is

$$\bar{Y} = 1 < Y^* = 2 \quad (178)$$

and is fully distributed between the two agents because, given the sharing rule,

$$w_i = \frac{1}{2} \quad (179)$$

Free riding reduces social output by one half. With  $N$  members of the team, the factor of reduction would be  $\frac{1}{N}$ .

## “Hiring a Principal” to solve the free riding problem

Holmstrom (1982) suggests that the emergence of hierarchies can be seen as dictated by the need to solve free-riding problems.

To be precise, what is needed is:

- a principal who can commit to inflict penalties and rewards;
- the willingness to give up the “balancing of the budget” between agents.

As long as output is fully shared among team members there is no way to avoid the free-riding problem in a one-shot game.

Moreover the presence of a principal who can enforce deviations from a full distribution of output is needed.

The principal must be prepared to assume the residual (positive or negative) of the non-budget-balancing sharing rules.

## A simple example

Suppose that

- $f(e_1 + e_2) = e_1 + e_2$
- $C = \frac{e^2}{2}$
- $U_i = w_i - C(e_i)$

and that a principal can commit to enforce the following compensation scheme:

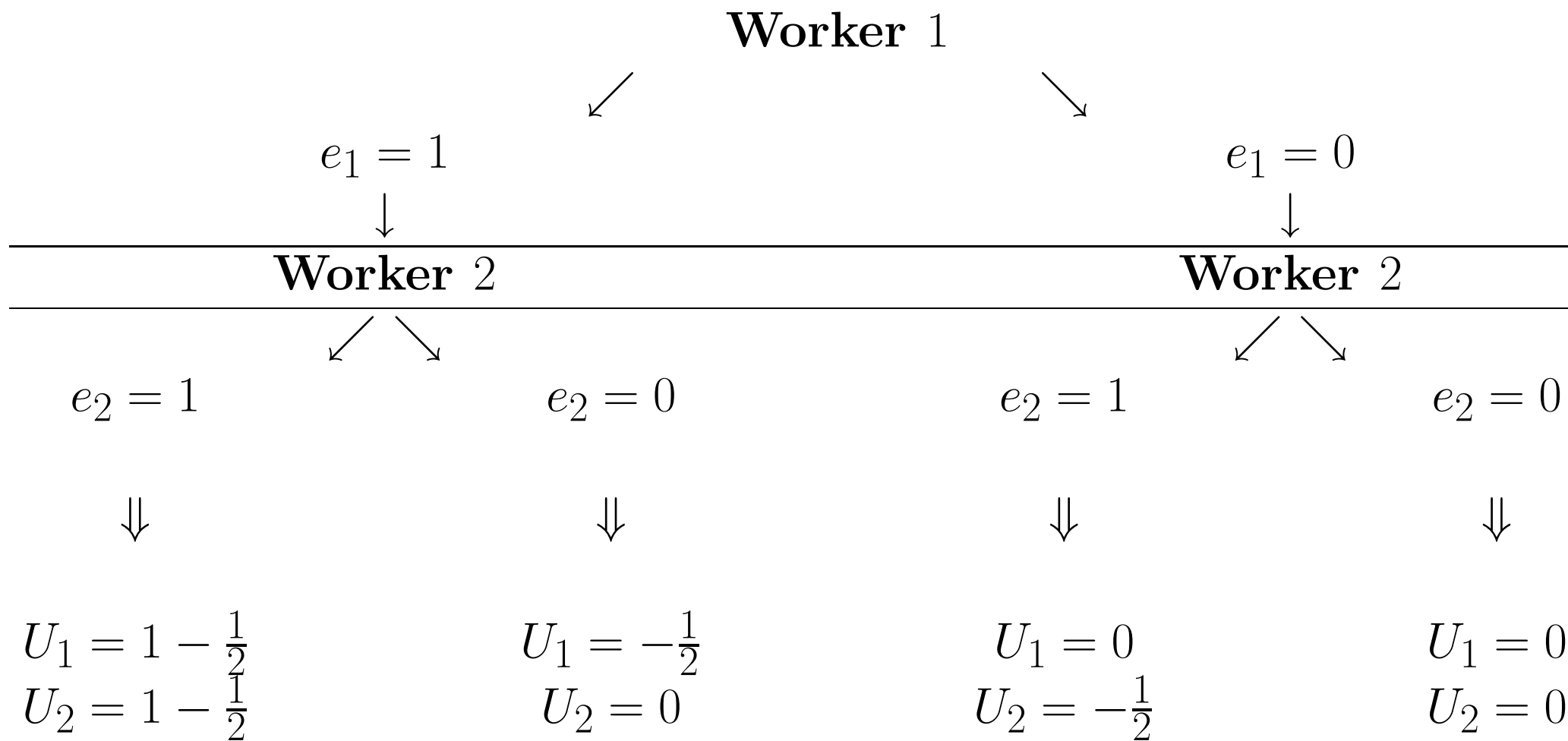
$$w_i = \begin{cases} \frac{Y}{2} & \text{if } Y \geq Y^* = e_1^* + e_2^* = 2 \\ 0 & \text{if } Y < Y^* = e_1^* + e_2^* = 2 \end{cases} \quad (180)$$

