Competition, Market Selection and Growth*

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Abstract

We study the effect of the competitive selection process on the incentive to innovate and the economy's rate of growth by extending standard quality-ladder models of endogenous growth to allow for the possibility that in each period several asymmetric firms (i.e., an endogenously determined number of past innovators) may be simultaneously active in an industry. Stronger competitive pressure has conflicting effects on the incentive to innovate, lowering prices but also selecting the more efficient firms. We show that the market selection effect of competition always increases the incentive to innovate and find circumstances in which it can outweigh the traditional negative Schumpeterian effect on growth.

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1 Introduction

One of the many reasons why economists praise competition is that it improves the process of output reallocation between producers, selecting the more efficient firms and weeding out the less efficient. This competitive selection process is especially important in growing economies, where productivity changes continuously with technical progress. In the growth literature, most Schumpeterian models of endogenous growth posit competition so intense that only the most productive firm is active in each industry at any time, implying that there is no room for further improvement in the process of market selection. But there is ample empirical evidence of micro-level heterogeneity in productivity across firms, which suggests that in real life competition may not be as intense.

If this is so, an increase in competitive pressure can still improve the process of market selection. This is certainly good for static efficiency, but what effect does it have on the incentive to innovate, and hence the growth rate of the economy? Our thesis is that stronger competitive selection is also good for growth, and its positive effect can be great enough to outweigh the traditional negative Schumpeterian effect. Thus, our analysis can help explain the positive or uncertain relationship between competition and growth found in the empirical literature.[1]

To demonstrate our results, we extend standard quality-ladder models of endogenous growth to allow for the possibility that several asymmetric firms (i.e., an endogenously determined number of successive innovators) can be active in an industry at the same time. In early Schumpeterian models, it is an equilibrium property that the technological leader should be the sole active firm. This follows from the postulate that innovations are drastic, or else firms compete *a là* Bertrand.[2] Our

[1] The many empirical studies of the relationship between competition and growth have generally found that competition tends to be positively associated with innovation, or that the relationship between the two is inverse U-shaped: see Aghion and Griffith (2005) for an excellent survey.

[2] See Aghion and Howitt (1992), Grossman and Helpman (1991), and Segerstrom et al. (1990). An innovation is "drastic" if it is so large that the innovator is effectively unconstrained by outside competition and can therefore engage in monopoly pricing.

extension is to assume that innovations are non-drastic and that firms compete *a là* Cournot. (We also use a conjectural-variations reduced-form model that encompasses the Bertrand and Cournot equilibria as special cases, yielding a continuous index of the intensity of competition.[3]) As a result, our model possesses a steady state in which $m + 1$ firms are active at each point in time, i.e., m past innovators plus the latest innovator. An innovator who does not engage in any further R&D will be displaced gradually, remaining active and reaping positive profits for $m + 1$ periods.[4] As further innovations arrive his market share shrinks, but he will not leave the market until after $m + 1$ successive innovations. The number of active firms and their respective market shares are determined endogenously as a function of the intensity of competition, the elasticity of demand, and the size of innovations (Proposition 1).

The analysis of a quality-ladder endogenous growth model with Cournot competition between asymmetric firms is a contribution that may be interesting in its own right,[5] but for our purposes it serves mainly as a term of comparison with the familiar case of Bertrand competition, since our objective is to analyze how more intense competition affects the incentive to innovate. In our framework where no firm innovates repeatedly, an innovator is the technological leader in the first period after his innovation, but becomes the second most efficient in the following period, and so on. Thus, over the various periods he obtains a certain weighted sum of all active firms' profits, with weights reflecting the expected duration of periods, growth, and

[3] In the endogenous growth literature, the intensity of competition is often measured by the elasticity of demand, which determines the size of the innovator's mark-up. Allowing for several active firms, one can instead focus on behavioural parameters, uncoupling changes in the degree of competition from those in taste and/or technology that ultimately determine the elasticity of demand. For a neat illustration of the shortcomings of measuring the intensity of competition through a structural determinant of the elasticity of demand, see Koeniger and Licandro (2006).

[4] A period is the random time interval between two innovations.

[5] Dinopoulos and Segerstrom (1999) also develop a Schumpeterian quality-ladder model with Cournot competition. However, they assume that patents expire when further innovation occurs, i.e., innovation k falls into the public domain as soon as innovation $k + 1$ arrives. With this assumption, the equilibrium price with Cournot competition is the same as with Bertrand competition, and even though the latest innovator does not produce all of the output, he reaps positive profits for only one period.

discounting. This weighted sum represents the incentive to innovate.

As the intensity of competition increases, the equilibrium price in innovative industries goes down (the *price effect*), while the number of active firms and their market shares change: specifically, the market shares of efficient firms increase and those of inefficient firms decrease, possibly driving some of the latter out of the market (*market selection*). The price effect unambiguously reduces profits and hence the incentive to innovate, but what is the effect of stronger market selection?

Since each innovator will inevitably become a relatively inefficient firm as new innovations arrive, at first sight the reallocation of market shares seems neutral for innovators' profits. On closer inspection, however, stronger market selection increases the incentive to innovate in two ways. First, the innovator gains more from his greater market share when he is relatively more efficient (hence with a larger price-cost margin) than he loses from the decrease when he is relatively less efficient and his margin is small. We call this the *productive efficiency effect.* Second, the reallocation of market shares has a *front-end-loading effect*, since it makes profits accrue to the innovator sooner. With Cournot competition, for instance, it may take several periods for an innovator to complete collecting his rents, whereas with Bertrand competition all profits are obtained in the first period after the innovation.

Can these positive effects of stronger competitive selection outweigh the negative price effect? To show that they can, we proceed in two steps. First we show that a sufficient condition is that more intense competition increases the unweighted sum of an innovator's profits over time periods (Proposition 2). This result is driven by the front-end-loading effect, and instructs us to focus on the effect of competition on the unweighted sum. As it turns out, the unweighted sum of an innovator's profits is in a one-to-one correspondence to industry profits. It may seem surprising that industry profits matter, since the incentive to innovate depends on the innovator's profits, not his competitors'. But over the successive time periods each innovator will occupy all the positions in the industry, from the most to the least efficient firm. Thus, an innovator's life cycle reflects the industry as a whole, and his total unweighted profits over time are proportional to industry profits. In the second step of our analysis, we identify two circumstances in which more intense competition increases the unweighted sum of an innovator's profits over time periods: large innovations (Proposition 3) and tough competition (Proposition 4). In these circumstances, competition is certainly good for growth.

Our qualitative results are local, but numerical calculations show that competition can be good for growth in a sizeable region of parameter values, which includes those usually considered realistic. This suggests that the mechanisms identified here are more than a theoretical possibility. Although we make no attempt to bring the model to the data, our analysis carries several implications for empirical work. For example, it suggests that strong market concentration may be taken as a proxy for more intense competition, not less. We discuss these issues more fully in the concluding section.

The rest of the paper is organized as follows. The next section discusses the literature. Section 3 outlines a simple quality-ladder model of endogenous growth. Section 4 focuses on the product market equilibrium, allowing for the possibility that several firms can be simultaneously active. Section 5 determines the value of an innovation, and hence the incentive to innovate, when innovators are displaced by subsequent innovations gradually. Section 6 analyzes how the intensity of product market competition affects the incentive to innovate and derives the main results. Section 7 summarizes and concludes. All proofs are collected in the Appendix.

2 The literature

We are not the first to incorporate the competitive selection process in an endogenous growth model.[6] To reconcile endogenous growth theory and empirical evidence that

[6] The market selection process has also been incorporated in models of exogenous growth. Acemoglu et al. (2006), for instance, assume that firms can be run by good or bad managers. The closer a firm is to the technological frontier, which shifts over time because of exogenous technical progress, the more valuable a good manager. As a result, institutions that facilitate the selection of good managers become more important as a country gets closer to the technological frontier.

competition is good for growth, Aghion et al. (2001) and others develop models of endogenous growth where in each industry only two incumbents can invest in R&D.[7] Since the two incumbents can be asymmetric, these models do capture the process of market selection, but the mechanism that drives a positive (or inverse-U shaped) relationship between competition and growth is quite different from ours. If, as they assume, only incumbents can innovate, the incentive to innovate is the incremental profit, i.e., the difference over the firm's current profit. An increase in product market competition can decrease both prospective and current profits, yet increase the difference. This is what generally happens when firms are neck-and-neck, producing an "escape-competition" effect that explains why more intense competition may be beneficial to growth in these models.[8]

We differ from this literature by assuming that outsiders can conduct research on an equal footing with incumbents, as in early Schumpeterian models. Because of Arrow's replacement effect, in equilibrium outsiders will conduct all of the research,[9] so the escape-competition effect cannot arise. From this viewpoint, the

[7] A simplified version of Aghion et al. (2001) is presented in Aghion et al. (1997). Aghion et al. (2005) is an attempt to bring the model to the data. These papers model innovation as a step-by-step process with no leapfrogging, which marks another difference from early Schumpeterian models. In an important contribution, however, Encaoua and Ulph (2004) show that a positive relationship between competition and growth may obtain in these models even allowing for leapfrogging. Generally speaking, whether it is more appropriate to model innovations as proceeding step-by-step or to allow for leapfrogging may depend on how innovations are protected. As Encaoua and Ulph (2004) discuss at some length, if innovators rely on patent protection, it seems reasonable to assume that laggards can leapfrog, because all innovative knowledge must in principle be disclosed in the patent specification. If instead innovators rely on secrecy or tacit knowledge, then laggards must duplicate the leader's innovative knowledge before moving up the quality ladder and we have step-by-step innovations.

