

Trading with a common agent under complete  
information: A characterization of Nash equilibria

Gabriella Chiesa

Vincenzo Denicolò\*

*Department of Economics, University of Bologna, Italy*

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\*Corresponding author at: Department of Economics, University of Bologna, Piazza Scaravilli 2,  
I-40126 Bologna, Italy. E-mail addresses: vincenzo.denicolo@unibo.it.

**Abstract.**

We analyze an abstract model of trading where  $N$  principals submit quantity-payment schedules that describe the contracts they offer to an agent, and the agent then chooses how much to trade with every principal. This represents a special class of common agency games with complete information. We study all the subgame perfect Nash equilibria of these games, not only truthful ones, providing a complete characterization of equilibrium payoffs. In particular, we show that the equilibrium that is Pareto-dominant for the principals is not truthful when there are more than two of them. We also provide a partial characterization of equilibrium strategies.

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# 1 Introduction

The theory of common agency applies to situations in which an agent makes a choice that affects  $N$  principals, each of whom tries to influence the agent's choice by offering payments contingent on it. This paper deals with a special class of complete information common agency games, those in which the agent's choice is an  $N$ -dimensional vector specifying the amount of a good to be traded with each principal. That is, principals first submit quantity-payment schedules describing the contracts they offer to the agent, and the agent then chooses how much to trade with every principal.

This abstract model of trading is flexible enough to encompass a wide variety of applications in economics. One, a natural extension of the standard model of Bertrand competition, is to the case of oligopolistic price discrimination, where the agent is a consumer who purchases a good from various firms that compete in non-linear prices (Spence [25], Spulber [26], Bhaskar and To [5]). Retaining the postulate that the agent *purchases* the good from the principals, other scenarios come readily to mind, such as vertical relationships (where several upstream firms share a common downstream agent that distributes the product),<sup>1</sup> split-award procurement (where a sponsor procures a good from various suppliers),<sup>2</sup> and markets for electricity or other intermediate goods (where a large buyer purchases from various firms).<sup>3</sup> The converse

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<sup>1</sup>See O'Brien and Shaffer [19, 20]. This case is complementary to that analyzed by Martimort and Stole [16] and Segal and Whinston [23], where competing retailers distribute the output of a common manufacturer. With competing retailers there are *direct* externalities between principals, since the price obtained by a retailer depends on the output sold by the others. Our model, by contrast, has only *contractual* externalities.

<sup>2</sup>See Anton and Yao [1], who assume that the sponsor is committed to purchase a pre-specified amount of the good and can only choose how to split the procurement between the suppliers. In our model, by contrast, the sponsor can choose the total amount to be purchased after suppliers have submitted their offers.

<sup>3</sup>Electricity markets are often modeled using the notion of supply function equilibrium proposed by Klemperer and Meyer [10]. In this approach, buyers are price-takers and the unique equilibrium price is determined by equating aggregate supply and demand. In our common agency model, by contrast, a centralized buyer exercises market power by maximizing along each principal's "supply function," and principals trade at personalized prices. Which model is more realistic depends on the institutional details of actual electricity markets, which vary considerably from country to country and over time.

case in which the agent *sells* the good to the principals, first studied by Stole [24] and Martimort [14], can apply to multi-unit package auctions (where buyers bid for variable amounts of a divisible good),<sup>4</sup> or the regulation of multinational enterprises by agencies in several countries (where each national agency offers the multinational a payment conditional on the quantity sold in its country, as in Calzolari [7]).

In all of these applications, one would like to determine the trades that occur, the payments that are made, and the structure of the offers that the principals submit in equilibrium. While the quantities traded determine the equilibrium allocation, the payments determine the distribution of the gains from trade; they also determine players' incentives (in an earlier stage) to invest in research, bear fixed entry costs, or make other kinds of investments that may affect those gains. Finally, the structure of equilibrium strategies determines the computational complexity of the equilibria.

These questions have been resolved for the special case in which principals are assumed to submit truthful offers.<sup>5</sup> With truthful strategies, the equilibrium allocation is efficient (Bernheim and Whinston [4]), each principal obtains his marginal contribution to social surplus (Bergemann and Välimäki [3]), and equilibrium offers are, by definition, truthful.<sup>6</sup> However, much less is known for the case of unrestricted offers. The literature has shown that under suitable regularity conditions the equilibrium allocation is always efficient (see footnote 12 below), but there is no general result on the structure of equilibrium payoffs and equilibrium strategies.