Note that given this compensation scheme and convex costs:

- any effort  $e > 1$  is dominated by an effort  $e = 1$ ;
- any effort  $0 < e < 1$  is dominated by an effort  $e = 0$ .

Consider the game played by the two agents given this compensation scheme.

The game played by the agents given the principal's proposal



## Equilibria of the game

There are two Nash equilibria:

- $e_1 = 1$  and  $e_2 = 1$ ;
- $e_1 = 0$  and  $e_2 = 0$ .

But the cooperative first best equilibrium is now a possible stable outcome.

In the equilibrium that sustains the first best, all the output is distributed and the principal gets nothing.

But in all other case the principal threatens to keep all the output.

Of course in the bad Nash Equilibrium no output is produced.

## Repeated interactions

The cooperative equilibrium can also be sustained by infinitely repeated interactions and by the following compensation scheme

$$w_{it} = \begin{cases} \frac{Y_t}{2} & \text{if } Y_{t-1} \geq Y^* \\ 0 \text{ forever} & \text{if } Y_{t-1} < Y^* \end{cases} \quad (181)$$

Under this scheme, if agents are not “too impatient” the benefit of a deviation from the first best is not larger than the cost of being punished for ever.

This is the argument which is used by Weitzman and Kruse (1990) to claim (with some empirical evidence) that profit sharing increases productivity.

An alternative view (Ichino (1994)) is that flexible compensation systems are introduced as a risk sharing device.

The trade off between incentives and risk sharing in the design of compensation systems.

# Compensation systems and financial markets

Ichino (1994) argues that:

- There is a correspondence between “labor contracts” and “financial contracts” .
- A fixed wage and the interest earned by debt holders are predetermined fixed claims on the value of the firm.
- A profit sharing compensation and the shareholders’ dividends are residual and variable claims on the value of the firm.
- A change from a fixed wage system to a profit sharing system is like a debt-to-equity swap.
- If the Modigliani-Miller theorem does not hold, changes in the ratio of fixed vs. residual claims on the financial side induce opposite changes on the labor side and viceversa.
- Firms with high debt to equity ratio should be more likely to introduce flexible compensations systems.

## Flexible compensation and industrial conflict

- This perspective suggests that profit sharing is not only a way to increase productivity but also a way to shift risk on workers.
- Profit sharing reduces the risk of defaulting on fixed wage claims, and thus reduces the cost of raising money on financial markets.
- Workers are more risk averse than financial markets, and it should be more costly to shift risk on them.
- But if the risk of bankruptcy is high, workers may be willing to accept more risk attached to compensation levels, if this reduces the risk of losing jobs.
- Nevertheless, should workers share risks without sharing decisions?
- When profit sharing is introduced as a risk sharing device, workers are likely to request more participation to entrepreneurial decisions.
- The alternative is industrial conflict.

### 3.10 Efficiency Wages as an incentive device (Gibbons and Waldmann (1999))

So far we have seen how:

- compensation based on individual performance;
- relative performance evaluation;
- the endogenous emergence of hierarchies;

can solve the problem of generating incentives.