[8] That more intense competition may increase incremental profits was first pointed out in a partial equilibrium framework by Delbono and Denicolò (1990). Following Beath et al. (1989), they distinguish between two notions of incremental profits, the "profit incentive" and the "competitive threat," and show that both can be greater under Bertrand than under Cournot competition when firms are neck-and-neck.

[9] This implication is admittedly unrealistic. There have been various attempts to extend early Schumpeterian models to allow for repeated inventions by incumbents. In one strand of the literature, incumbents are assumed to have a limited advantage in conducting research, such as a first-mover advantage, and to conduct all research preemptively. Denicolò (2001) and Etro (2004) show that in such a preemptive equilibrium the level of innovative activity is still determined by the outsiders' incentive to innovate. This means that our results could be extended to such a framework, as we

contribution of this paper is to show that stronger competitive selection can generate a positive relationship between competition and growth even in the absence of an escape-competition effect.

That stronger competitive selection can increase the incentive to innovate is also the thesis of part of the industrial organization literature. In partial equilibrium models of horizontal product differentiation with asymmetric firms, Aghion and Schankerman (1999, 2004), Boone (2001), and Zanchettin (2006) show that the competitive selection effect, which increases the market shares of the more efficient firms, can actually increase their profits. As a result, innovators, which naturally tend to be more efficient, can benefit from stronger competition. These papers focus on stand-alone innovations, however, where the incentive to innovate is simply given by the (incremental) profit of a successful innovator. In this case, the fact that more efficient firms hold larger market shares when competition is more intense directly increases the incentive to innovate. In our general equilibrium model, by contrast, there is an infinite sequence of innovations, and innovators are displaced gradually, so a firm's life cycle re-produces the industry pattern and the value of an innovation is thus a weighted sum of all active firms' profits, where the coefficients reflect the expected length of time periods, discounting, and growth. In this framework, the fact that more efficient firms hold larger market shares operates more subtly, through the front-end-loading and the productive efficiency effect, as discussed above.[10]

showed in a previous version of this paper. However, a more general analysis, in which neither insiders nor outsiders are precluded from innovating, has yet to be done.

[10] In this respect, our analysis is closer to Segal and Whinston (2007), who study a partial equilibrium model of successive innovations in which each innovator may be active for several periods. However, their model differs from ours in several other respects; for example, it posits that there can be at most two active firms, and that an innovation drives the incumbent out of the market after a period whose length is exogenously given. Moreover, they do not compare Bertrand and Cournot competition, but focus on various business practices that may or may not be anti-competitive.

3 The model

We present a simple quality-ladder model of endogenous growth. We use a one-sector version of Barro and Sala-i-Martin (2003, ch. 7), but our results are more general and can be reproduced in many other models, with or without scale effects.

3.1. *Preferences and technology*

The economy is populated by identical, infinitely-lived individuals whose mass is normalized to one. Each individual inelastically supplies one unit of labour and has linear intertemporal preferences:

$$u(c) = \int_0^\infty c(t)e^{-rt}dt \tag{1}$$

so that the equilibrium rate of interest is fixed and coincides with the rate of time preference r.[11] Time is continuous but can be divided into periods: as in Aghion and Howitt (1992), a period is the random time interval between two successive innovations.

There is a unique final good in the economy that can be consumed, used to produce intermediate goods, or used in research. This good is taken as the numeraire. It is produced in a perfectly competitive market using labour (which is in fixed supply) and an intermediate good whose quality increases over time because of technical progress. We normalize the quality of the intermediate good at time zero to 1 and denote by $\theta > 1$ the size of each innovation, so that the quality of the intermediate good of vintage k is θ^k.

In period k, (k being the number of past innovations) the final good y_k can be

[11] One can easily allow for more general preferences. If the instantaneous utility function is concave, the Euler equation gives an increasing relationship between the interest rate and the economy's rate of growth. The analysis below provides another, decreasing relationship. These two equations can be solved simultaneously to determine the equilibrium interest rate and growth rate. Any effect of the intensity of competition on the incentive to innovate now affects both the interest rate and the rate of growth. Quantitatively, the effects identified in this paper are attenuated, but their sign does not change.

produced according to the following constant-returns production function:

$$y_k = \left(\sum_{i=0}^{k} \theta^{k-i} x_{i,k} \right)^{\alpha} L^{1-\alpha} \qquad 0 < \alpha < 1 \tag{2}$$

where L is labour input, $(1 - \alpha)$ is the income share of labour, and $x_{i,k}$ denotes the input of the intermediate good of vintage $k - i$. Hence, $\sum_{i=0}^{k} \theta^{k-i} x_{i,k}$ is a quality-adjusted index of a composite good that combines all past generations of intermediate goods, which are perfect substitutes at fixed ratios. It is convenient to rewrite $\sum_{i=0}^{k} \theta^{k-i} x_{i,k}$ as $\theta^k X_k$, where $X_k = \sum_{i=0}^{k} \theta^{-i} x_{i,k}$ measures the input of the composite intermediate good in efficiency units relative to the last vintage, k. Setting labour input equal to one, the production function (2) becomes

$$y_k = \theta^{\alpha k} X_k^{\alpha} \tag{3}$$

Profit maximization by the final good sector implies the following demand for the intermediate good, measured in efficiency units:

$$X_k = \alpha^{\frac{1}{1-\alpha}} \theta^{\frac{\alpha}{1-\alpha} k} p_k^{-\frac{1}{1-\alpha}} \tag{4}$$

where p_k is the price for the intermediate good in period k. The demand function (4) has a constant elasticity $\frac{1}{1-\alpha}$: the greater α, the more elastic the demand.[12] Each innovation shifts the demand function up by a constant factor $\theta^{\frac{\alpha}{1-\alpha}}$: as will be shown below, in a steady state $\theta^{\frac{\alpha}{1-\alpha}}$ is the growth factor between periods.

Independently of its vintage, the intermediate good is produced using the final good with a constant rate of transformation that is normalized to 1, which means that the marginal production cost, in terms of the numeraire, is 1. Since intermediate

[12] The elasticity $\frac{1}{1-\alpha}$ captures the effect of a change in the price of the intermediate good on the supply of the final good by competitive firms. The assumption that a producer of the intermediate good internalizes the effect of its choice on national income may seem farfetched, but in our model (4) is the true, objective demand function, and any other "perceived" demand function would be hard to justify. Alternatively, one could model a continuum of intermediate goods; then, the producer of a single capital good would not internalize the macroeconomic effects of its choice, which are indeed negligible, but only the substitution between different capital goods. In such a multi-sector model, the elasticity of demand would reflect the elasticity of substitution but all of our results would continue to hold provided that the elasticity is lower than one. For simplicity, we posit a one-sector economy.

goods of different vintages are perfect substitutes at constant rates, firms producing different intermediate goods can be treated as if all were producing the same good, measured in *efficiency units*, and facing the same market demand function (4), but with different costs that reflect the quantity of final good needed to make one efficiency unit. Specifically, the effective marginal production cost of vintage $k-i$, in efficiency units relative to the last vintage, is $c_i = \theta^i$, since one unit of the intermediate good of vintage k is as productive as θ^i units of the good of vintage $k-i$.

Innovations are non-drastic, meaning that the latest innovator cannot engage in monopoly pricing without being underpriced. In other words, the monopoly price must be greater than θ, the effective unit cost of the latest innovator's most efficient competitor. With a constant elasticity of demand $\frac{1}{1-\alpha}$ and a marginal cost equal to 1, the monopoly price is $\frac{1}{\alpha}$. Therefore, we henceforth assume $\theta < \frac{1}{\alpha}$.

3.2. *Technical progress*

In each period k there is a patent race for innovation $k+1$. The size of innovations is exogenous, θ, but the timing is a probabilistic function of the amount invested in R&D by research firms, so innovations occur at a rate determined by R&D effort. Like traditional Schumpeterian models, we assume that incumbents have no advantage over outsiders in conducting research and that there is free entry by outsiders. Arrow's replacement effect then implies that research is conducted only by outsiders,[13] so in each period the current leader is systematically overtaken. However, each innovator may stay active in the product market for several periods, being displaced only gradually by subsequent innovations.