This paper provides the first complete characterization of the set of equilibrium

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<sup>4</sup>The auction literature (see e.g. Krishna and Tranaes [12] and Milgrom [18]) has typically posited that the seller is committed to sell a pre-specified total amount, but Milgrom [18] argues the importance of the case of an uncommitted seller, which corresponds to our common agency model.

<sup>5</sup>Generally speaking, a strategy is said to be truthful relative to a given action if it truly, and for all cases, reflects the principals' marginal preferences for another action relative to the given action. This notion was originally developed for the case of public common agency, but it can be extended immediately to the present framework of private common agency with no direct externalities. In such a framework, truthfulness means that each principal can ask for payments that differ from his true valuations of the proposed trades only by a constant.

<sup>6</sup>These properties of truthful equilibria actually hold in a larger class of common agency models of which ours is a special case.

payoffs and a partial characterization of equilibrium strategies. We show that the set of the principals' equilibrium payoffs is a hyper-rectangle: every principal, that is to say, can obtain any positive payoff below an upper bound that is independent of the other principals' payoffs and can be calculated explicitly (Proposition 1). Remarkably, with  $N > 2$  the maximum payoff for any given principal exceeds his marginal contribution to social surplus (Proposition 2).

The characterization of equilibrium strategies is more difficult, since principals may very well offer many non-serious contracts without disrupting an equilibrium. To narrow the field, we focus on the equilibrium sets of contracts of minimum cardinality and show that to support any equilibrium, in addition to the contract that will eventually be accepted by the agent, at least two principals must offer at least one other contract that will *not* be accepted. Conversely, any equilibrium outcome can be supported by strategies in which all principals offer only the contract that will be accepted, apart from just two principals each of whom also offers just one other contract (Proposition 3).

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 collects useful preliminary results that have already been demonstrated in the literature, and Section 4 covers new ground by characterizing the set of equilibrium payoffs. Section 5 analyzes the structure of equilibrium strategies, and Section 6 offers concluding remarks.

## 2 The model

This Section sets up our common agency model of trading. In the common agency literature, ours is known as a private, delegated common agency model. Common agency is *private* since a principal cannot condition payments due to him on the quantities traded by others, either because this is prohibited by antitrust law, or because trades with others cannot be observed or verified by a third party (Billette de Villemeur and Versaevel [6]). Common agency is *delegated* in that the agent is

allowed to trade with just a subset of the principals – even though in equilibrium she trades with all.

## 2.1 Notation and basic assumptions

There are  $N$  principals, indexed by  $i \in N = \{1, 2, \dots, N\}$ , who trade a homogeneous good with a common agent, indexed by 0. For ease of exposition, we posit that the agent (female) *purchases* the good from the principals (males). (The converse interpretation where the agent *sells* the good to the principals is obvious.) Trade is modeled as a first-price auction in which principals simultaneously submit a menu of contracts and the agent then chooses the quantity she will purchase from each. The game is one of complete information.

Since the quantities traded by the agent with other principals are not contractible, a strategy for a generic principal  $i$  is a set  $B_i \subset \mathbb{R}_+^2$  of quantity-payment pairs  $b_i = (x_i, P_i)$ , where  $x_i \geq 0$  is the quantity that principal  $i$  is willing to supply and  $P_i \geq 0$  is the corresponding total payment requested from the agent. We shall refer to a quantity-payment pair as a *contract* and to the menu of all contracts offered by a principal as a *supply schedule*.<sup>7</sup>

To keep the model as general as possible, we set almost no restrictions on feasible supply schedules. In particular, we do not require principals to submit one and only one offer for each output level but allow them for certain output levels not to submit any offer. This is both analytically convenient and realistic.<sup>8</sup> We only assume that supply schedules are compact (to guarantee the existence of an optimal choice for the agent), and that the null contract  $b^0 = (0, 0)$  belongs to any feasible supply schedule (this is just for notational convenience). Thus, letting  $\Gamma$  denote the set of feasible

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<sup>7</sup>The taxation principle implies that there is no substantial loss of generality in assuming that principals submit a set of quantity-payment pairs: see Peters [21] and Martimort and Stole [15]. Moreover, Peters [22] shows that in a framework like ours the assumption that each principal offers a single incentive scheme to the agent (i.e., only one supply schedule) is not restrictive.

