Trading with a common agent under full information: The minimum rent equilibrium *

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May 29, 2008

Abstract

We analyze an abstract model of trading where N principals submit quantity-payment schedules that describe the contracts they offer to an agent, and the agent then chooses how much to trade with every principal. Among the equilibria of this game, we focus on the one in which the agent's rent is minimized, contrasting it with the truthful equilibrium. Minimum rent and truthful equilibria coincide when there are only two principals, but differ with three or more principals. In truthful equilibria the competition among the principals is strongest when they are symmetric, and a merger between principals always hurts the agent. In minimum rent equilibria, by contrast, a certain degree of asymmetry may improve competition, and the agent may benefit from a merger even in the absence of efficiency gains.

1 Introduction

Many models in auction theory and industrial organization can be viewed as instances of an abstract game of trading, in which N principals first submit quantity-payment schedules describing the contracts they offer to the agent, and the agent then chooses how much to trade with every principal and at what prices. Common agency games of trading include models of oligopolistic price discrimination, where the agent is

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a consumer who purchases a good from various firms that compete in non-linear prices (Spence, 1976; Spulber, 1979; Bhaskar and To, 2004); vertical relationships, where several upstream firms share a common downstream agent that distributes the product (O'Brien and Shaffer, 1997, 2005); split-award procurement, where a sponsor procures a good from various suppliers (Anton and Yao, 1989); and markets for intermediate goods, where a large buyer purchases from various firms. In these applications, the agent purchases the good from the principals. The converse case in which the agent sells the good to the principals, first studied by Stole (1991) and Martimort (1992), can apply to multi-unit package auctions (Khrisna and Tranaes, 2003; Milgrom, 2004) and the regulation of multinational enterprises by agencies in several countries, where each national agency offers the multinational a payment conditional on the quantity sold in its country (Calzolari, 2001).

With complete information, in such games the equilibrium allocation is unique and efficient, but there is a large multiplicity of equilibrium payments and payoffs. In a companion paper (Chiesa and Denicolò, 2008) we provide a complete characterization of equilibrium payments and payoffs, showing that the set of the principals' payoffs is a hyper-rectangle: every principal, that is to say, can obtain any positive payoff below an upper bound that is independent of the other principals' payoffs and can be calculated explicitly.

Payoffs are important, as they determine not only the distribution of the gains from trade, but also players' incentives (in an earlier stage) to invest in research, bear fixed entry costs, or make other kinds of investments that may affect those gains. This paper develops a comparative statics analysis of equilibrium payoffs, contrasting the equilibrium that is Pareto-dominant for the principals (i.e., the north-east vertex of the hyper-rectangle) with the truthful equilibrium.[1] There are several reasons to fo-

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[1] Generally speaking, a strategy is said to be truthful relative to a given action if it truly, and for all cases, reflects the principals' marginal preferences for another action relative to the given action (Bernheim and Whinston, 1986). In our framework, truthfulness means that each principal can ask for payments that differ from his true valuations of the proposed trades only by a constant.

cus on these equilibria. First, both Pareto dominance and truthfulness are appealing criteria for selecting equilibria, which coincide when there are two principals ($N = 2$), but differ when $N > 2$. In the wake of a commonly accepted theory of equilibrium selection, knowledge of the different comparative statics properties of different equilibria may provide valuable guidance to applied theorists. An additional reason for interest in truthful equilibria is that most of the common agency literature so far has focused on them. The Pareto-dominant equilibrium is also of special interest. First, it fully determines the set of equilibrium payoffs since it corresponds to the north-east vertex of the hyper-rectangle, while the south-west vertex is always the origin. Moreover, in the equilibrium that is Pareto-dominant for the principals the agent's payoff is minimized, so the analysis of such equilibrium illustrates the determinants of the rent that is guaranteed to the agent just because of the competition among the principals (Laussel and Le Breton, 2001).

In our comparative statics analysis, we focus on the classic question of the effects of market concentration. To this end, we specialize the model assuming that principals differ only in size – productive capacity if principals are suppliers, "absorptive" capacity if they are purchasers. In equilibrium, each principal's capacity fully determines his market share, i.e., his share of the aggregate trade. We then analyze the effects of the distribution and redistribution of capacity on equilibrium payments and payoffs.

Our main results are as follows. First, while in the truthful equilibrium the payment to or from a principal i depends only on his size, in the minimum rent equilibrium it depends also on the the size of his "pivotal" competitor, i.e., the principal who plays a special disciplining role by posing the most serius threat to replace principal i. Under our assumptions, a principal's pivotal competitor turns out to be his largest competitor, implying that in the minimum rent equilibrium two principals play a special disciplining role: the largest principal, who is the pivotal competitor of the remaining $N - 1$ principals, and the second largest principal, who is the pivotal competitor of the largest principal.

This has important consequences for equilibrium payoffs. While in truthful equilibria asymmetry always hurts the agent, whose rent is largest when principals are symmetric, in the minimum rent equilibrium a certain degree of asymmetry can benefit the agent. For example, in a market with two large principals and several smaller ones, the disciplining principals are stronger than if all principals were symmetric, and hence the market can be more competitive. Remarkably, this can provide some support to certain antitrust policies adopted in practice. European antitrust authorities, for instance, routinely look at the difference between the market shares of the largest and the second-largest firm to ascertain whether the former holds a dominant position. Standard oligopoly models, where the two largest firms do not play any special role in determining equilibrium outcomes, are unable to explain such a focus.

The analysis of mergers between principals also provides interesting insights. In the minimum rent equilibrium, when some principals merge, the insiders always gain, whereas outsiders never gain and may even lose. In standard oligopoly theory, by contrast, outsiders typically gain more than insiders from a merger. More strikingly, in the minimum rent equilibrium the agent can benefit from a merger between principals even in the absence of efficiency gains. Such an outcome, which can never happen in truthful equilibria, may occur in the minimum rent equilibrium when a merger narrows the gap between the market shares of the two largest firms, thereby strengthening the dominant firm's pivotal competitor. It may occur even if the merger creates a new dominant firm, provided that the merged entity is not too much larger than the previously dominant firm. This may help explain why antitrust authorities are sometimes sympathetic to mergers that strengthen the dominant firm's strongest competitor, even though standard oligopoly models would predict that such mergers are anti-competitive.