An alternative strategy is offered by the combination of

- a “carrot”: workers get rents with respect to alternative opportunities;
- a “stick”: workers are threatened of being fired if caught shirking.

This strategy is feasible only if effort can be observed at least imperfectly:

- either by inferring from output;
- or by monitoring.

## The literature on efficiency wages

Under the heading of “efficiency wages” goes a large, mainly macro, literature in which the common ingredient is that raising wages may increase profits:

- Development economics: see references in Katz (1986).
- Union threat: Dickens (1986).
- Adverse selection: Gush and Weiss (1981); we have seen this in Section 2.
- Unemployment as a discipline device: Shapiro and Stiglitz (1984).

Most of this literature focuses on the unemployment consequences of efficiency wages from a macro perspective, in particular Shapiro and Stiglitz (1984)

Here instead we focus on the incentive effects of the provision of efficiency wages following Gibbons and Waldman (1999).

## A simple model of efficiency wages

Consider a risk neutral firm that offers to a risk neutral worker a wage  $w$  in each period and the workers can accept or reject.

If the worker rejects she gets the alternative wage  $w_a$ .

If the worker accepts she can choose:

- a high level of effort  $e_H$  which entails a cost  $c$ ;
- a low level of effort  $e_L$  which entails no cost.

Output  $Y$  takes two values  $H$  and  $L$  with  $H > L$ .

- if  $e = e_H$  then  $Y = H$ ;
- if  $e = e_L$  then:
  - $Y = H$  with probability  $p$
  - $Y = L$  with probability  $1 - p$

The firm observes output and can only infer imperfectly the level of effort.

## The one shot game

Assume further that

$$H - c > w_a > pH + (1 - p)L \quad (182)$$

which means that it is efficient for the worker to be employed at the firm and choose high effort.

However, in a one shot interaction the efficient outcome will not be reached:

- If the firm pays ex-post the worker does not accept because it fears that the firm reneges.
- If the firm pays ex-ante it anticipates that the worker will not exert effort.
- Thus it will offer a wage

$$w_{os} = pH + (1 - p)L < w_a \quad (183)$$

and the worker rejects because the alternative opportunity is better.

## The outcome with repeated interactions

Let  $r$  be the players' common discount rate.

Consider the following wage offer of the firm:

$$w_t = \begin{cases} w^* > w_a & \text{if } w_{t-1} = w^* \text{ and } Y_{t-1} = H \\ 0 \text{ for ever} & \text{otherwise} \end{cases} \quad (184)$$

Note that this implies reverting to the one shot outcome forever in case of both a firm's or a worker's deviation.

If the worker does not exert effort

- she gains the effort cost  $c$ ;
- but faces an expected PDV loss of

$$(1 - p) \frac{w^* - w_a - c}{r} \quad (185)$$

## The efficiency wage $w^*$

The two parties will then set  $w^*$  equal to the lowest level that avoid shirking:

$$w^* = w_a + c + \frac{rc}{1-p} \quad (186)$$

The key point is that the efficiency wage pays not only the opportunity cost of the worker  $w_a + c$  but also a rent  $\frac{rc}{1-p}$ .

The risk of losing this rent in case of firing, induces the worker to exert effort.

So the efficiency wage  $W^*$  is larger than the walrasian equilibrium wage  $w_a + c$ , and in general equilibrium may generate unemployment (Shapiro-Stiglitz (1984)).

But as long as  $H > w^*$  it is mutually optimal for the worker and the firm to implement the efficiency wage level.

## The “bonding critique”

Suppose that:

- The firm forces the worker to post a bond equal to  $\frac{rc}{1-p}$  before being hired.
- The two parties agree that the bond is kept by the firm if effort is inferred to be low.
- If effort is never low the worker gets back his bond (plus interests) at the end of the relationship.

This agreement induces the worker to exert effort, even if the wage is set at the Walrasian equilibrium, without perverse macroeconomic consequences.

The posting of bonds is rarely seen in labor market, but upward sloping wage profiles may be thought as an implicit way to post bonds.

On the other hand the solution based on bonds is not robust to the possibility of firms renegeing, absent reputational concerns.