In period k, each outside firm ℓ in the patent race decides its R&D effort $n_{\ell k}$, in terms of the numeraire, to obtain the $k+1$-th innovation. The R&D effort is a flow cost that determines the expected time of successful completion of the R&D project according to a Poisson discovery process with a hazard rate equal to $\lambda_k n_{\ell k}$,

[13] Other things being equal, incumbents have less incentive to innovate than outsiders, because part of the prospective profits would just replace their current profits.

with $\lambda_k > 0$. The projects of different firms are independent, so that the aggregate instantaneous probability of success is simply the sum of the individual probabilities. Let $n_k = \sum_{\ell} n_{\ell k}$ denote aggregate R&D investment per unit of time in period k. Then, the $k+1$-th innovation occurs according to a Poisson process with a hazard rate $z_k = \lambda_k n_k$.

Innovative technological knowledge is proprietary: there is perfect, infinitely-lived patent protection, meaning that nobody can imitate an innovation. Thus, in each period only the k-th innovator, who holds a patent on the k-th innovation, can produce the intermediate good of vintage k.[14] Since all innovations are obtained by outsiders, nobody holds multiple patents.

3.3. *Steady state*

Our model economy may exhibit a transitional dynamics or persistent cyclical growth.[15] Abstracting from any such dynamics, we consider only the steady state. A steady state is defined as a situation in which the expected duration of time periods is constant and the economy grows proportionally from one period to the next.[16] This requires that all quantities and the wage rate are constant within each period and grow by a constant factor across periods, whereas the hazard rate, the price of the intermediate good, and the number of active firms are constant both within and between periods.

[14] We follow the vast majority of endogenous growth models in ruling out patent licensing. The standard justification for this assumption is that licensing agreements between successive innovators would have anti-competitive effects and thus would be prohibited by antitrust authorities. In our model, however, such licensing agreements could be drafted so as to improve productive efficiency with no anti-competitive effects. Thus ruling them out is not an innocent assumption. However, innovative technological knowledge may be difficult to codify and transmit to others and a variety of transaction costs impede licensing, so it is unlikely that licensing will be widespread enough to nullify the market selection effect.

[15] A transitional dynamics arises if the initial conditions do not conform to the steady state properties. If the technology of vintage 0 is in the public domain, for instance, innovator 1 will compete against a competitive fringe instead of m past innovators. For a discussion of cyclical growth, see Aghion and Howitt (1992).

[16] Had we allowed for a continuum of intermediate goods, in a steady state, by the law of large numbers, the economy would grow smoothly at a constant rate instead of jumping up at random intervals.

Assume for the time being that a steady state exists. Since the price of the intermediate good is constant, it is clear from (4) that X_k must grow at rate $\theta^{\frac{\alpha}{1-\alpha}}$ from one period to the next. It follows immediately from (3) that y_k, the wage rate, etc. will also grow at rate $\theta^{\frac{\alpha}{1-\alpha}}$. Thus, $\theta^{\frac{\alpha}{1-\alpha}}$ is the growth factor between periods; it will be denoted by g.

To guarantee the existence of a steady state, we must ensure that the expected duration of time periods can be constant. Since in a steady state R&D investment n_k grows at rate g then in order for the hazard rate $z_k = \lambda_k n_k$ to be constant the productivity of R&D, λ_k, must decline at rate g. This requires the knife-edge assumption $\lambda_k = \lambda g^{-k}$, which is common to all R&D-driven endogenous growth models. Finally, note that the following condition must hold (Barro and Sala-i-Martin, 2003):

$$r > z(g-1) \tag{5}$$

If this condition is violated, the utility u is unbounded.

4 Product market equilibrium

Now we derive the product market equilibrium, accounting for the possibility that in each period m past innovators, plus the technological leader, may be active. This means, conversely, that each innovator may stay active and profitable for $m+1$ periods: in the first period, he is the technological leader and produces the intermediate good at a unit cost equal to 1; in the next period he has an effective unit cost of θ, being the second most efficient firm in the market; and so forth. The number of active firms, or periods for which an innovator stays active, is determined endogenously; with Bertrand competition, for instance, $m = 0$.

Since innovations are non-drastic and in a leapfrogging equilibrium the second-best quality is available to a non-leading firm, the leader is generally constrained by outside competition. The product market equilibrium depends on the strength of this competitive pressure.

4.1. *Bertrand competition*

In this case the outcome is a limit-pricing equilibrium where the leader prices at $p^B = \theta$, the unit cost of his most efficient competitor, and drives all his competitors out of the market. In this equilibrium, only the most productive vintage is produced, so there is no productive inefficiency. Equilibrium profits are $\pi^B_{0,k} = (\theta - 1)\, \theta^{-\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} g^k$ and $\pi^B_{i,k} = 0$ for all $i \geq 1$, where $\pi_{i,k}$ denotes the flow of profit earned by innovator $k-i$ in period k and the superscript B stands for Bertrand.

4.2. *Cournot competition*

In a steady state, in each period there is a long sequence of past innovators that can produce the intermediate good at cost $\theta, \theta^2, ...$, respectively. In the Cournot equilibrium, the latest m^C of these past innovators will be active. To determine m^C and the Cournot equilibrium price, p^C, let us take an arbitrary value of m and compute the associated Cournot equilibrium price as $\frac{1+\theta+...+\theta^m}{m+\alpha}$.[17] For $m+1$ to be the equilibrium number of active firms, however, this candidate equilibrium price must be lower than θ^{m+1} (otherwise more firms would enter) and greater than θ^m (otherwise some supposedly active firms would make negative profits and exit). It follows that m^C is the largest integer such that $p^C \geq \theta^{m^C+1}$, where the Cournot equilibrium price p^C is :

$$p^C = \frac{1 + \theta + \theta^2 + ... + \theta^{m^C}}{m^C + \alpha} \tag{6}$$

[17] The calculations are simpler if we exploit the well-known property of the Cournot equilibrium that the equilibrium price depends only on the unweighted average of the individual unit costs and is independent of the distribution of costs across firms: see Bergstrom and Varian (1985). This property guarantees that the Cournot equilibrium price is the same as if all the $m+1$ active firms shared the same unit cost $c = \frac{1+\theta+...+\theta^m}{m+1}$. With an iso-elastic demand and a symmetric unit cost c, it is a standard exercise to calculate the Cournot equilibrium price as

$$\frac{c(m+1)}{(m+1) - \frac{1}{\varepsilon}}$$

where ε is the elasticity of demand. Since in our model $\varepsilon = \frac{1}{1-\alpha}$, the expression in the text follows immediately.

Clearly, the Cournot equilibrium price is higher than the Bertrand,[18] and both are independent of the shift parameters in the demand curve (this is a consequence of the curve being iso-elastic).

Aggregate output X_k^C is obtained by substituting (6) into (4). Individual outputs $x_{i,k}^C$ can be easily calculated exploiting the fact that the ratio between any two active firms' market shares is equal to that between their price-cost margins.[19] We obtain:

$$x_{i,k}^C = \frac{X_k^C}{(1-\alpha)} \frac{\left(p^C - \theta^i\right)}{p^C} \tag{7}$$

where it is understood that $x_{i,k}^C = 0$ when $p^C \leq \theta^i$.

4.3. *Productive inefficiency*

Equation (7) shows that low-cost firms have larger market shares than high-cost firms. But in the Cournot equilibrium different vintages of the intermediate good are produced at each date, even though the older vintages are less productive. This means that the market selection process is less effective than under Bertrand competition, so the product market equilibrium exhibits productive inefficiency, in that total output is not produced at the lowest possible social cost. More precisely, in the intermediate good industry the average unit cost is $\bar{c}_k = \sum_{i=0}^m \frac{x_{i,k}}{X_k} \theta^i$, i.e., it is an average of

[18] To show that $p^C > p^B = \theta$, note that

$$\begin{aligned} p^C &= \frac{1+\theta+\theta^2+...+\theta^{m^C}}{m^C+\alpha} \\ &> \frac{\alpha\theta+\theta+\theta^2+...+\theta^{m^C}}{m^C+\alpha} \quad \text{because } \alpha\theta < 1 \text{ (innovations are non-drastic)} \\ &> \frac{\alpha\theta + m^C\theta}{m^C+\alpha} \quad \text{because } \theta > 1 \\ &= \theta. \end{aligned}$$

[19] This property of the Cournot equilibrium follows directly from the first-order conditions $p'(X^C)x_i^C + p^C = c_i$, where $p(X)$ is the inverse demand function and to simplify the notation we have dropped the time indices. Given the equilibrium price (6), the ensuing conditions

$$\frac{x_i^C}{x_j^C} = \frac{p^C - \theta^i}{p^C - \theta^j}$$

provide m^C independent equations in the m^C+1 unknowns $x_0^C, ..., x_m^C$. Together with the adding up condition $X^C = \sum_{i=0}^{m^C} x_i^C$, they comprise a system of m^C+1 linearly independent equations, the unique solution of which is (7).

individual firms' unit costs weighted by market shares.