The remainder of the paper is organized as follows. Section 2 presents the model and some preliminary results. Section 3 characterizes the equilibrium payments and asks whether asymmetry benefits or harms the agent. Section 4 develops a simple linear-quadratic example, and Section 5 analyzes the effects of mergers. Section 6 offers concluding remarks.

2 The model

This section sets up a specialized version of the common agency model of trading analyzed in Chiesa and Denicolò (2008), to which we refer the reader for omitted proofs and further details.

There are N principals, indexed by $i \in N = \{1, 2, ...N\}$, who trade a homogeneous good with a common agent, indexed by 0. For ease of exposition, we posit that the agent (female) purchases the good from the principals (males). (The converse interpretation where the agent sells the good to the principals is sketched in footnote 3.) Trade is modeled as a first-price auction in which principals simultaneously submit a menu of contracts and the agent then chooses the quantity she will purchase from each.[2] The game is one of complete information.

The quantities traded by the agent with other principals are assumed not to be contractible, so a strategy for a generic principal i is a set $B_i \subset \Re^2_+$ of quantity-payment pairs $b_i = (x_i, P_i)$, where $x_i \geq 0$ is the quantity that principal i is willing to supply and $P_i \geq 0$ is the corresponding total payment requested from the agent. We shall refer to a quantity-payment pair as a contract and to the menu of all contracts offered by a principal as a supply schedule. We set almost no restrictions on feasible supply schedules, assuming only that supply schedules are compact (to guarantee the existence of an optimal choice for the agent), and that the null contract $b^0 = (0, 0)$ belongs to any feasible supply schedule (this is just for notational convenience). Thus, letting Γ denote the set of feasible supply schedules (the same for all principals):

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[2] A popular modeling alternative is the notion of supply function equilibrium proposed by Klemperer and Meyer (1989). In this approach, buyers are price-takers and the unique equilibrium price is determined by equating aggregate supply and demand. In our common agency model, by contrast, a centralized buyer exercises market power by maximizing along each principal's "supply function," and principals trade at personalized prices.

Assumption 1 $\Gamma = \left\{ B \subset \Re_{+}^{2} \mid B \text{ is compact and } (0,0) \in B \right\}$.

Given a profile of supply schedules $\mathbf{B} = (B_1, B_2, ..., B_N) \in \Gamma^N$, in the second stage of the game the agent selects a contract from each supply schedule, trading is conducted, and payoffs are realized. Notice that since we have included the null contract in any feasible supply schedule, the agent can effectively refuse to trade with some principals by selecting their null contracts. A strategy for the agent is thus a function $\boldsymbol{\beta}(\mathbf{B}) : \Gamma^N \to \left(\Re_{+}^{2}\right)^N$ such that $\boldsymbol{\beta}(\mathbf{B}) \in \times_{i=1}^{N} B_i$ for all $\mathbf{B} \in \Gamma^N$. In other words, $\boldsymbol{\beta} = (\beta_1, \beta_2, ..., \beta_N)$ is the profile of contracts accepted by the agent, one for each principal, and the function $\boldsymbol{\beta}(\mathbf{B})$ maps Γ^N, the set of all profiles of feasible supply schedules, to the set of admissible trades.

For any given $\boldsymbol{\beta}$, the agent's payoff is

$$\pi_0(\boldsymbol{\beta}) = U(X) - \sum_{i=1}^{N} P_i \tag{1}$$

where $X = \sum\limits_{i=1}^{N} x_i$ is the total quantity traded and the function $U(X) : \Re_{+} \to \Re_{+}$ denotes its value to the agent, in monetary terms. The principals' payoffs are

$$\pi_i(\boldsymbol{\beta}) = \pi_i(\beta_i) = P_i - C(x_i, k_i). \tag{2}$$

where $C(x_i, k_i) : \Re_{+}^{2} \to \Re_{+}$ is the cost function, the same for all principals, which depends on output x_i and productive capacity k_i.[3] Aggregate capacity is fixed, and without any further loss of generality it is normalized 1, so that $\sum\limits_{i=1}^{N} k_i = 1$.

We make the following regularity assumptions.

Assumption 2 *The function $U(X)$ is differentiable, strictly increasing and strictly concave (and hence a.e. twice differentiable) with $U(0) = 0$.*

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[3] If the agent sold the good to the principals, $U(X)$ would be a cost function and would be convex, while $C(x_i, k_i)$ would represent the value of the good to principals and would be concave in x_i. In this case, one could think of k_i as the "absorptive" capacity of principal i. For example, if the agent sells a risky asset and all principals share the same von Neumann-Morgenstern utility function C, principal i's propensity to purchase the risky asset x_i may depend on his wealth k_i.

Assumption 3 *The cost function* $C(x_i, k_i)$ *is differentiable, strictly increasing in* x_i*, strictly decreasing in* k_i*, and strictly convex (and hence a.e. twice differentiable) in* x_i*, with* $C(0, k_i) = 0$*, for all* $i \in N$ *and all* $k_i > 0$*.*

Assumption 4 *(Inada conditions)* $\lim_{X\to 0} U'(X) > 0$, $\lim_{X\to\infty} U'(X) = 0$*, and, for all* $i \in N$ *and all* $k_i > 0$*,* $\lim_{x_i\to 0} C'_{x_i}(x_i, k_i) = 0$ *and* $\lim_{x_i\to\infty} C'_{x_i}(x_i, k_i) = \infty$*.*

Assumptions 2 and 3 guarantee that the equilibrium allocation is efficient, so we can focus on equilibrium payoffs. The assumption $C(0, k_i) = 0$ means that all fixed costs are sunk. The assumption that marginal costs are strictly increasing rules out the trivial case of Bertrand competition with constant marginal costs. Finally, the Inada conditions ensure that efficiency requires every principal to trade a strictly positive and finite quantity with the agent.