<sup>8</sup>In many economic applications, such as multi-unit auctions or markets for electricity, principals usually offer a finite number of contracts. Our general framework allows, but does not force, principals to offer a contract for each non-negative output level below some arbitrarily large finite upper bound.

supply schedules (the same for all principals):

**Assumption 1**  $\Gamma = \{B \subset \mathfrak{R}_+^2 \mid B \text{ is compact and } (0, 0) \in B\}$ .

Given a profile of supply schedules  $\mathbf{B} = (B_1, B_2, \dots, B_N) \in \Gamma^N$ , in the second stage of the game the agent selects a contract from each supply schedule, trading is conducted, and payoffs are realized. Notice that since we have included the null contract in any feasible supply schedule, the agent can effectively refuse to trade with some principals by selecting their null contracts. A strategy for the agent is thus a function  $\beta(\mathbf{B}) : \Gamma^N \rightarrow (\mathfrak{R}_+^2)^N$  such that  $\beta(\mathbf{B}) \in \times_{i=1}^N B_i$  for all  $\mathbf{B} \in \Gamma^N$ . In other words,  $\beta = (\beta_1, \beta_2, \dots, \beta_N)$  is the profile of contracts accepted by the agent, one for each principal, and the function  $\beta(\mathbf{B})$  maps  $\Gamma^N$ , the set of all profiles of feasible supply schedules, to the set of admissible trades. Equilibria are *outcome-equivalent* if they lead to the same outcome  $\beta$ .

For any given  $\beta$ , the agent's payoff is

$$\pi_0(\beta) = U(X) - \sum_{i=1}^N P_i \quad (1)$$

where  $X = \sum_{i=1}^N x_i$  is the total quantity traded and the function  $U(X) : \mathfrak{R}_+ \rightarrow \mathfrak{R}_+$  denotes its value to the agent, in monetary terms. The principals' payoffs are

$$\pi_i(\beta) = \pi_i(\beta_i) = P_i - C_i(x_i). \quad (2)$$

where  $C_i(x_i) : \mathfrak{R}_+ \rightarrow \mathfrak{R}_+$  is  $i$ 's cost function. Since each principal's cost depends only on own output, there are no *direct* externalities: there are only the *contractual* externalities between principals, which arise because the agent's marginal willingness to pay for the good depends on the quantities traded with all principals.

We make the following regularity assumptions.

**Assumption 2** *The function  $U(X)$  is differentiable, strictly increasing and strictly concave (and hence a.e. twice differentiable) with  $U(0) = 0$ .*

**Assumption 3** Each principal's cost function  $C_i(x_i)$  is differentiable, strictly increasing and strictly convex (and hence a.e. twice differentiable), with  $C_i(0) = 0$ , for all  $i \in N$ .

**Assumption 4** (Inada conditions)  $\lim_{X \rightarrow 0} U'(X) > 0$ ,  $\lim_{X \rightarrow \infty} U'(X) = 0$ , and, for all  $i \in N$ ,  $\lim_{x_i \rightarrow 0} C'_i(x_i) = 0$  and  $\lim_{x_i \rightarrow \infty} C'_i(x_i) = \infty$ .

Assumptions 2 and 3 guarantee that the equilibrium allocation is efficient, so we can focus on equilibrium payoffs and strategies.<sup>9</sup> The assumption  $C_i(0) = 0$  means that all fixed costs are sunk. The assumption that marginal costs are strictly increasing rules out the trivial case of Bertrand competition with constant marginal costs. Finally, the Inada conditions ensure that efficiency requires every principal to trade a strictly positive and finite quantity with the agent.

## 2.2 Efficiency

The *efficient allocation* is the vector  $\mathbf{x}^* = (x_1^*, x_2^*, \dots, x_N^*)$  that maximizes the sum of the players's payoffs  $S(\mathbf{x}) = U(X) - \sum_{i=1}^N C_i(x_i)$  (starred variables denote the efficient allocation). Given Assumptions 2 and 3, the social surplus  $S(\mathbf{x})$  is bounded above and globally, strictly concave; thus, the efficient allocation is the unique solution to the following system of first-order conditions:

$$U'(X) = C'_i(x_i) \quad \forall i \in N. \quad (3)$$

By Assumption 4,  $\mathbf{x}^*$  is strictly positive.