As we shall see below in greater detail, not only is such productive inefficiency a source of social costs, it also decreases the innovator's profits. To see why, note that over his life cycle, innovator k produces the intermediate good, measured in efficiency units, at an average unit cost of $\sum_{i=0}^{m} \frac{x_{i,k+i}}{X_{k+i}} \theta^i$, which in a steady state coincides with $\bar{c}_k$. Thus, other things being equal, productive inefficiency translates directly into higher costs and hence lower profits for an innovator.

4.4. *Conjectural variations*

Many contributions in the industrial organization literature have analyzed cases intermediate between Bertrand and Cournot competition. These may arise, for instance, in the presence of capacity constraints or adjustment costs; in addition, partially collusive outcomes can be sustained when firms interact repeatedly. To cover these cases, we use a reduced-form model, the conjectural variations equilibrium, which encompasses all intermediate degrees of competition between Bertrand and Cournot. The conjectural variations parameter ν, which measures the hypothetical change in the competitors' aggregate output associated with a unit change in own output, is assumed to be the same for all firms. Although ν is apparently a behavioral parameter, it may also be taken as a metaphor for policies and institutions that intensify product market competition.[20]

Proceeding as for Cournot competition, we obtain the following equilibrium price:[21]

$$p(\nu) = \frac{1 + \theta + \theta^2 + ... + \theta^{m(\nu)}}{[m(\nu) + 1] - (1 - \alpha)(1 - \nu)} \tag{8}$$

[20] With capacity constraints or adjustment costs, for instance, the intensity of competition is related negatively to the slope of the marginal cost curve beyond the capacity level (Maggi, 1994), or to the size of the costs firms face when scaling output up or down (Dockner, 1992). The intensity of competition is also related negatively to the degree of collusion (Cabral, 1995), and positively to firms' aggressiveness when they choose simultaneously quantities and prices (d'Aspremont et al., 2007). We discuss different meanings of the intensity of competition in the concluding section.

[21] In a conjectural variations equilibrium, the first-order conditions for profit maximization become $(1-\nu)p'(X)x_i + p = c_i$. The factor $(1-\nu)$ that multiplies the first term on the left-hand side changes the perceived elasticity of demand, which is now effectively $\frac{1}{(1-\alpha)(1-\nu)}$. With this insight, equation (8) follows immediately by the same logic as in footnote 17.

where $m(\nu)$ is the largest integer such that $p(\nu) \geq \theta^{m(\nu)+1}$. Differentiating (8), it is clear that the equilibrium price decreases with ν.[22] The Cournot solution is obtained for $\nu = 0$ and the Bertrand solution for $\nu = \nu^B$, where[23]

$$\nu^B = \frac{1 - \alpha\theta}{(1 - \alpha)\theta}. \tag{9}$$

The equilibrium aggregate output and individual outputs can be calculated as for Cournot competition. For future reference, notice the following relationship between any two firms' profits:

$$\frac{\pi_{i,k}}{\pi_{j,k}} = \left[\frac{p(\nu) - \theta^i}{p(\nu) - \theta^j}\right]^2 \tag{10}$$

The reason why the ratio between firms' profits is proportional to the squared ratio between price-cost margins is that in any conjectural variations equilibrium market shares are proportional to margins, as in the Cournot equilibrium. This latter property also implies that a rise in competitive pressure reallocates output from less to more efficient firms. This is an important feature of the competitive selection process, one that drives most of our results.

4.5. *Competition and the number of active firms*

Our first proposition highlights another, related aspect: as competitive pressure mounts, past innovators are driven out of the market more quickly and the number of active firms decreases. Actually, the number of active firms is piecewise constant because of the integer number problem: when ν rises, m almost always remains constant and falls to the integer immediately below only when ν passes certain critical thresholds, which we calculate explicitly below. The same is true of changes in θ and α. With this caveat, we can state:

[22] The equilibrium price p^C depends on ν both directly and through m. Since $m(\nu)$ jumps down at certain critical points as ν increases, the equilibrium price is differentiable with respect to ν only piecewise. But the price is continuous in ν at these critical points. As a result, if the partial derivative of p^C with respect to ν is negative, as it indeed is, the equilibrium price is necessarily monotonically decreasing in the intensity of competition. A similar logic applies to the derivatives with respect to ν of other variables, which depend also on m.

[23] In the symmetric case ($\theta = 1$), the Bertrand solution is obtained when $\nu = 1$, but when firms are asymmetric the critical value of ν that reproduces the Bertrand equilibrium is less than 1 and depends on the magnitude of innovations and the elasticity of demand.

Proposition 1. *The number of active firms is a step-wise decreasing function of the intensity of competition, the magnitude of innovations, and the elasticity of demand.*

The negative effect of sharper competition on the equilibrium number of firms has been emphasized in the industrial organization literature since Demsetz (1972) and Vickers (1995), but the endogenous growth literature has tended to overlook it. For any given intensity of competition, a swifter replacement of past innovators is also caused by an increase in the magnitude of innovations (which accentuates cost asymmetry between successive innovators), and in the elasticity of demand (which lowers the price-cost margin, leaving less room for inefficient firms).

Using (8), we can calculate explicitly the critical thresholds mentioned above: there are exactly $m+1$ active firms when[24]

$$\frac{m\theta^m - (1+\theta+...+\theta^{m-1})}{\theta^m} \leq (1-\alpha)(1-\nu) \leq \frac{(m+1)\theta^{m+1} - (1+\theta+...+\theta^m)}{\theta^{m+1}} \tag{11}$$

[insert Figure 1 around here]

To get a feeling for the magnitude of m, we consider Cournot competition ($\nu = 0$). Barro and Sala-i-Martin (2003) note that reasonable values for α, capital's share in national income, range from 0.30 if capital is interpreted as physical capital only to 0.70 if it includes human capital. Stokey (1995) observes that if innovations occur every few years, a reasonable range for θ is 1.02 to 1.04; if instead innovations occur only a couple of times per century, then a reasonable range for θ is 1.25 to 1.50. Take a reasonable value of θ such as $\theta = 1.15$.[25] Then, there will be two active firms for

[24] When one of these expressions holds as an equality, the equilibrium price is exactly equal to the unit cost of one of the firms, which will then produce zero output. Whether such a marginal firm should be considered active or non-active is a matter of terminology.

[25] With $\theta = 1.15$, the average gross markup (the ratio of price to the industry's average cost) can be computed as 1.22 when $\alpha = 0.7$ and 1.40 when $\alpha = 0.3$. These values seem consistent with empirical estimates of the gross markup: see e.g. Basu (1996).

$0.87 \geq \alpha \geq 0.63$ and three active firms for $0.63 \geq \alpha \geq 0.28$; only for $\alpha > 0.87$ are innovations drastic, and only one firm is active. More generally, Figure 1 illustrates how the number of active firms varies with the elasticity of demand and the step size of innovations. From (11), the decreasing curves have equation $\alpha = \frac{1+\theta+...+\theta^m - m\theta^{m+1}}{\theta^{m+1}}$ for successive values of m. The grey area in Figure 1 corresponds to the "reasonable" range $\theta \in [1.02, 1.50]$ and $\alpha \in [0.30, 0.70]$. Over this range, innovations are typically non-drastic, so with Cournot competition the displacement of incumbents is gradual.

5 Equilibrium growth

In this section we derive the equilibrium growth rate as a function of an innovator's prospective profits. This will allow us to analyze, in section 6, the effect of the intensity of competition on the economy's growth rate. We begin by determining the incentive to innovate in an economy where innovators are displaced gradually.

5.1. *The incentive to innovate*

When innovators are displaced gradually, their rents are not terminated immediately by the subsequent innovation. The expected value of innovation $k+1$, $E(V_{k+1})$, is determined by the following asset condition:

$$rE(V_{k+1}) = \pi_{0,k+1} - z_{k+1}\left[E(V_{k+1}) - E(V_{k+1}^1)\right] \tag{12}$$

where $E(V_{k+1}^h)$ is the value of innovation $k+1$ after h periods, i.e., in period $k+1+h$. This equation says that securities issued by the $k+1$-th innovator pay the flow profit $\pi_{0,k+1}$ in period $k+1$, less the expected capital loss $z_{k+1}\left[E(V_{k+1}) - E(V_{k+1}^1)\right]$ that will be incurred when the next innovation occurs. This capital loss is the difference between the value of being leader and that of being the second most efficient firm, i.e. $E(V_{k+1}) - E(V_{k+1}^1)$. The value of being the second most efficient firm in the market, $E(V_{k+1}^1)$, is in turn determined by the asset condition

$$rE(V_{k+1}^1) = \pi_{1,k+2} - z_{k+2}\left[E(V_{k+1}^1) - E(V_{k+1}^2)\right] \tag{13}$$

and so on. Eventually, after $m+1$ innovations, the $k+1$-th innovator leaves the market, so $E(V_{k+1}^{m+1}) = 0$. Consequently, we have

$$rE(V_{k+1}^{m}) = \pi_{m,k+m+1} - z_{k+m+1}E(V_{k+1}^{m}) \tag{14}$$

This system is recursive, as the last equation solves for $E(V_{k+1}^{m})$, and then given $E(V_{k+1}^{m})$ the penultimate equation solves for $E(V_{k+1}^{m-1})$, and so on. We obtain:

$$\begin{aligned} E(V_{k+1}) &= \frac{\pi_{0,k+1}}{r+z_{k+1}} + \frac{z_{k+1}}{(r+z_{k+1})}\frac{\pi_{1,k+2}}{(r+z_{k+2})} + ... \\ &\quad + \left[\prod_{i=1}^{m}\frac{z_{k+i}}{(r+z_{k+i})}\right]\frac{\pi_{m,k+m+1}}{(r+z_{k+m+1})} \\ &= \sum_{i=0}^{m}\left[\frac{\pi_{i,k+i+1}}{(r+z_{k+i+1})}\prod_{h=1}^{i}\frac{z_{k+h}}{(r+z_{k+h})}\right] \end{aligned} \tag{15}$$