The bonding critique is a challenge for efficiency wage theories, but does not kill its validity if we abstract from macro and focus on the incentives problem.

### 3.11 Career concerns (Holmstrom (1999))

We have already considered the worse possible case for incentives, when both effort and output are not observable or in any case not contractible.

We have seen (see Sections 3.6 and 3.7) that the emergence of hierarchies may solve the problem of incentives in these cases.

However, Fama (1980) proposes the idea (formalized by Holmstrom(1999)) that concerns about reputation for future career prospects induce agents to choose efficient effort levels even in the absence of incentive schemes.

The intuition is that agents who are:

- heterogenous with respect to ability;
- and can exert effort to compensate for lack of ability.

have the incentive to exert effort in order to induce the principal to think that they have high ability.

This incentive is stronger at the beginning of a career.

## Setup and notation

Output has three components:

$$Y_t = \eta + e_t + \epsilon_t \quad (187)$$

- $\eta \sim N(m_0, h_0)$  is ability.
- $e_t$  is effort which is exerted at a cost  $C(e_t)$  increasing and convex.
- $\epsilon_t \sim N(0, h_\epsilon) \forall t$  is noise.
- $N(a, b)$  denotes the normal distribution with mean  $a$  and precision (inverse of the variance)  $b$ .
- Ability and noise are independent.
- The principal and the worker are risk neutral.
- The firm, the worker and the market share the prior on the distribution of ability and noise.
- The firm, and the market observe output but not effort.
- Only the worker observes effort and knows ability.

## Payoffs

The firm's payoff is

$$\Pi = \sum_{t=0}^T \delta^t (Y_t - w_t) \quad (188)$$

where  $\delta$  is the discount factor and  $w_t$  is the compensation level.

The worker's payoff is

$$U = \sum_{t=0}^T \delta^t (w_t - C(e_t)) \quad (189)$$

where the discount factor is assumed to be the same for all players.

Note that the first best level of effort is

$$e_{fb} \text{ which solves } C'(e) = 1 \quad (190)$$

## A simple two periods case

In period 2 there is no reason for the agent to exert effort and therefore

$$e_2^* = 0 \quad (191)$$

and thus  $Y_2 = \eta + \epsilon_2$ .

In period 2 the firm pays a wage equal to expected output in the second period, where the expectation is based on the information obtained in period 1.

$$w_2(Y_1) = E(Y_2 | Y_1) = E(\eta | Y_1) \quad (192)$$

Given the distributional assumptions, the posterior distribution of  $\eta$  is given by the following updating rule (see De Groot (1970)):

$$E(\eta | Y_1) = \frac{h_0 m_0 + h_\epsilon (\eta + \epsilon_1)}{h_0 + h_\epsilon} = \frac{h_0 m_0 + h_\epsilon (Y_1 - e_1^*)}{h_0 + h_\epsilon} \quad (193)$$

The agent anticipates that the firm will make inference about ability in period 2 on the basis of this rule, and therefore exerts effort in period 1 because this will increase earnings in period 2 via the effect on  $E(\eta | Y_1)$ .

## Optimal effort decision in period 1

The agent in period 1 chooses effort  $e_1$  to maximize

$$\text{Max } [w_1 - C(e_1)] + \delta[E(w_2(Y_1) | e_1) - C(e_2)] \quad (194)$$

Noting that in 193  $e_1^*$  is given and  $e_1$  appears in  $Y_1$ , the f.o.c. is :

$$C'(e_1) = \delta \frac{dE(w_2(Y_1) | e_1)}{de_1} \quad (195)$$

$$C'(e_1) = \delta \frac{h_\epsilon}{h_0 + h_\epsilon}$$

which gives the optimal period 1 effort  $e_1^*$ .

The firm anticipates this behaviour of the worker in period 1 and offers

$$w_1 = E(Y_1) = E(\eta + e_1^* + \epsilon_1) = m_0 + e_1^* \quad (196)$$

Note that the worker exerts effort in period 1 not only to increase  $w_1$  but also to increase  $w_2$  by inducing the firm to believe that she has high ability.