Since ours is a two-stage game of complete information, we employ the solution concept of subgame perfect Nash equilibrium. A subgame perfect Nash equilibrium (henceforth, an equilibrium) is a list of strategies, $\left\langle \hat{\boldsymbol{\beta}}(\mathbf{B}), \hat{B}_1, \hat{B}_2, ..., \hat{B}_N \right\rangle$, one for each player, such that

$$\hat{\boldsymbol{\beta}}(\mathbf{B}) \in \arg \max_{\boldsymbol{\beta} \in \times_{i=1}^{N} B_i} \pi_0(\boldsymbol{\beta}) \qquad \forall \mathbf{B} \in \Gamma^N \tag{3}$$

and

$$\hat{B}_i \in \arg\max_{B_i \in \boldsymbol{\Gamma}} \pi_i(\hat{\boldsymbol{\beta}}(\hat{\mathbf{B}}_{-i}, B_i)) \qquad \forall i \in N \tag{4}$$

where $(\hat{\mathbf{B}}_{-i}, B_i) \equiv \left(\hat{B}_1, ..., \hat{B}_{i-1}, B_i, \hat{B}_{i+1} ..., \hat{B}_N\right)$.

The *efficient allocation* is the vector $\mathbf{x}^* = (x_1^*, x_2^*, ..., x_N^*)$ that maximizes the sum of the players's payoffs $S(\mathbf{x}) = U(X) - \sum_{i=1}^{N} C(x_i, k_i)$. Given Assumptions 2 and 3, the social surplus $S(\mathbf{x})$ is bounded above and globally, strictly concave; thus, the efficient allocation is the unique solution to the following system of first-order conditions:

$$U'(X) = C'_x(x_i, k_i) \qquad \forall i \in N. \tag{5}$$

By Assumption 4, $\mathbf{x}^*$ is strictly positive.

Under Assumptions 1-4, the following result holds:

Lemma 1 *In any equilibrium,* $\hat{\mathbf{x}} = \mathbf{x}^*$.

To guarantee that any redistribution of capacity is efficiency neutral and can affect only the division of the total surplus among players, we further specialize the model by making the following:

Assumption 5 *The cost function* $C(x_i, k_i)$ *is homogeneous of degree one in output* x_i *and capacity* k_i.

Assumption 5 allows us to define the normalized cost function $c(\chi_i) = \frac{C(x_i,k_i)}{k_i} = C(\chi_i, 1)$, which expresses the cost per unit of capacity as a function of $\chi_i = \frac{x_i}{k_i}$, the output per unit of capacity. By Euler's theorem we have $C'_x(x_i, k_i) = c'(\chi_i)$ and $C'_k(x_i, k_i) = c(\chi_i) - \chi_i c'(\chi_i)$. Another consequence of Assumption 5 is that the higher the capacity of a principal, the larger is his equilibrium market share. More precisely:

Lemma 2 *Equilibrium total output* $X^* = \sum x_i^*$ *and aggregate costs* $\sum C(x_i^*, k_i)$ *are independent of the distribution of capacity across principals. Moreover, the equilibrium market share of principal* i *equals his share of aggregate capacity:*

$$x_i^* = k_i X^*. \tag{6}$$

Proof. From (5) it follows immediately that in equilibrium χ_i is the same for all principals, so from Lemma 1 we get $\chi_i = X^*$. Equations (5) then all collapse to $U'(X^*) = c'(X^*)$, which uniquely determines X^* independently of the capacity distribution. Aggregate costs $\sum C(x_i^*, k_i)$ are $\sum C(x_i^*, k_i) = \sum k_i c(X^*) = c(X^*)$, so they too are independent of the capacity distribution. ■

Relabel principals such that $k_1 \geq k_2 \geq ... \geq k_N$. Then, principals are unambiguously ranked in terms of their marginal costs when individual output exceeds efficient output: principal 1 is the most efficient, 2 the second most efficient, etc.[4]

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[4] That is, Assumption 5 implies the No-crossing condition employed by Chiesa and Denicolò (2008).

Now we turn to equilibrium payoffs, which are not unique. The multiplicity of equilibrium payoffs arises because each principal i can offer many contracts other than the one that will be accepted in equilibrium. These other contracts do not affect i's own payoff, but they may constrain the payments that i's competitors can request for their prescribed equilibrium quantities, and hence their payoffs. In particular, the more aggressively principal i bids for quantities greater than x_i^*, the lower his competitors' payoffs.

This suggests that principals can tacitly "coordinate" their offers in order to increase their payoffs. What is the maximum degree of coordination among principals in a non-cooperative equilibrium? The limits to coordination are due to the fact that if a principal i asks for an excessive payment, it may become profitable for other principals to offer a contract that crowds i out. That is, the maximum payment that principal i can obtain for x_i^* is determined by the threat of being replaced by somebody else. More precisely, it can be shown that each principal i has a *pivotal competitor*, denoted by v_i, who poses the most serious threat to replace him. The maximum payment i can request, $\bar{P}_i$, makes his pivotal competitor v_i just indifferent between supplying his prescribed equilibrium quantity or proposing to the agent to unilaterally replace i, given that all the other principals but i continue to supply their equilibrium quantities. Formally, let:

$$V_i(X_{-i}) = \max_{x_i}[U(X_{-i} + x_i) - C(x_i, k_i)] \tag{7}$$

be the maximum joint payoff that the agent and principal i can get as a function of the quantity supplied by the other principals, X_{-i},[5] and gross of any payments to them. By Assumptions 2-4, the functions $V_i(X_{-i})$ are well defined for all $i \in N$, strictly increasing, and strictly concave. Then, Chiesa and Denicolò (2008) show that the maximum payment that principal i can ask in equilibrium for his prescribed

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[5] Hereafter $X_{-\Omega}$ denotes $\sum_{j \notin \Omega} x_j$ for all $\Omega \subset N$; with a slight abuse of notation, we write X_{-i} instead of $X_{-\{i\}}$.

equilibrium quantity x_i^* is

$$\bar{P}_i = V_{v_i}(X^*_{-v_i}) - V_{v_i}(X^*_{-\{i,v_i\}}), \qquad \forall i \in N \tag{8}$$

where the pivotal competitor of principal i is his largest competitor, i.e.,

$$v_1 = 2 \qquad \text{and} \qquad v_i = 1 \qquad \text{for } i = 2, 3, ..., N. \tag{9}$$

The associated maximum equilibrium payoff is

$$\bar{\pi}_i = \bar{P}_i - C(x_i^*, k_i), \qquad \forall i \in N. \tag{10}$$

Summarizing:

Lemma 3 *A vector of payoffs* $(\pi_0, \pi_1, \pi_2, ..., \pi_N)$ *is a vector of equilibrium payoffs if and only if it satisfies* $\pi_0 + \pi_1 + \pi_2 + ... + \pi_N = S^*$ *and* $0 < \pi_i \leq \bar{\pi}_i \quad \forall i \in N$.