When $m=0$, this expression reduces to the standard formula $E(V_{k+1}) = \frac{\pi_{0,k+1}}{r+z_{k+1}}$, which says that the value of the $k+1$-th innovation is obtained by discounting the innovator's flow profits by an interest rate augmented by the factor z_{k+1}, which captures the hazard rate at which the innovator's leadership ends. In general, equation (15) tells us that the value of an innovation is the expected present value of all future profits that the innovator will earn in the $m+1$ periods for which he will be active in the product market. In each period, the discount factor is augmented to keep account of the probability that the current flow of profits will be terminated by the occurrence of the next innovation. Moreover, future profits are weighted by the factors $\prod_{h=1}^{i}\frac{z_{k+h}}{(r+z_{k+h})}$, which can be interpreted as the "discount-adjusted probabilities" that future innovations will be achieved: with a Poisson discovery process, each future innovation eventually occurs with probability 1, but since there is discounting, delayed success counts less than instant success.

In a steady state, z is constant and profits grow at rate g between periods: $\pi_{i,k} \equiv \pi_i g^k$. Equation (15) then reduces to:

$$E(V_{k+1}) = \sum_{i=0}^{m}\frac{z^i g^{k+i}\pi_i}{(r+z)^{i+1}} \tag{16}$$

5.2. *Equilibrium*

The expected discounted profit of an outsider firm that invests $n_{\ell k}$ units of the final good in period k to obtain innovation $k+1$ is $\frac{\lambda_k n_{\ell k} E(V_{k+1}) - n_{\ell k}}{r + n_k \lambda_k}$. Because there is free entry, in equilibrium the expected profit must vanish; hence the following condition must hold:

$$\lambda_k E(V_{k+1}) = 1 \tag{17}$$

This zero-profit condition determines the aggregate R&D effort and hence the aggregate hazard rate z_k.[26] In a steady state, inserting (16) into the zero-profit condition (17) we obtain

$$\sum_{i=0}^{m} \frac{z^i g^i \pi_i}{(r+z)^{i+1}} = \frac{1}{\lambda} \tag{18}$$

Equation (18) determines the equilibrium steady state hazard rate, z^*. Given the growth factor between periods, g, the expected rate of growth depends only on the expected length of the periods, which in turn depends on the speed of technical progress. With an exponential distribution of the timing of success, the equilibrium expected waiting time for each innovation is $\frac{1}{z^*}$. Thus, the equilibrium hazard rate z^* fully determines the economy's expected rate of growth, which is thus $z^* \log g$.

Lemma 1. *Assume that* $\frac{\pi_0}{r} > \frac{1}{\lambda}$. *Then a unique, strictly positive steady state equilibrium hazard rate* z^* *exists.*

Condition $\frac{\pi_0}{r} > \frac{1}{\lambda}$ ensures that innovation is sufficiently profitable that in equilibrium some research will be conducted. If this inequality is reversed, there is no innovative activity and the economy stagnates indefinitely. By implicit differentiation, it can be checked that the steady-state level of research z^* is a decreasing function of the rate of time preference r and an increasing function of both the productivity of R&D effort λ and the magnitude of innovations θ.

[26] With constant returns to research, the equilibrium number of research firms and individual R&D investments are indeterminate, and only aggregate R&D investment is determinate. Our results extend immediately to the case $z_k = \lambda_k n_k^{\beta}$ with $\beta < 1$, provided that β reflects the presence of *external* diseconomies in research.

6 Competition and growth

We are now ready to analyze the effect of an increase in the intensity of competition on the economy's rate of growth. Let us denote by $V \equiv \sum_{i=0}^{m} \frac{z^i g^i \pi_i}{(r+z)^{i+1}}$ the left-hand side of equation (18). The function V, which represents the incentive to innovate, is decreasing in z (see the proof of Lemma 1 in the Appendix). By the implicit function theorem, it follows that the sign of $\frac{\partial z^*}{\partial \nu}$ equals the sign of $\frac{\partial V}{\partial \nu}$. Hence:

Lemma 2. *An increase in the intensity of competition increases the equilibrium growth rate if and only if it increases the incentive to innovate, V.*

Lemma 2 suggests that we consider how the intensity of competition affects the incentive to innovate V. As we have seen, V is a weighted sum of the profits obtained by the innovator over all the periods in which he is active. As such, it depends both on the level of the profits and their distribution over time. To disentangle the effect on the level of an innovator's prospective profits from that on their timing, we introduce some further notation. Let $\Pi_s = \sum_{i=0}^{s} \pi_i$ denote the cumulative undiscounted profits that would accrue to an innovator over the first $s+1$ periods of activity in a stationary environment with no growth, with $\Pi = \Pi_m$ denoting the corresponding total profits. Since without discounting, growth, or variable time periods Π_s is an unweighted sum, it may be viewed, perhaps more conveniently, as the cumulative profits of the $s+1$ most efficient firms. Similarly, Π may be viewed as an index of industry profits (to be precise, industry profits in period k are $g^k \Pi$). As was observed in the introduction, the incentive to innovate is related to such aggregate measures of profitability because an innovator's life cycle reflects the pattern of the industry.

With this notation, the incentive to innovate V is rewritten as

$$V = \underbrace{\frac{z^m g^m}{(r+z)^{m+1}} \Pi}_{\text{profit level}} + \underbrace{\sum_{s=0}^{m-1} [r - z(g-1)] \frac{z^s g^s}{(r+z)^{s+2}} \Pi_s}_{\text{profit timing}} \tag{19}$$

The first term on the right-hand side of (19) captures the level of the innovator's profits, the second term their timing.

6.1. *Competition and the timing of profits*

Deferred profits are increased by economic growth, but they must also be discounted: by the "transversality" condition (5), the term inside square brackets, $r - z(g-1)$, is positive, hence discounting prevails over growth. Equation (19) thus means that early profits have a greater effect on the incentive to innovate than late profits. This observation is formalized in the following:

Proposition 2. *If total profits* Π *increase weakly with the intensity of competition* ν*, then the incentive to innovate* V *increases strictly with* ν*.*

Proposition 2 follows from the fact that the timing of profits improves with the intensity of competition, as stronger competitive pressure has a front-end-loading effect. Mathematically, this is clear from (10). The intuitive reason is twofold. First, stronger competition increases the market shares of the more efficient firms (the latest innovators) and decreases those of the less efficient (the oldest innovators). Second, sharper competition lowers the equilibrium price, but for any given fall in the price, the percentage decrease in the price-cost margin is greater for inefficient than for efficient firms. These effects imply that when the market becomes more competitive, new innovators gain and old innovators lose, in relative terms. Condition (5) implies that this front-end-loading effect is always good for the incentive to innovate.

6.2. *Competition and total profits*

We now look for conditions under which more intense competition increases the unweighted sum of profits Π or, equivalently, industry profits; given Proposition 2, these conditions will guarantee that competition is good for growth.

One may wonder whether it is at all possible for more intense competition to increase total profits. As we have seen, more intense competition decreases the equilibrium price and this generally tends to lower total profits when all active firms have the same unit cost. This is the standard *price effect*, which explains why in early Schumpeterian models of endogenous growth the incentive to innovate decreases with the intensity of competition. With asymmetric firms, however, there is also a countervailing effect: more intense competition reduces the market shares of old innovators and augments those of latecomers. Since the latest innovators have wider price-cost margins than the older ones, this effect tends to increase the total profits an innovator earns over his life cycle.