## Comments and comparative static

The marginal return to exerting effort in period 1 is

$$\delta \frac{h_{\epsilon}}{h_0 + h_{\epsilon}} \quad (197)$$

which suggests that:

- The larger is the initial uncertainty (i.e. the lower is the precision  $h_0$ ) the larger is effort  $e_1^*$  in period 1.
  - Without uncertainty there is no way that the agent can convince the principal to have high ability by exerting more effort.
- The lower the variance of the noise (i.e. the higher is the precision  $h_{\epsilon}$ ), the higher is the return to effort.
  - This also makes sense, because with a lot of noise it does not pay to send the signal.
- The firm anticipates that the worker is trying to “fool” the market (see 193), but the worker is trapped in supplying the effort that is expected from her.

## Extension to more than two periods

With  $T > 2$  periods the updating rules are as follows:

$$E(\eta | Y_{t+1}) = \frac{h_t m_t + h_\epsilon (\eta + \epsilon_t)}{h_t + h_\epsilon} = \frac{h_0 m_0 + h_\epsilon \sum_{s=1}^t (Y_s - e_s^*)}{h_0 + t h_\epsilon} \quad (198)$$

The precision of the estimate of ability  $h_t$  increases with time:

$$h_{t+1} = h_t + h_\epsilon = h_0 + t h_\epsilon \quad (199)$$

The optimal amount of effort  $e_t^*$  therefore solves the following: condition

$$C'(e_t) = \sum_{s=t+1}^T \delta^{s-t} \frac{h_\epsilon}{h_0 + t h_\epsilon} \quad (200)$$

## Comments

When uncertainty on ability is high at the beginning of a career it pays for the agent to exert effort in order to fool the firm about her ability.

With time however, the past history of output increases the precision of the estimate of ability.

Thus the agent's incentive to exert effort decreases, and effort goes to zero in the limit.

Even career concerns do not solve completely the problem effort incentives, particularly later on in a career.

#### 4 End of the employment relationship: to be completed

## 5 References

- Aghion, Philippe and Benjamin Hermalin (1990), "Legal restrictions on private contracts can enhance efficiency" *Journal of Law, Economics and Organization*, 6(2):381-409.
- Akerlof, George (1970), "The market for lemons: Quality uncertainty and the market mechanism" *Quarterly Journal of Economics*, 89:488-500.
- Alchian, A., and H. Demsetz (1972), "Production, Information Costs, and Economic Organization" *American Economic Review*, Vol. 62.
- Baker, George P. (1992), "Incentive Contracts and Performance Measurement" *Journal of Political Economy*, 100(3):598-614.
- Baker, G., M. Jensen and K. J. Murphy (1988), "Compensation and incentives: practice vs. theory" *Journal of Finance*, XLIII: 593-616.
- Cho, I-K, and D. M. Kreps (1987), "Signaling games and stable equilibria" *Quarterly Journal of Economics*, 102:179-221.
- Cowen, T. and A. Glazer (1996), "More monitoring induces less effort" , *Journal of Economic Behaviour and Organizations*, 145(4), 627-642.
- Dickens, W.T. (1986), "Wages, employment and the threat of collective action by workers", mimeo, Berkeley University.
- De Groot, M. (1970) "Optimal Statistical Decisions", New York, McGraw-Hill.
- Dubey, P. and C.W. Wu (2001), "Competitive Prizes: When Less Scrutiny Induces More Effort", *Journal of Mathematical Economics*, 36, 311-336.
- Fama, E. F. (1980), "Agency Problems and the Theory of the Firm", *Journal of Political Economy* 88, 288-307.
- Gibbons, Robert and Kevin Murphy (1990), "Relative Performance Evaluation for Chief Executive Officers", *Industrial and Labor Relations Review* 43(supplement), 30-51.
- Gibbons, Robert and Michael Waldman (1999), "Careers in Organizations: Theory and Evidence", in Orey Ashenfelter and David Card, "Handbook of Labor Economics", Volume 3, ch. 36, Amsterdam, Elsevier Science.
- Guash, Luis and Andrew Weiss (1981), "Self-selection in the labor market" *American Economic Review*, June, 275-284.
- Holmstrom, Bengt and Roger B. Myerson (1983), "Efficient and Durable Decision Rules with incomplete information" *Econometrica*, 51(6):1799-1820.
- Holmstrom, Bengt (1979), "Moral hazard and observability" *Bell Journal of Economics*, 9: 74-91.
- Holmstrom, Bengt (1982), "Moral hazard in teams" *Bell Journal of Economics*, 13: 324-340.
- Holmstrom, Bengt (1999), "Managerial Incentive Problems: A Dynamic Perspective" *Review of Economic Studies*, 66: 169-182.