The set of the principals' equilibrium payoffs is a semi-open hyper-rectangle. The equilibrium that is Pareto-dominant for the principals, i.e., the north-east vertex of the hyper-rectangle, is the *minimum rent* equilibrium since it minimizes the agent's rent. It is interesting to contrast the minimum rent equilibrium with the truthful equilibrium, where each principal earns exactly his marginal contribution to social surplus (Bergemann and Välimäki, 2003). In the truthful equilibrium, equilibrium payments are

$$\tilde{P}_i = S^* - S^*_{-i} + C(x_i^*, k_i), \qquad \forall i \in N \tag{11}$$

where $S^* = S(\mathbf{x}^*)$ is the maximized social surplus, and S^*_{-i} is the maximum social surplus attainable when principal i is inactive, i.e.

$$S^*_{-i} = \max_{\substack{x_j \\ j \neq i}} \left[U\left(X_{-i}\right) - \sum_{j \neq i} C\left(x_j, k_j\right) \right]. \tag{12}$$

Lemma 4 *When* $N = 2$*, the minimum rent equilibrium is truthful. When* $N > 2$*, the two equilibria differ, and in the minimum rent equilibrium every principal's payoff exceeds his marginal contribution to social surplus:* $\bar{P}_i > \tilde{P}_i$.

The intuition is simple. A principal's marginal contribution to social surplus is the difference between maximized social surplus and the social surplus that is attained when that principal is *optimally* replaced by the remaining $N-1$ principals. However, Lemma 3 shows that in a non-cooperative equilibrium each principal's maximum payoff is determined by the threat of being *unilaterally* replaced by one of his competitors – the pivotal competitor. Because marginal costs are increasing, when $N > 2$ unilateral replacement by one principal is more costly than cooperative replacement by $N-1$ principals. This explains why in the minimum rent equilibrium each principal can earn more than his marginal contribution to social surplus.

3 Comparative statics

Now we analyze the effects of a change in the distribution of aggregate capacity across principals on the players' payoffs, contrasting the comparative statics properties of the truthful equilibrium with those of the minimum rent equilibrium. In particular, we are interested in the determinants of the agent's rent $\pi_0 = U(X^*) - \sum_{i=1}^{N} P_i$. In the limiting case in which $k_1 = 1$, the agent is effectively facing a single principal, so her rent vanishes. This simple result highlights the fact that with complete information, since principals have all the bargaining power, any rent earned by the agent must be due to the competition among the principals. From this viewpoint, the problem we address is how a redistribution of capacity affects the intensity of the competition among the principals.

The answer depends on which equilibrium prevails. Let us consider the truthful equilibrium first. In a truthful equilibrium, the payment to principal i depends only on his market share and is independent of the distribution of the residual capacity $1-k_i$ across the remaining $N-1$ principals.

Proposition 1 *In the truthful equilibrium, $\tilde{P}_i = \tilde{P}(k_i)$ where the function $\tilde{P}(.)$ is strictly increasing and strictly convex, with $\tilde{P}(0) = 0$.*

Proof. See the Appendix. ■

In the minimum rent equilibrium, by contrast, when $N > 2$ equilibrium payments $\bar{P}_i$ depend not only on i's market share, but also on his pivotal competitor's one. However, $\bar{P}_i$ is still independent of the distribution of the residual capacity $1 - k_i - k_{v_i}$ across the remaining $N - 2$ principals.

Proposition 2 *When* $N > 2$*, in the minimum rent equilibrium* $\bar{P}_i = \bar{P}(k_i, k_{v_i})$*, where the function* $\bar{P}(k_i, k_{v_i})$ *is strictly increasing and strictly convex in* k_i *with* $\bar{P}(0, k_{v_i}) = 0$*, and is strictly decreasing in* k_{v_i}*.*

Proof. See the Appendix. ■

Using Propositions 1 and 2, we can easily trace out the effects of a capacity redistribution on the agent's rent. In the truthful equilibrium, the competition between the principals is stronger, and hence the agent's rent is larger, the more equal is the distribution of capacity. Formally, define the Lorenz curve of the capacity distribution $\kappa = (k_1, k_2, ..., k_N)$ as $L_\kappa(\frac{h}{N}) = \sum_{i=N-h}^{N} k_i$. It shows the proportion of aggregate capacity owned by any given percentage of the population of principals, starting from the smallest principal. Comparing two capacity distributions κ and κ', κ is more equal than κ' according to the Lorenz dominance criterion if $L_\kappa(\frac{h}{N}) \geq L_{\kappa'}(\frac{h}{N})$ for all $h = 1, 2, ..., N - 1$, with at least one strict inequality.

Proposition 3 *In a truthful equilibrium, if the capacity distribution becomes more equal according to the Lorenz dominance criterion, the agent's rent increases. Thus, the agent's rent is largest when she faces* N *symmetric principals.*

Proof. By Proposition 1, in the truthful equilibrium the agent's rent can be rewritten as $\tilde{\pi}_0 = U(X^*) - \sum_{i=1}^{N} \tilde{P}(k_i)$. Since $\tilde{P}(.)$ is strictly convex, $\tilde{\pi}_0$ is strictly concave. By Atkinson's theorem, it follows that if capacity distribution κ Lorenz dominates distribution κ', the agent's rent under distribution κ is greater than under distribution κ' (see Atkinson, 1970). Since the equal distribution Lorenz dominates any other distribution, the agent's rent is maximized when capacity is equi-distributed. ■

Things are different in the minimum rent equilibrium when $N > 2$. (When $N = 2$, the minimum rent equilibrium is truthful). Using Proposition 2, we can express the agent's minimum rent as

$$\bar{\pi}_0 = U(X^*) - \sum_{i=1}^{N} \bar{P}(k_i, k_1) - \left[\bar{P}(k_1, k_2) - \bar{P}(k_1, k_1)\right] \tag{13}$$