To identify these effects analytically, note that in a steady state an innovator's unweighted total profit is $\Pi = (p - \bar{c})X$, where X is his total output over all time periods (which, abstracting from growth, coincides with the aggregate output produced by all firms in one period) and $\bar{c} = \sum_{i=0}^{m} \frac{x_i}{X}\theta^i$ is his average cost, i.e., the weighted average of his effective unit costs over his lyfe cycle, with weights given by his successive market shares. The equilibrium price, aggregate output, and average cost $\bar{c}$ are all affected by the intensity of competition ν. (In particular, $\bar{c}$ depends on ν because competition affects an innovator's successive market shares over his life cycle.) The change in total profits can then be written as

$$\frac{d\Pi}{d\nu} = \underbrace{\left[X + (p - \bar{c})\frac{dX}{dp}\right]\frac{dp}{d\nu}}_{\text{price effect}} \qquad \underbrace{-\frac{d\bar{c}}{d\nu}X}_{\text{productive efficiency effect}} \tag{20}$$

Provided that Π is concave in p and the price is lower than the monopoly price – two properties that always hold in our model – the price effect is negative. If the average cost $\bar{c}$ were independent of the intensity of competition, the change in total profits would be negative. However, it can be shown that $\frac{d\bar{c}}{d\nu} < 0$, i.e., stronger competition improves an innovator's average productivity over his lyfe cycle. To be sure, competition has no direct impact on the technology; it affects $\bar{c}$ only via market shares. For example, with Bertrand competition all the output is produced by the latest innovator, whereas under Cournot competition old innovators have positive market shares.

Can this *productive efficiency effect* dominate the price effect, so that more intense competition increases total profits? We identify two cases in which the answer is affirmative: large innovations and tough competition. Let us consider the case of large innovations first. Recall that in our model innovations are non-drastic if $\theta < \frac{1}{\alpha}$ and the conjectural variations parameter ranges from 0 (Cournot competition) to $\nu^B = \frac{1-\alpha\theta}{(1-\alpha)\theta}$ (Bertrand competition).

Proposition 3. *When θ is close to $\frac{1}{\alpha}$, Π increases with ν over the interval $0 \leq \nu \leq \nu^B$. In other words, if innovations are sufficiently large total profits increase with the intensity of competition.*

An immediate corollary of Proposition 3 is that when θ is close to $\frac{1}{\alpha}$, total profits are greater with Bertrand than with Cournot competition. The intuition is that at $\theta = \frac{1}{\alpha}$ both Bertrand and Cournot yield the monopoly solution since the latest innovator is not constrained by outside competition. Let us consider the effect of decreasing θ starting from $\theta = \frac{1}{\alpha}$. The next-to-latest innovator now constrains the technological leader, who must price at $p = \theta$ with Bertrand competition. However, when θ is close to the monopoly price $\frac{1}{\alpha}$ the effect of this additional competitive pressure on the leader's profit is second-order – his profit function is flat at the monopoly price. With Cournot competition, a fall in θ lowers the equilibrium price by less than with Bertrand competition, but it also guarantees a positive market share to the next-to-latest innovator, thereby raising the average unit cost $\bar{c}$. Since this effect is first-order, our result follows. A similar intuition holds for marginal changes in the intensity of competition.

Consider next a change in the intensity of competition when the economy is close to the Bertrand equilibrium – the equilibrium that most quality-ladder models of endogenous growth with non-drastic innovations consider.

Proposition 4. *When ν is close to ν^B, Π increases with ν for any value of $\theta < \frac{1}{\alpha}$. In other words, if competition is sufficiently intense total profits increase with the intensity of competition.*

The intuition here is that when ν is close to ν^B, only two firms are active and the old innovator's market share is already small. Now, a small increase in the intensity of competition, which slightly reduces the equilibrium price, has a dramatic percentage effect on the market share of the old innovator, implying that the productive efficiency effect is very large.

[insert Figure 2 around here]

Figures 2 illustrates these two Propositions. With $\alpha = \frac{1}{2}$, it shows the regions in which industry profits increase or decrease with the intensity of competition as θ ranges from 1 to 2 and ν ranges from 0 to $\nu^B = \frac{2-\theta}{\theta}$. Although our qualitative results are local, Figure 2 show that the productive efficiency effect can prevail over the price effect in a sizeable range of parameter values.

6.3. *Competition and growth*

Given Proposition 2, Propositions 3 and 4 have an immediate corollary: the economy's rate of growth increases with the intensity of competition when innovations are large (θ is close to $\frac{1}{\alpha}$), competition is already relatively strong, but not so strong as to exclude all but the most efficient firm (ν is close to ν^B), or both. Although we make no attempt to bring the model to the data here, it is interesting to ask whether these outcomes can result only for extreme values of the parameters, in which case they could hardly help explain the empirically positive relationship between competition and growth.

To get ideas in order, let us focus on the comparison between Cournot and Bertrand competition. The parameter region in which $\Pi^B > \Pi^C$, and hence Bertrand competition is certainly associated with faster growth, is shown in Figure 3. The shaded area again corresponds to the "reasonable" range: $\theta \in [1.02, 1.50]$ and $\alpha \in [0.30, 0.70]$. Competition is certainly good for growth in a sizeable region that includes parameter values that are commonly regarded as realistic, suggesting that our results are more than theoretical possibilities.

[insert Figure 3 around here]

6.4. *Competition and welfare*

Although a detailed welfare analysis is beyond the scope of this paper, a few remarks are in order. More intense competition has both a static and a dynamic effect on social welfare. The static effect is unambiguously positive: for any given state of the technology, the sharper the competition, the lower the price and the greater the output. Further, more intense competition improves the economy's productive efficiency. The dynamic effect, which operates via the incentive to innovate, is more complex. As we have seen, competition can be growth-enhancing or growth-inhibiting. In addition, the equilibrium rate of growth may exceed the socially optimal rate, which means that faster growth is not necessarily socially beneficial. Therefore, the overall welfare effect of more intense competition is in principle ambiguous; a more detailed analysis of the issue is left for future work.

7 Conclusion

After a recapitulation, we shall offer some brief remarks on the robustness of our analysis and its implications for policy and empirical work.

7.1. *Synopsis*

We have studied the effect of the competitive selection process on the incentive to innovate and the economy's rate of growth, by extending a standard Schumpeterian model of endogenous growth with improvements in the quality of products to allow for several asymmetric firms to be active simultaneously. Specifically, we have assumed that innovations are non-drastic and that competition is less intense than Bertrand competition, focusing on Cournot competition (and, more generally, on cases intermediate between Bertrand and Cournot). We have shown that competition can be good for growth when the magnitude of innovations is large or when competition is already moderately strong but not so strong as to leave room for only the most efficient firm. This conclusion is driven by two effects of the competitive selection process that we highlight: the front-end-loading effect and the productive efficiency effect.

7.2. *Robustness*

Although for concreteness we have used a specific endogenous growth model, we believe our main results can be re-produced in most if not all quality-ladder models that exhibit a positive relationship between the incentive to innovate and the economy's rate of growth. Many general equilibrium details of the model, that is to say, can be modified without changing the qualitative conclusions. We believe that the same holds for certain assumptions on the microeconomic structure of the innovative industry: even though we have focused on a model where the product is differentiated only vertically and there are no fixed costs, the front-end-loading and productive efficiency effects also operate under more general conditions. Another approach, for instance, would be to introduce horizontal product differentiation. Sticking to Bertrand competition, the intensity of competition could then be measured by the degree of product substitutability. The results of Aghion and Schankerman (2004) and Zanchettin (2006) suggest that a positive relationship between competition and growth can also emerge within such a framework. As for fixed costs, they generally speed up the displacement of innovators by subsequent innovations. However, as long as several asymmetric firms are active simultaneously, the effects we have highlighted continue to operate.

7.3. *Implications for empirical work*

For the purposes of empirical analysis, the main implication of our model is that some indices that are commonly taken as negative measures of the intensity of competition may in fact be positively related to it. This is true, for instance, of any concentration index, suggesting caution in interpreting certain empirical results. On the positive side, the analysis suggests that the average price-cost margin of an industry is a proper measure of the intensity of competition: it is easy to see that there is, indeed, a one-to-one negative relationship between ν and the average price-cost margin. Other proxies of the intensity of competition used in empirical work, such as exogenous changes in antitrust policy, can also be naturally mapped into changes in the conjectural variations parameter ν (Aghion et al., 2005).

Another testable implication of our analysis is that sharper competition tends to be positively associated with growth when the step-size of innovations is large. Since in equilibrium large innovations will also be more frequent, this suggests that a positive association between competition and innovation is more likely to be found in highly innovative industries.

7.4. *Policy implications*

For policy purposes, the model suggests that one must distinguish carefully between the effects of more competition in the innovative sector (in our model, the intermediate good sector), the traditional sector (the final good sector), and the research sector. More competition in R&D and in the traditional sector is always growth-enhancing, but its effect in the innovative sector is less obvious. Ideally, competition policy should allow the latest innovator to compete vigorously against past innovators, since this improves the process of market selection; at the same time, however, it should combat the entrenchment of monopoly, preventing incumbents from exploiting their market power to impede entry by subsequent innovators. Striking a balance between these objectives is perhaps the main current challenge for competition policy.

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Appendix

This Appendix contains all proofs that are omitted in the text.