Let  $S^* = S(\mathbf{x}^*)$  be the maximized social surplus. The *marginal contribution* of principal  $i$  is  $S^* - S_{-i}^*$ , where  $S_{-i}^*$  is the maximum social surplus attainable when

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<sup>9</sup>In particular, Assumption 2 rules out the case in which the agent is committed to purchase a fixed quantity  $\bar{X}$  (which corresponds to a function  $U(X)$  that is zero for  $X < \bar{X}$  and is an arbitrarily large constant for  $X \geq \bar{X}$ ). Anton and Yao [1] and Krishna and Tranaes [12] show that when the agent is committed to purchase (or to sell) a pre-set total quantity, there will generally be a large number of inefficient equilibria.

principal  $i$  is inactive, i.e.

$$S_{-i}^* = \max_{\substack{x_j \\ j \neq i}} \left[ U(X_{-i}) - \sum_{j \neq i} C_j(x_j) \right]. \quad (4)$$

(Hereafter  $X_{-\Omega}$  denotes  $\sum_{j \notin \Omega} x_j$  for all  $\Omega \subset N$ ; with a slight abuse of notation, we write  $X_{-i}$  instead of  $X_{-\{i\}}$ .)

The following condition is not really needed but simplifies the exposition:

**Assumption 5** (*no crossing*) *After possible relabelling of principals,*

$$C'_1(x_1^* + \xi) \leq C'_2(x_2^* + \xi) \leq \dots \leq C'_N(x_N^* + \xi) \quad \forall \xi \geq 0 \quad (5)$$

This means that principals can be unambiguously ranked in terms of their marginal costs when individual output exceeds efficient output: principal 1 is the most efficient, 2 the second most efficient, etc.

### 2.3 Equilibrium

Since ours is a two-stage game of complete information, it seems appropriate to employ the solution concept of subgame perfect Nash equilibrium.<sup>10</sup> Denoting equilibrium variables by a circumflex, a subgame perfect Nash equilibrium (henceforth, an equilibrium) is a list of strategies,  $\langle \hat{\beta}(\mathbf{B}), \hat{B}_1, \hat{B}_2, \dots, \hat{B}_N \rangle$ , one for each player, such that

$$\hat{\beta}(\mathbf{B}) \in \arg \max_{\beta \in \times_{i=1}^N B_i} \pi_0(\beta) \quad \forall \mathbf{B} \in \Gamma^N \quad (6)$$

and

$$\hat{B}_i \in \arg \max_{B_i \in \mathbf{I}} \pi_i(\hat{\beta}(\hat{\mathbf{B}}_{-i}, B_i)) \quad \forall i \in N \quad (7)$$

where  $(\hat{\mathbf{B}}_{-i}, B_i) \equiv (\hat{B}_1, \dots, \hat{B}_{i-1}, B_i, \hat{B}_{i+1}, \dots, \hat{B}_N)$ .

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<sup>10</sup>Essentially, any outcome  $(\beta_1, \beta_2, \dots, \beta_N)$  where  $C_i(x_i) < P_i < U(X) - U(X_{-i})$  for all  $i \in N$  could be supported as a Nash equilibrium outcome. To see this, suppose that the agent pledges to refuse to trade with principal  $i$  if  $i$  does not offer his prescribed contract  $\beta_i$ , and each principal offers only his prescribed contract (plus the null contract). Clearly, these strategies constitute a Nash equilibrium of the game. Subgame perfection rules out any such commitment on the part of the agent and so restricts the set of equilibrium outcomes considerably.

### 3 Equilibrium allocation

We start our analysis with a useful implicit characterization of the equilibria. Observe that, given the supply schedules of the other  $N - 1$  principals, every principal  $i$  effectively plays a bargaining game with the agent, in which he has all the bargaining power and so must leave the agent with a rent that does not exceed her disagreement payoff, which is the maximum rent she can obtain by trading optimally with the remaining  $N - 1$  principals. This implies that in an equilibrium the agent can exclude any one principal and still earn her equilibrium payoff:

$$\hat{\pi}_0 = \max_{\substack{(x_j, P_j) \in \hat{B}_j, \\ j \neq i}} \left[ U(X_{-i}) - \sum_{j \neq i} P_j \right] \quad \forall i \in N. \quad (8)$$