The first term on the right hand side of (13), $U(X^*)$, is a constant. The second term, $\sum_{i=1}^{N} \bar{P}(k_i, k_1)$, is the aggregate payment that the agent would make for X^* if each principal had a pivotal competitor of size k_1. Since $\bar{P}(k_i, k_1)$ is convex in k_i, this term can be interpreted as an index of market concentration. For example, in the linear-quadratic example developed below, $\sum_{i=1}^{N} \bar{P}(k_i, k_1)$ is a linear transformation of the Hirschman-Herfindahl concentration index. This term highlights the fact that principal 1 plays a special disiciplining role, because the maximum payoff that a principal can get in the minimum rent equilibrium is determined by the size of his pivotal competitor, and because principal 1, being the largest principal, is the pivotal competitor of all the other $N-1$ principals. The third term on the right hand side of (13) accounts for the fact that principal 1's pivotal competitor actually has a market share of k_2, not k_1. That is, the pivotal competitor of the largest principal is the second largest principal. Since $\bar{P}\left(k_i, k_{v_i}\right)$ is decreasing in k_{v_i}, the difference between the market shares of the two largest principals impacts the agent's rent negatively. Summarizing, in equilibrium two principals play a special role: the largest principal (who is the pivotal competitor of the remaining $N-1$ principals) and the second largest principal (who is the pivotal competitor of the largest principal).

For any given k_1 and k_2, the convexity of $\sum_{i=1}^{N} \bar{P}(k_i, k_1)$ implies that the agent prefers the residual capacity to be distributed evenly across the other $N-2$ principals. But now a symmetric distribution of capacity across principals is no longer optimal for the agent.

Proposition 4 *In the minimum rent equilibrium, the capacity distribution that maximizes the agent's rent is asymmetric when $N > 2$.*

Proof. Starting from a symmetric capacity distribution, we calculate the effect of a small increase in the market share of principal 1 and a corresponding parallel decrease in the market shares of principals 3, 4, ..., N, leaving the size of principal 2 unchanged. That is, we posit that $dk_i = -\frac{1}{N-2}dk_1$ for $i = 3, 4, ...N$. Differentiating (13) we then get:

$$\begin{aligned} \frac{d\bar{\pi}_B}{dk_1} &= -\left[-(N-2)\frac{\partial \bar{P}}{\partial k_i}\frac{1}{(N-2)} + (N-1)\frac{\partial \bar{P}}{\partial k_{v_i}} + \frac{\partial \bar{P}}{\partial k_i}\right] \\ &= -(N-1)\frac{\partial \bar{P}}{\partial k_{v_i}} > 0. \end{aligned}$$

This means that a small move away from the symmetric distribution increases the agent's rent, which therefore is not maximized when capacity is equi-distributed. ■

The intuitive reason is that an increase in k_1 has two opposite effects on the agent's rent. On one hand, the equilibrium payment to principal 1 increases at an increasing rate with his market share, and those to all the other principals decrease at a decreasing rate with their respective market shares. As a result, the agent's total payment tends to increase, and hence her rent tends to decrease with k_1. On the other hand, principal 1 is the pivotal competitor of all the other $N-1$ principals, so in the minimum rent equilibrium an increase in k_1 tends to reduce the equilibrium payments to them, and hence to increase the agent's rent. Starting at equi-distribution, the negative effect of an increase in k_1 is second order while the positive effect is first order, so the agent wants to increase $k_1$somewhat.

For similar reasons, the agent might want to somewhat increase k_2, since principal 2 is the pivotal competitor of principal 1, so principal 2's size limits principal 1's market power. Unfortunately, we have not found any simple characterization of the distribution of capacity that maximizes the agent's rent in the general case.

4 The linear-quadratic case

To illustrate some of the results obtained so far, in this Section we develop a linear-quadratic example. We assume a quadratic utility function:

$$U = aX - \frac{1}{2}bX^2, \tag{14}$$

and a quadratic, homogeneous of degree one cost function:

$$C\left(x_i, k_i\right) = \frac{1}{2}\frac{c}{k_i}x_i^2. \tag{15}$$

where a, b and c are positive parameters. Although strictly speaking (14) does not obey the Inada conditions, it is sufficiently well behaved to yield an economically meaningful solution of the game.

The efficient allocation is the unique solution to the system of first order conditions (5), yielding

$$x_i^* = k_i\frac{a}{b+c}. \tag{16}$$

Clearly, $X^* = \frac{a}{b+c}$ is independent of the distribution of total capacity across principals. The maximized social surplus is

$$S^* = \frac{a^2}{2\left(b+c\right)}. \tag{17}$$

Let us consider the truthful equilibrium first. Straighforward calculation shows that

$$S_{-i}^* = \frac{(1-k_i)a^2}{2\left[(1-k_i)b+c\right]}, \tag{18}$$

so equilibrium payments are

$$\tilde{P}_i = \frac{a^2}{4\left(b+c\right)^2}ck_i\frac{b(1+k_i)+c}{\left[(1-k_i)b+c\right]}. \tag{19}$$

This implies that the principals' equilibrium payoffs are

$$\pi_i = \frac{c}{(1-k_i)b+c}k_iS^*. \tag{20}$$

Note that each principal obtains a share of S^* lower than his market share k_i (unless $k_i = 1$, in which case there is in fact only one principal who makes a takes-it-or-leave-it offer, extracting all the rent). This implies that the agent obtains a positive rent, given by

$$\tilde{\pi}_0 = S^* \left[1 - \sum_{i=1}^{N} \frac{ck_i}{[(1-k_i)b + c]} \right] \tag{21}$$

It can be easily checked that the agent's rent is maximized at $k_1 = k_2 = .. = k_N = \frac{1}{N}$, i.e., when capacity is equi-distributed.