Proof of Proposition 1. Define $\Omega(m) \equiv \frac{1+\theta+\theta^2+...+\theta^m}{(m+1)-(1-\alpha)(1-\nu)} - \theta^{m+1}$ and $\tilde{m}$ as the (necessarily unique) solution to $\Omega(m) = 0$. Then, the equilibrium number of firms is the highest integer not larger than $\tilde{m}$. The comparative statics properties of m mirror those of $\tilde{m}$. To be precise, since m is piecewise constant because of the integer number problem, when any arbitrary parameter a that influences $\tilde{m}$ increases, m jumps up (or down) to the next highest (or lowest) integer as a passes certain critical thresholds if and only if $\frac{\partial \tilde{m}}{\partial a}$ is positive (or negative). Next, note that

$$\frac{\partial \Omega}{\partial m} |_{m=\tilde{m}} = -\frac{\theta^m}{(m+1)-(1-\alpha)(1-\nu)} \{\theta - \log\theta + \theta\log\theta\,[(m+1)-(1-\alpha)(1-\nu)]\}$$

and is always negative for $\theta > 1$. It follows that for any arbitrary parameter a that influences $\tilde{m}$, the sign of $\frac{\partial \tilde{m}}{\partial a}$ equals the sign of $\frac{\partial \Omega}{\partial a}$. Using this fact, it is immediate to verify that $\tilde{m}$ decreases with α and ν, and hence m is a step-wise decreasing function of α and ν. As for θ, note that

$$\frac{\partial \Omega}{\partial \theta} = \frac{1+2\theta+3\theta^2+...+m\theta^{m-1}}{(m+1)-(1-\alpha)(1-\nu)} - (m+1)\theta^m < 0$$

whence the result follows. ■

Proof of Lemma 1. Define $V \equiv \sum_{i=0}^{m} \frac{z^i g^i \pi_i}{(r+z)^{i+1}}$, so that the equilibrium condition (18) becomes $V = \frac{1}{\lambda}$. First of all, we show that V is monotonically decreasing in z. Differentiating V we get:

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}\sum_{i=0}^{m} \frac{z^i g^i \pi_i}{(r+z)^{i+1}} = -\sum_{i=0}^{m-1} \frac{(i+1)z^i g^i(\pi_i - g\pi_{i+1})}{(r+z)^{i+2}} - \frac{(m+1)z^m g^m \pi_m}{(r+z)^{m+2}}$$

A sufficient condition for $\frac{\partial V}{\partial z}$ to be negative is that $\pi_i \geq g\pi_{i+1}$, or, in view of (10),

$$(p-\theta^i)^2 \geq g(p-\theta^{i+1})^2$$

A sufficient condition for this inequality to hold is

$$(p-\theta^{i}) \geq g(p-\theta^{i+1})$$

or

$$p(g-1) \leq \theta^{i}(\theta g-1)$$

This inequality is satisfied for all $i=0,...,m-1$ if it holds for $i=0$, that is if

$$p(g-1) \leq (\theta g-1)$$

which reduces to

$$p \leq \frac{\theta^{\frac{1}{1-\alpha}}-1}{\theta^{\frac{\alpha}{1-\alpha}}-1}$$

Clearly, it suffices to show that this inequality holds true when p equals the monopoly price $\frac{1}{\alpha}$, which always exceeds the equilibrium price $p(\nu)$ for $0 \leq \nu \leq \nu^{B}$. Therefore, we must prove that

$$\frac{1}{\alpha} \leq \frac{\theta^{\frac{1}{1-\alpha}}-1}{\theta^{\frac{\alpha}{1-\alpha}}-1}$$

which reduces to

$$\theta^{\frac{\alpha}{1-\alpha}}-1-\alpha\left(\theta^{\frac{1}{1-\alpha}}-1\right) \leq 0$$

At $\theta=1$, the expression is satisfied as an equality. To conclude the proof, it suffices to show that the derivative with respect to θ of the left-hand side of the above inequality is negative. Differentiating, we get

$$\frac{d}{d\theta}\left[\theta^{\frac{\alpha}{1-\alpha}}-1-\alpha\left(\theta^{\frac{1}{1-\alpha}}-1\right)\right]=-\frac{\alpha}{(1-\alpha)\,\theta}\left(\theta^{\frac{1}{1-\alpha}}-\theta^{\frac{\alpha}{1-\alpha}}\right) \leq 0.$$

This completes the proof that $\frac{\partial V}{\partial z}<0$, which implies that the equilibrium, if it exists, is unique. To show existence, note that $V(0)=\frac{\pi_0}{r}>\frac{1}{\lambda}$ by assumption, and $\lim_{z\to\infty} V(z)=0<\frac{1}{\lambda}$. Because $V(z)$ is continuous, an equilibrium exists. ■

Proof of Proposition 2. Differentiating (19) we get

$$\frac{dV}{d\nu}=\frac{z^{m}g^{m}}{(r+z)^{m+1}}\frac{d\Pi}{d\nu}+\sum_{s=0}^{m-1}[r-z(g-1)]\frac{z^{s}g^{s}}{(r+z)^{s+2}}\frac{d\Pi_{s}}{d\nu}$$

To prove the Proposition, it suffices to show that

$$\frac{d\Pi}{d\nu} \geq 0 \qquad \Longrightarrow \qquad \frac{d\Pi_s}{d\nu} > 0$$

for all $s = 0, 1, ..., m-1$.

To proceed, note that from (10) we get

$$\frac{\pi_{i+1}}{\pi_i} = \left(\frac{p - \theta^{i+1}}{p - \theta^i}\right)^2 \qquad i = 0, 1, ..., m-1$$

whenever firms i and $i+1$ are active at equilibrium. Differentiating the above expression we obtain

$$\frac{d}{d\nu}\left(\frac{\pi_{i+1}}{\pi_i}\right) = 2\frac{(p-\theta^{i+1})}{(p-\theta^i)^3}(\theta^{i+1} - \theta^i)\frac{dp}{d\nu} < 0$$

This means that the ratio between any active firm's profit and that of its next most efficient competitor decreases with the intensity of competition. It follows that

$$\frac{d\pi_j}{d\nu} \geq 0 \qquad \Longrightarrow \qquad \frac{d\pi_i}{d\nu} > 0 \qquad \forall i < j. \tag{A}$$

Obviously,

$$\frac{d\Pi_s}{d\nu} = \frac{d\pi_0}{d\nu} + \frac{d\pi_1}{d\nu} + ... + \frac{d\pi_s}{d\nu}$$

If $\frac{d\Pi}{d\nu} \geq 0$, the profit of at least one firm must increase with ν. Let j be the least efficient firm whose profits weakly increase with the intensity of competition. By definition, the profit of all firms $i > j$ will then decrease with the intensity of competition, whereas by condition (A) the profit of all firms $i \leq j$ will increase with the intensity of competition. This obviously implies that $\frac{d\Pi_s}{d\nu} > 0$ for $s \leq j$. When $s > j$, write

$$\frac{d\Pi_s}{d\nu} = \frac{d\Pi}{d\nu} - \frac{d\pi_{s+1}}{d\nu} - \frac{d\pi_{s+2}}{d\nu} - ... - \frac{d\pi_m}{d\nu}$$

Since $\frac{d\pi_{s+1}}{d\nu}, \frac{d\pi_{s+2}}{d\nu}, ..., \frac{d\pi_m}{d\nu}$ are negative for $s > j$, it is clear that if $\frac{d\Pi}{d\nu} \geq 0$ we have $\frac{d\Pi_s}{d\nu} > 0$ even when $s > j$. ■

Proof of Proposition 3. Assume for the time being that there are exactly two active firms ($m = 1$). Then, equilibrium price, output and profits are given by

$$p = \frac{1+\theta}{2 - (1-\alpha)(1-\nu)}$$

$$X = \alpha^{\frac{1}{1-\alpha}} p^{-\frac{1}{1-\alpha}}$$

$$\pi_0 = \frac{(p-1)^2}{2p-(1+\theta)} X$$

and

$$\pi_1 = \frac{(p-\theta)^2}{2p-(1+\theta)} X$$

respectively. Hence, industry profits are

$$\Pi = \frac{(p-1)^2 + (p-\theta)^2}{2p-(1+\theta)} X$$

Note that the intensity of competition ν affects industry profits Π only through the equilibrium price p (both directly and through X). Thus, we can use the chain rule of differentiation to obtain

$$\frac{d\Pi}{d\nu} = \frac{X}{[2p-(1+\theta)]^2} \left\{ 2[2p-(1+\theta)]^2 - [(p-1)^2 + (p-\theta)^2] \left[\frac{2p-(1+\theta)}{(1-\alpha)p} + 2 \right] \right\} \frac{dp}{d\nu}$$

It follows that

$$\frac{d\Pi}{d\nu} \propto [(p-1)^2 + (p-\theta)^2] \left[\frac{2p-(1+\theta)}{(1-\alpha)p} + 2 \right] - 2[2p-(1+\theta)]^2 \qquad \text{(B)}$$

where the symbol $\propto$ means "has the same sign as".