We shall call this condition *individual excludability*. Individual excludability in turn means that given  $\hat{\mathbf{B}}_{-i}$  the agent's equilibrium payoff  $\hat{\pi}_0$  is independent of  $B_i$ . As a consequence, principal  $i$ 's strategy must maximize his joint payoff with the agent given  $\hat{\mathbf{B}}_{-i}$ , since this amounts to maximizing his own payoff. This implies *bilateral efficiency*, meaning that the equilibrium joint payoff of the agent and principal  $i$  is the maximum joint payoff given  $\hat{\mathbf{B}}_{-i}$ :<sup>11</sup>

$$\hat{\pi}_0 + \hat{\pi}_i = \max_{\substack{x_i \\ (x_j, P_j) \in \hat{B}_j, \\ j \neq i}} \left[ U(X_{-i} + x_i) - \sum_{j \neq i} P_j - C_i(x_i) \right] \quad \forall i \in N. \quad (9)$$

These two properties – individual excludability and bilateral efficiency – essentially characterize the equilibria of our game. This characterization result recurs in the common agency literature (see e.g. Lemma 2 in Bernheim and Whinston [4] and the fundamental equations of Laussel and Le Breton [13]), and its proof need not be repeated here.

**Lemma 1** *In any equilibrium, individual excludability (8), bilateral efficiency (9), and the agent best response property (6) must hold. The converse also holds: any list of strategies  $\langle \beta(\mathbf{B}), B_1, B_2, \dots, B_N \rangle$  satisfying (6), (8), and (9) is an equilibrium.*

<sup>11</sup>See Martimort and Stole [16]; Segal and Whinston [23] call this condition *pairwise stability*.

It can be shown that bilateral efficiency implies that individual outputs  $x_i$  are set in such a way as to satisfy the equalities (3). Because the social surplus function is globally concave, this ensures that the social surplus is maximized in equilibrium.<sup>12</sup>

**Lemma 2** *In any equilibrium,  $\hat{\mathbf{x}} = \mathbf{x}^*$ .*

Lemma 2 implies that in equilibrium each principal  $i$  must offer a contract for his efficient quantity  $x_i^*$  that will certainly be accepted. This contract will be called the *efficient contract* and will be denoted by  $b_i^* = (x_i^*, P_i^*)$ .

## 4 Equilibrium payoffs

Lemma 2 fully characterizes the equilibrium allocation. But what are the payments and payoffs in equilibrium? To address this question, define

$$V_i(y) = \max_{x_i} [U(y + x_i) - C_i(x_i)] \quad (10)$$

as the maximum joint payoff that the agent and principal  $i$  can get as a function of the quantity supplied by the other principals,  $y$ , and gross of any payments to them. By Assumptions 2-4, the functions  $V_i(y)$  are well defined for all  $i \in N$ ; it is easy to check that they are strictly increasing and strictly concave.<sup>13</sup> Then, let

$$\bar{\pi}_i = V_{v_i}(X_{-v_i}^*) - V_{v_i}(X_{-\{i, v_i\}}^*) - C_i(x_i^*), \quad \forall i \in N \quad (11)$$

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<sup>12</sup>This efficiency result dates back to Spence [25] and Spulber [26], who however assume that the equilibrium payoffs to principals are equal to their marginal contributions. The first proof of the efficiency of the equilibrium allocation that does not rely on this assumption is that of O'Brien and Shaffer [19], showing that in the two-principal case the equilibrium is efficient and principals cannot obtain more than their marginal contribution to social surplus. The general case is covered by O'Brien and Shaffer [20], which analyzes a bargaining game in which the agent bargains simultaneously but separately with  $N$  principals. In the special case in which the principals have all the bargaining power, their framework is equivalent to a common agency game.

<sup>13</sup>To see this, compute

$$\frac{dV_i}{dy} = U'(y + x_i) > 0$$

by the envelope theorem, and

$$\frac{d^2V_i}{dy^2} = U''(y + x_i) \left[ 1 - \frac{U''(y + x_i)}{U''(y + x_i) - C_i''(x_i)} \right] < 0$$

by the implicit function theorem.

where

$$v_1 = 2 \quad \text{and} \quad v_i = 1 \quad \text{for } i = 2, 3, \dots, N. \quad (12)$$