Next consider the minimum rent equilibrium. Using (7) and (8), one calculates:

$$\bar{P}_i = \frac{ca^2}{2\left(b+c\right)^2} k_i \left[\frac{2c + b\left(2k_{v_i} + k_i\right)}{c + bk_{v_i}} \right] \tag{22}$$

The agent's rent is $\bar{\pi}_0 = U(X^*) - \sum\limits_{i=1}^{N} \bar{P}_i$, or

$$\bar{\pi}_0 = \frac{ca^2}{\left(b+c\right)^2} \left[\frac{b+2c}{2} - 1 - \frac{b}{2(c+bk_1)} HHI - \frac{b^2 k_1^2 \left(k_1 - k_2\right)}{2\left(c+bk_2\right)\left(c+bk_1\right)} \right], \tag{23}$$

where $HHI = \sum\limits_{i=1}^{N} k_i^2$ is the Hirschman-Herfindahl concentration index.

Even in this linear-quadratic case, it is difficult to analytically characterize the market structure most favorable to the agent in the minimum rent equilibrium, i.e., the one that maximizes $\bar{\pi}_0$. However, one can confirm that a certain degree of asymmetry benefits the agent by resorting to numerical calculations. Let us consider, for instance, the case $N = 3$. Then, the optimal degree of asymmetry positively depends on the ratio $\frac{b}{c}$. When $\frac{b}{c}$ is close to zero, the agent's rent is maximized when capacity is equi-distributed, just as in the truthful equilibrium. When $\frac{b}{c} = 1$, however, the most competitive market structure is approximately $k_1 = 0.363$, $k_2 = 0.343$ and $k_3 = 0.296$; and as the ratio $\frac{b}{c}$ grows arbitrarily large, the most competitive market structure converges to $k_1 = 0.412$, $k_2 = 0.402$ and $k_3 = 0.186$. In this example, both k_1 and k_2 increase monotonically with $\frac{b}{c}$.

5 Mergers

Now we analyze the effects of mergers, assuming that when two or more principals merge, the merged entity has a productive capacity which equals the sum of the productive capacities of the insiders.

Let $M \subset N$ be the set of the insiders. The merged entity will behave as a single principal whose productive capacity is $k_M \equiv \sum\limits_{i \in M} k_i$. Our assumption that variable costs are homogeneous of degree one in output and capacity rules out any efficiency effect of mergers. That is, total output, total costs, and hence the maximized social surplus, are all independent of the distribution of productive capacity. However, mergers can affect the equilibrium payments, and hence the principals' payoffs and the agent's rent.

Let us consider the truthful equilibrium first. By Proposition 1, outsiders are unaffected by a merger. To determine the effect on the insiders, it is convenient to think of the merger as a redistribution of capacity from the smaller insiders to the largest one, i.e., as a set of regressive Pigou-Dalton transfers. This means that a merger shifts down the Lorenz curve of the capacity distribution. It then follows immediately from Proposition 3 that the agent's rent is reduced. Since the agent loses, outsiders are unaffected, and the maximized social surplus is unchanged, a merger always benefits the insiders.

That insiders always benefit from a merger is always true also in the minimum rent equilibrium. To see this, note that while the insiders' aggregate output and costs do not change, the total payment to them increases. The reason is that $\bar{P}(k_i, k_{v_i})$ is strictly convex in k_i and strictly decreasing in k_{v_i}. Since $k_M \equiv \sum\limits_{i \in M} k_i$ and $k_{v_M} \leq k_{v_i}$ for all $i \in M$ (the pivotal competitor of the merged entity cannot be larger than the pivotal competitor of any insider), it follows that $\bar{P}(k_M, k_{v_M}) > \sum\limits_{i \in M} \bar{P}(k_i, k_{v_i})$. By the same logic, the equilibrium payments to outsiders cannot increase, since after the merger the outsider are of the same size as before the merger, and their pivotal competitors cannot be smaller. Summarizing, in the minimum rent equilibrium a merger increases the insiders' aggregate payoff and cannot increase the payoff of any outsider.

What is the effect of a merger on the agent's minimum rent? From the above discussion it is clear that if the merger does not increase the size of the outsiders' pivotal competitors, it necessarily hurts the agent. Therefore, a merger between small principals cannot be pro-competitive. However, things are more complicated when a merger involves large principals. As it turns out, a merger can benefit the agent if it increases the market share of the largest principal, or lowers the difference between the market shares of the two largest principals.

Proposition 5 *In the minimum rent equilibrium, the agent can benefit from a merger between principals even in the absence of efficiency gains.*

Proof. See the Appendix. ■

The intuitive reason why the agent may gain when the merger increases the size of the largest principal is that the largest principal will then exert a stronger disciplining effects on his rivals. The agent may benefit from a merger also when it reduces the difference between the market shares of the two largest firms, thereby strengthening the largest firm's pivotal competitor. This latter possibility (which is the one we have exploited to prove Proposition 5 in the Appendix) seems especially interesting for antitrust policy. It may help explain why antitrust authorities are sometimes sympathetic to mergers that strengthen the dominant firm's strongest competitor, even though standard oligopoly models would predict that such mergers are anti-competitive.[6]

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[6] In Europe, for instance, until the enactment of the 2004 Merger Regulation, a merger should not have been prohibited unless it created or strengthened a dominant position.

6 Conclusions

We have studied a complete information, common-agency game where an agent trades a good with N principals. Principals submit price-quantity schedules that describe the contracts they offer to the agent, and the agent then maximizes her rent by choosing how much to trade with every principal. We have contrasted two equilibria, the truthful equilibrium and the minimum rent equilibrium (i.e., the equilibrium that is Pareto dominant for principals), showing that they exhibit remarkably different comparative statics properties. Although it is hard to tell which equilibrium is most likely to prevail, the comparative statics properties of the minimum rent equilibrium are new and noteworthy.

The key difference between the truthful equilibrium and the minimum rent equilibrium lies in the fact that with increasing marginal costs, the efficient replacement of a principal requires all the other principals to increase their outputs, whereas the upper bound on any one principal's payoff is determined by the threat of unilateral replacement. This intuition suggests that our results may extend to the case of differentiated products with constant or increasing marginal costs. Indeed, if products are substitutes and the agent's preferences are strictly convex, the cooperative replacement of one principal is less costly than unilateral replacement even if marginal costs are constant. The analysis of competition in non-linear prices by firms supplying differentiated products is an interesting topic for future research.