Next, note that when θ is close to $\frac{1}{\alpha}$ there are indeed only two active firms for any $\nu \in [0, \nu^B)$ (see condition (11) in the text), so relation (B) applies. To determine the sign of $\frac{d\Pi}{d\nu}$, note further that as θ tends to $\frac{1}{\alpha}$, the equilibrium price p must also tend to $\frac{1}{\alpha}$, since p is greater than θ for all $\nu < \nu^B$ and cannot be greater than the monopoly price $\frac{1}{\alpha}$. It follows that

$$\frac{d\Pi}{d\nu} |_{\theta=\frac{1}{\alpha}} \propto \left(\frac{1}{\alpha} - 1 \right)^2 > 0$$

Thus, industry profits increase monotonically with the intensity of competition. ■

Proof of Proposition 4. When ν is close to ν^B, for any value of θ only two firms will be active, i.e. the latest innovator and his immediate predecessor (see condition (11) in the text), and the equilibrium price will be close to θ. Using again relation

(B), exploiting the fact that when ν is close to ν^B the equilibrium price is close to θ we now obtain

$$\frac{d\Pi}{d\nu} \propto \frac{(\theta-1)^3}{(1-\alpha)\theta} > 0$$

Thus, industry profits increase with the intensity of competition for ν close to ν^B. ■

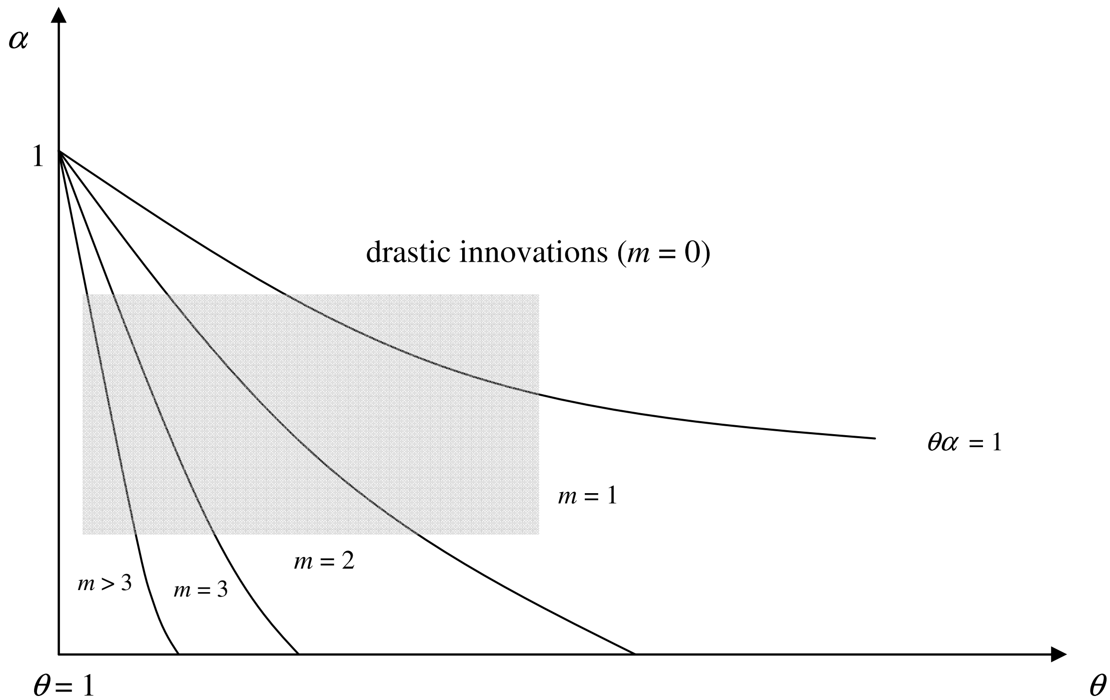

Figure 1. The equilibrium number of active firms as a function of the elasticity of demand α and the magnitude of innovations θ with Cournot competition.

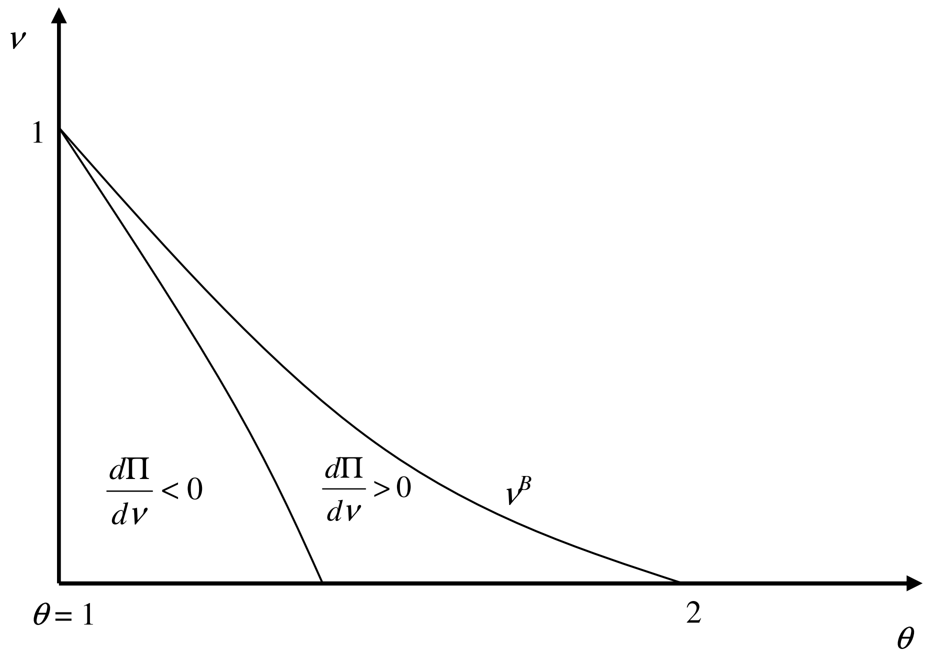

Figure 2. The effect of tougher competition on industry profits as a function of the magnitude of innovation, θ, and the intensity of competition, ν, for $\alpha = ½$.

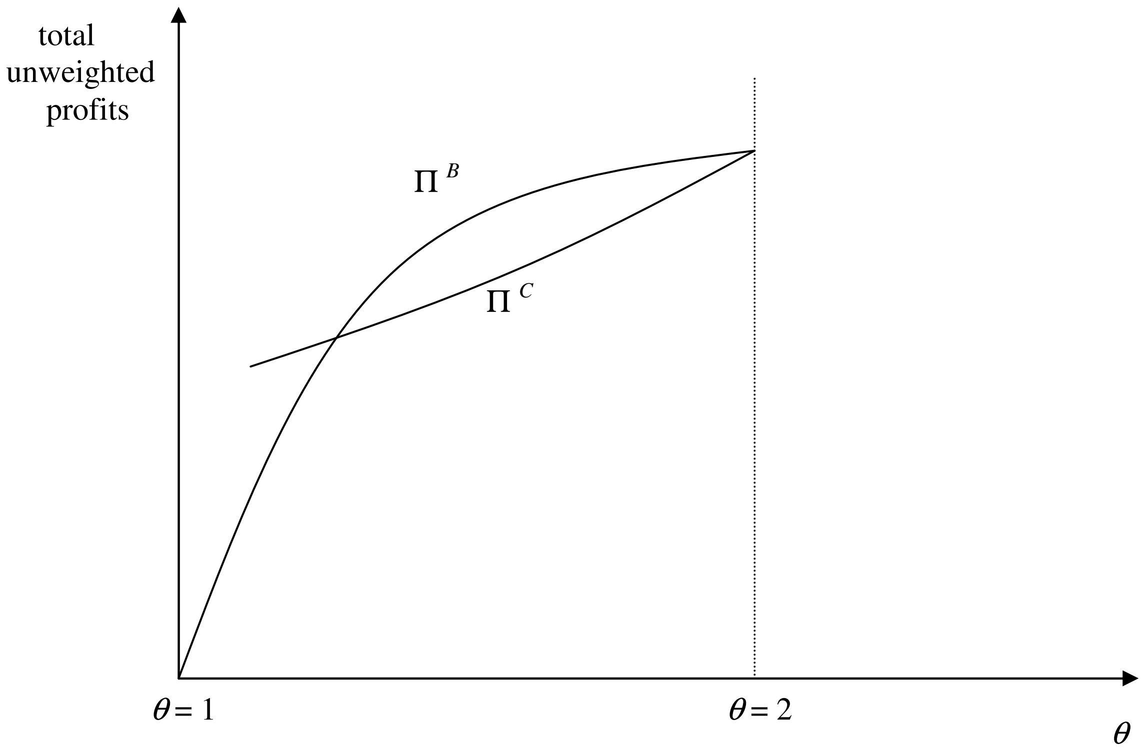

Figure 2. Total unweighted profits under Bertrand and Cournot competition as a function of the magnitude of innovations, θ, for $\alpha = ½$.