**Proposition 1** *A vector of payoffs  $(\pi_0, \pi_1, \pi_2, \dots, \pi_N)$  is a vector of equilibrium payoffs if and only if it satisfies  $\pi_0 + \pi_1 + \pi_2 + \dots + \pi_N = S^*$  and  $0 < \pi_i \leq \bar{\pi}_i \quad \forall i \in N$ .*

Equilibrium payoffs are not unique: the set of the principals' equilibrium payoffs is, indeed, a semi-open hyper-rectangle. To gain some intuition for this multiplicity of equilibrium outcomes, observe that each principal  $i$  can offer many contracts other than the one that will be accepted in equilibrium. As long as the agent selects  $i$ 's efficient contract  $b_i^*$ , these other contracts do not affect  $i$ 's own payoff, but they may constrain the payments that  $i$ 's competitors can request for their prescribed equilibrium quantities, and hence their payoffs. In particular, the more aggressively principal  $i$  bids for quantities greater than  $x_i^*$ , the lower his competitors' payoffs. This explains the multiplicity of equilibrium payoffs, and suggests that principals can tacitly "coordinate" their offers in order to increase their payoffs.

What is the maximum degree of coordination among principals that is consistent with playing the game non-cooperatively? In other words, supposing that principals tacitly agree to request very large payments in exchange for any quantities except their respective equilibrium quantities  $x_i^*$ , what are the maximum payments for  $x_i^*$  that make the tacit agreement stable? Clearly, no principal  $i$  could request a payment higher than  $U(X^*) - U(X_{-i}^*)$  for his equilibrium quantity  $x_i^*$ , for the agent would refuse. But the maximum equilibrium payments are in fact lower than  $U(X^*) - U(X_{-i}^*)$ , because if a principal  $i$  asks for an excessive payment it may be profitable for other principals to offer a contract that crowds  $i$  out. That is, it is the threat of being replaced by somebody else that determines the maximum payment that principal  $i$  can obtain for  $x_i^*$ .

More precisely, Proposition 1 shows that each principal  $i$  has one *pivotal competitor*, denoted by  $v_i$ , who poses the most serious threat to replace him. The maximum

payment  $i$  can request is thus

$$\bar{P}_i^* = V_{v_i}(X_{-v_i}^*) - V_{v_i}(X_{-\{i,v_i\}}^*) < U(X^*) - U(X_{-i}^*),^{14} \quad (13)$$

i.e., it makes  $v_i$  just indifferent between supplying his prescribed equilibrium quantity or proposing to the agent to unilaterally replace  $i$ , given that all the other principals but  $i$  continue to supply their equilibrium quantities. Proposition 1 shows that this is the most serious menace to principal  $i$ , and also that the pivotal competitor is  $i$ 's most efficient competitor (this follows from Assumption 5).

The multiplicity of equilibrium outcomes raises the question of equilibrium selection. Our next result contrasts two common criteria for selecting equilibria – Pareto dominance and truthfulness. Given the timing of moves, it is clear that the appropriate notion of Pareto dominance is restricted to the principals. We shall call the equilibrium that is Pareto-dominant for the principals, namely  $(\bar{\pi}_0, \bar{\pi}_1, \bar{\pi}_2, \dots, \bar{\pi}_N)$ , the *minimum rent* equilibrium since it minimizes the agent's rent.

**Proposition 2** *When  $N = 2$ , the minimum rent equilibrium is outcome-equivalent to the truthful equilibrium. When  $N > 2$ , however, the two equilibria are not outcome-equivalent, and in the minimum rent equilibrium every principal's payoff exceeds his marginal contribution to social surplus.*

The intuition is simple. A principal's marginal contribution to social surplus is the difference between maximized social surplus and the social surplus that is attained when that principal is *optimally* replaced by the remaining  $N - 1$  principals. However, Proposition 1 shows that in a non-cooperative equilibrium each principal's maximum payoff is determined by the threat of being *unilaterally* replaced by one

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<sup>14</sup>To prove the inequality, note that

$$\begin{aligned} V_{v_i}(X_{-v_i}^*) - V_{v_i}(X_{-\{i,v_i\}}^*) &= U(X^*) - C_{v_i}(x_{v_i}^*) - \max_{x_{v_i}} [U(X_{-\{i,v_i\}}^*) - C_{v_i}(x_{v_i})] \\ &< U(X^*) - U(X_{-i}^*) \end{aligned}$$

where the inequality is strict because  $x_i^* > 0$ , which implies  $U'(X_{-i}^*) > C'_{v_i}(x_{v_i}^*)$  and hence  $\arg \max_{x_{v_i}} [U(X_{-\{i,v_i\}}^*) - C_{v_i}(x_{v_i})] > x_{v_i}^*$ .