References

Anton, J. J., and D. A. Yao (1989) "Split Awards, Procurement, and Innovation," *RAND Journal of Economics* 20, 538-552.

Atkinson, A. "On the Measurement of Inequality" *Journal of Economic Theory* 2, 244-263.

Bergemann, D. and J. Välimäki (2003), "Dynamic Common Agency," *Journal of Economic Theory* 111, 23-48.

Bernheim, B.D., and M. D. Whinston (1986) "Menu Auctions, Resource Allocation, and Economic Influence," *Quarterly Journal of Economics* 101, 1-31.

Bhaskar, V. and T. To (2004) "Is Perfect Price Discrimination Really Efficient? An Analysis of Free Entry," *RAND Journal of Economics* 35, 762-776.

Calzolari G. (2001), "Incentive Regulation of Multinational Enterprises," *International Economic Review* 45, 257-282.

Chiesa, G. and V. Denicolò (2008), "Trading with a Common Agent: A Characterization of the Nash Equilibria," *Journal of Economic Theory*, forthcoming.

Klemperer, P. and M. Meyer (1989) "Supply Function Equilibrium in Oligopoly with Uncertainty," *Econometrica*, 57, 1283-1298.

Krishna, K., and T. Tranaes (2002) "Allocating Multiple Units," *Economic Theory*, 20, 733-750.

Martimort, D. (1992) "Multi-principaux Avec Anti-selection," *Annales d'Economie et de Statistique*, 28, 1-37.

O'Brien D.P. and G. Shaffer (1997) "Nonlinear Supply Contracts, Exclusive Dealing, and Equilibrium Market Foreclosure," *Journal of Economics and Management Strategy*, 6, 755-785.

O'Brien D.P. and G. Shaffer (2005) "Bargaining, Bundling, and Clout: The Portfolio Effects of Horizontal Mergers," *RAND Journal of Economics* 36, 573-595.

Spence, A. M. (1976), "Product Selection, Fixed Costs and Monopolistic Competition," *Review of Economic Studies* 43, 217-235.

Spulber, D. (1979) “Noncooperative Equilibrium with Price Discriminating Firms,” *Economics Letters* 4, 221-227.

Stole, L. (1991), “Mechanism Design under Common Agency,” mimeo, University of Chicago.

Wilson, R. (2004) *Putting Auction Theory to Work*, Cambridge, Cambridge University Press.

Appendix

Proof of Proposition 1. In a truthful equilibrium, payments are given by (11). As in the proof of Lemma 2, it is clear that under Assumption 5 S^*_{-i} depends only on the total residual capacity $1 - k_i$, and does not depend on how it is distributed among the remaining $N - 1$ principals. Since also $C(x^*_i, k_i)$ depends only on k_i, it follows that $\tilde{P}_i$ depends only on k_i. It is also clear that $\tilde{P}_i(0) = 0$, since x^*_i vanishes when $k_i = 0$.

Next, we show that $\tilde{P}\left(k_i\right)$ is strictly increasing and strictly convex in k_i. Since S^* is independent of the capacity distribution, differentiating (11) we get

$$\frac{d\tilde{P}}{dk_i} = -\frac{dS^*_{-i}}{dk_i} + C'_k(x^*_i, k_i) + C'_x(x^*_i, k_i)\frac{dx^*_i}{dk_i}.$$

We have just argued that the distribution of $1 - k_i$ among the remaining $N - 1$ principals is a matter of indifference, so we can posit that $k_{v_i} = 1 - k_i$ and $k_j = 0$ for all $j \neq i, v_i$ without any loss of generality. This implies that $\dfrac{dk_{v_i}}{dk_i} = -1$. By the envelope theorem we then have $\frac{dS^*_{-i}}{dk_i} = -C'_k(\tilde{x}^i_{v_i}, k_{v_i})\dfrac{dk_{v_i}}{dk_i} = C'_k(\tilde{x}^i_{v_i}, k_{v_i})$, where

$$\tilde{x}^i_{v_i} \equiv \underset{x_{v_i}}{\arg\max}\left[U\left(x_{v_i}\right) - C\left(x_{v_i}, k_{v_i}\right)\right].$$

By implict differentiation,

$$\frac{d\tilde{x}^i_{v_i}}{dk_{v_i}} = \frac{C''_{xk}(\tilde{x}^i_{v_i}, k_{v_i})}{U''(\tilde{x}^i_{v_i}) - C''_{xx}(\tilde{x}^i_{v_i}, k_{v_i})} > 0.$$

Also, we have $\frac{dx^*_i}{dk_i} = X^*$ by (6). Using the normalized cost function $c(\chi)$, the derivative becomes:

$$\frac{d\tilde{P}}{dk_i} = -c(\tilde{\chi}^i_{v_i}) + \tilde{\chi}^i_{v_i}c'(\tilde{\chi}^i_{v_i}) + c(X^*),$$

where $\tilde{\chi}^i_{v_i} = \dfrac{\tilde{x}^i_{v_i}}{k_{v_i}}$. The derivative is positive because $c(\chi)$ is convex (since $c''(\chi) = kC''_{xx}(x, k) > 0$) and $c(0) = 0$.

Differentiating again and keeping in mind that X^* does not depend on the distribution of capacity, we obtain:

$$\frac{d^2\tilde{P}}{d^2k_i} = -\tilde{\chi}^i_{v_i}c''(\tilde{\chi}^i_{v_i})\frac{d\tilde{\chi}^i_{v_i}}{dk_{v_i}},$$

which is positive because $c''(\tilde{\chi}^i_{v_i}) > 0$ and

$$\frac{d\tilde{\chi}^i_{v_i}}{dk_{v_i}} = -\frac{\tilde{\chi}^i_{v_i} U''(\tilde{x}^i_{v_i})}{k_{v_i} U''(\tilde{x}^i_{v_i}) - c''(\tilde{\chi}^i_{v_i})} < 0. \blacksquare$$

Proof of Proposition 2. From (7) and (8) it is immediate that $\bar{P}_i$ can be rewritten as:

$$\bar{P}_i = \left[U\left(X^*\right) - U\left(X^*_{-\{i,v_i\}} + \bar{x}^i_{v_i}\right)\right] + \left[C(\bar{x}^i_{v_i}, k_{v_i}) - C(x^*_{v_i}, k_{v_i})\right],$$

where

$$\bar{x}^i_{v_i} \equiv \arg\max \left[U\left(X^*_{-\{i,v_i\}} + x_{v_i}\right) - C(x_{v_i}, k_{v_i})\right].$$

Clearly, $\bar{x}^i_{v_i} > x^*_{v_i}$, whence it follows $\dfrac{\bar{x}^i_{v_i}}{k_{v_i}} > X^*$.