- Holmstrom, Bengt and Paul Milgrom (1991), "Multitask Principal Agent Analyses: Linear Contracts, Asset Ownership and Job Design" *Journal of Law, Economics and Organization*, 7 (suppl.)(6): 24-52.
- Kahn, Charles and Gur Huberman (1988), "Two-sided uncertainty and "up-or-out" contracts" *Journal of Labor Economics*, 6(4): 423-444.
- Katz, Lawrence F. (1986), "Efficiency Wage Theories: A partial Evaluation" in the "NBER Macro Annual.
- Kerr, Steven (1975), "On the folly of rewarding A, while hoping for B" *Academy of Management Journal*, 43: 769-783.
- Ichino, Andrea (1994), "Flexible Labour Compensation, Risk Sharing and Company Leverage", *European Economic Review*.
- Ichino, Andrea and Gerd Muehlheusser (2004), "How Often Should You Open the Door? Optimal Monitoring to Screen Heterogeneous Agents", mimeo, EUI
- Lazear, Edward P. and Sherwin Rosen (1981), "Rank-Order Tournaments as Optimum Labor Contract", *Journal of Political Economy* 89(5), 841-864.
- Lazear, Edward P. (1995), "Personnel Economics", Cambridge, MIT press.
- Lazear, Edward P. (1999), "Output-Based Pay: Incentives or Sorting?", NBER Working Paper 7419.
- Lazear, Edward P. (2004), "The Peter Principle: A theory of Decline", *Journal of Political Economy* 112(1), s141-163.
- Mas-Colell, Andreu, Mark Whinston, and Jerry Green (1995), "Microeconomic theory", Oxford, Oxford University Press.
- Mirrlees, Jim (1974), "Notes on economic behavior under uncertainty", in M. Blch, D. McFadden and S. Wu, (eds.) "Essays on economic behaviour under uncertainty", North Holland, Amsterdam, 243-258.
- Mirrlees, Jim (1976) "The optimal structure of incentives and authority within an organization", *Bell Journal of Economics*, 7: 105-131.
- Peter, Lawrence J. and Raymond Hull (1969), *The Peter Principle: Why Things Always Go Wrong*, New York, Morrow.
- Prendergast, Canice (1993), "The role of promotion in inducing specific human capital investment" *Quarterly Journal of Economics*, 110(5):1071-1102.
- Prendergast, Canice (1999), "The Provision of Incentives in Firms" *Journal of Economic Literature*, XXXVII(March):7-63.
- Prendergast, Canice (2002), "The Tenuous Trade-off between Risk and Incentives" *Journal of Political Economy*, 110(5):1071-1102.
- Salanié, Bernard (2002), "The economics of contracts", Cambridge, The MIT Press.
- Shapiro, Carl and Joseph Stiglitz (1984), "Equilibrium Unemployment as a Discipline device", *American Economic Review*, 74, 433-444.
- Schavell, S. (1979), "Risk sharing and incentives in the principal and agent relationship" *Bell Journal of Economics*, 10: 55-73.

Spence, A. M. (1973), "Job Market Signaling" *Quarterly Journal of Economics*, 87:355-374.

Rawls, JOHN (1971), "A theory of justice", 1971.

Wang, R. and A Weiss (1998), "Probation, layoffs and wage-tenure profiles: A sorting explanation", *Labour Economics*, 5(3), 359-383.

Weitzman, Martin and Douglas Kruse (1990), *Profit Sharing and Productivity*, in Alan Blinder, (ed.), "Paying for Productivity", The Brookings Institution, Washington.

Weiss A (1980) "Job queues and layoffs in labor markets with flexible wages" *Journal of Political Economy*, 88:526-538.