We first show that $\bar{P}_i$ does not depend on the entire capacity distribution, but only on k_i and k_{v_i} (as long as a change in the capacity distribution does not change v_i). To see this, note that when variable costs are homogeneous of degree one in output and capacity, by Lemma 2 the aggregate efficient output of the remaining $N-2$ principals is $X^*_{-\{i,v_i\}} = (1 - k_i - k_{v_i}) X^*$. This implies that $\bar{P}_i$ depends only on k_i and k_{v_i}. Thus, we can write $\bar{P}_i = \bar{P}(k_i, k_{v_i})$.

Next, we show that for any given fixed k_{v_i}, $\bar{P}(k_i, k_{v_i})$ is strictly increasing and strictly convex in k_i. First note that

$$\frac{\partial \bar{P}}{\partial k_i} = X^* U'\left(X^*_{-\{i,v_i\}} + \bar{x}^i_{v_i}\right) - U'\left(X^*_{-\{i,v_i\}} + \bar{x}^i_{v_i}\right) + C'_x(\bar{x}^i_{v_i}, k_{v_i}),$$

for X^* and $C(x^*_{v_i}, k_{v_i})$ are independent of k_i. Since $U'\left(X^*_{-\{i,v_i\}} + \bar{x}^i_{v_i}\right) = C'_x(\bar{x}^i_{v_i}, k_{v_i})$, it follows

$$\frac{\partial \bar{P}}{\partial k_i} = X^* U'\left(X^*_{-\{i,v_i\}} + \bar{x}^i_{v_i}\right) > 0.$$

Next, we calculate

$$\frac{\partial^2 \bar{P}}{\partial k_i^2} = X^* U''\left(X^*_{-\{i,v_i\}} + \bar{x}^i_{v_i}\right)\left(-X^* + \frac{\partial \bar{x}^i_{v_i}}{\partial k_i}\right)$$

where

$$\begin{aligned}\frac{\partial \bar{x}_{v_i}^i}{\partial k_i} &= -X^* \frac{\partial \bar{x}_{v_i}^i}{\partial X_{-\{i,v_i\}}^*} \\ &= X^* \frac{U''(.)}{U''(.) - C_{xx}''(.)}.\end{aligned}$$

It follows

$$\frac{\partial^2 \bar{P}}{\partial k_i^2} = X^{*2} U''(.) \frac{C_{xx}''(.)}{U''(.) - C_{xx}''(.)} > 0$$

Finally, we show that $\bar{P}$ is decreasing in k_{v_i}. Since X^* is independent of k_{v_i} and $U'\left(X_{-\{i,v_i\}}^* + \bar{x}_{v_i}^i\right) = C_x'(\bar{x}_{v_i}^i, k_{v_i})$, we have

$$\frac{\partial \bar{P}}{\partial k_{v_i}} = X^* \left[U'\left(X_{-\{i,v_i\}}^* + \bar{x}_{v_i}^i\right) - C_x'(x_{v_i}^*, k_{v_i})\right] + C_k'(\bar{x}_{v_i}^i, k_{v_i}) - C_k'(x_{v_i}^*, k_{v_i}).$$

Using the normalized cost function $c(\chi)$ and setting $\bar{\chi}_{v_i}^i = \dfrac{\bar{x}_{v_i}^i}{k_{v_i}} < X^*$, the derivative becomes:

$$\frac{\partial \bar{P}}{\partial k_{v_i}} = c\left(\bar{\chi}_{v_i}^i\right) - c(X^*) - \left(\bar{\chi}_{v_i}^i - X^*\right) c'\left(\bar{\chi}_{v_i}^i\right),$$

which is negative by the convexity of $c(\chi)$. ■

Proof of Proposition 5. An example suffices to prove this Proposition. We use the linear-quadratic example developed in Section 4, assuming that there are $N \geq 3$ principals. We posit that principal 1 is larger than his competitors, who are symmetric and hold a market share of $\frac{1-k_1}{N-1}$ each. Now suppose principals $i = 2, 3, ..., N$ merge. After the merger, the industry then consists of two principals with market share k_1 and $1 - k_1$, respectively.

In the post-merger minimum rent equilibrium, the agent's total payment is

$$\sum_{i=1}^{N} \bar{P}_i = aX^* \left\{1 + \frac{\tau}{2(1+\tau)} \left[\frac{k_1^2}{1+\tau(1-k_1)} + \frac{(1-k_1)^2}{1+\tau k_1} - 2\right]\right\}.$$

In the pre-merger equilibrium, the agent's total payment is

$$\sum_{i=1}^{N} \bar{P}_i = aX^* \left\{1 + \frac{\tau}{2(1+\tau)} \left[\frac{k_1^2(N-1)}{(N-1)+\tau(1-k_1)} + \frac{(1-k_1)^2}{(1+\tau k_1)(N-1)} - 2\right]\right\}$$

It follows that the merger benefits the agent if

$$\frac{k_1^2}{1+\tau(1-k_1)} + \frac{(1-k_1)^2}{1+\tau k_1} < \frac{k_1^2(N-1)}{(N-1)+\tau(1-k_1)} + \frac{(1-k_1)^2}{(1+\tau k_1)(N-1)},$$

where $\tau \equiv \frac{b}{c}$. To illustrate, when $k_1 = \frac{1}{2}$, this inequality holds (and hence the agent benefits from the merger) whenever

$$\tau > \frac{2\left(N-1\right)}{N-2}.$$

If $k_1 = \frac{2}{3}$, the inequality holds provided that

$$\tau > \frac{3\left[\left(41N^2 - 92N + 52\right)^{\frac{1}{2}} - \left(3N-4\right)\right]}{2\left(8N-9\right)}.$$

The greater k_1 and N, the more likely the merger benefits the agent. ■