of his competitors – the pivotal competitor. Because marginal costs are increasing, when  $N > 2$  unilateral replacement by one principal is more costly than cooperative replacement by  $N - 1$  principals. This explains why in equilibrium a principal can obtain more than his marginal contribution to social surplus. It also implies that the minimum rent equilibrium is not truthful, since in a truthful equilibrium each principal earns exactly his marginal contribution (Bergemann and Välimäki [3]).

Proposition 2 highlights a tension between Pareto dominance and truthfulness.<sup>15</sup> Both criteria are appealing, and taken together they suggest that principals are unlikely to get less than their marginal contributions. However, neither one alone seems compelling, so it is hard to rule out the possibility that principals may coordinate on an equilibrium in which they receive more than their marginal contributions.

This possibility highlights a source of inefficiency that seems to have been neglected so far. Often, before trading principals can make decisions that affect the eventual benefits, such as how much to invest in research or whether to bear fixed entry costs to participate in trading. To analyze the incentives for these investments, one can embed our common agency game in a two-stage game in which the principals first make some sort of investment and then trade with the agent. One can then distinguish between *ex post* efficiency, which is restricted to the common agency game, and *ex ante* efficiency, which also takes the first-stage investments into account. Clearly, the incentives to make such investments depend on the payoffs that principals obtain in the common agency stage. In the absence of strategic externalities, *ex ante* efficiency would be guaranteed if in the second stage principals got their marginal contributions to social surplus.<sup>16</sup> But the fact that principals can get more

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<sup>15</sup>Konishi, Le Breton and Weber [11] provide an early example of a common agency game where the Pareto-dominant equilibrium is not truthful.

<sup>16</sup>Farrell and Katz [8] and Bhaskar and To [5] note that in the presence of strategic externalities, inefficiencies may arise even if principals earn just their marginal contributions. In their framework, there are two strategic choices to be made sequentially before entering the market competition stage: entry, then R&D investment (Farrell and Katz) or location (Bhaskar and To). They argue that a principal's entry affects the investment or location choices of others, implying excessive incentives to enter.

than their marginal contributions means that ex ante efficiency is not guaranteed; for instance, there may be excessive entry, or distorted R&D investments.<sup>17</sup>

## 5 Equilibrium Strategies

Now we focus on the structure of the principals' equilibrium strategies. Since each principal can add many non-serious contracts to an equilibrium supply schedule without disrupting the equilibrium, a more specific question is in order, namely what the structure of the simplest equilibrium supply schedules is. Even so, the problem is not well specified, since there are various reasonable definitions of complexity. In our framework, a natural approach is to define complexity by the cardinality of supply schedules. The problem then becomes, What is the *minimum cardinality* of equilibrium supply schedules?

We already know that each principal must necessarily offer his efficient contract and the null contract. If a principal submits only those contracts, he is effectively making a take-it-or-leave-it (TIOLI) offer. The following example shows that in general, equilibria cannot be supported by TIOLI offers.<sup>18</sup>

*Example.* Assume that there are two symmetric principals with  $C_i(x_i) = \frac{1}{2}x_i^2$  for  $i = 1, 2$ . Assume further that  $U(X) = X - \frac{1}{2}X^2$ . By Lemma 2, at any equilibrium, necessarily  $x_1 = x_2 = \frac{1}{3}$ . Now, suppose there exists an equilibrium in which principal 1, say, submits a supply schedule that consists only of an efficient contract  $b_1^* = (\frac{1}{3}, P_1^*)$  and the null contract  $b^0$ . By individual excludability, if the agent refuses to trade with

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<sup>17</sup>The robustness of the findings of papers using the truthfulness requirement is also questioned by the results of Martimort and Stole [17], who in a common agency game in a public good context study the equilibria that can be rationalized as limits of equilibria of the corresponding asymmetric information game. Even with this refinement, they too find that the set of equilibrium payoffs is larger than the set of truthful equilibrium payoffs.

<sup>18</sup>If the revelation principle held true, it would indeed be possible to sustain any equilibrium with TIOLI offers (with full information, the set of the agent's possible types has dimension zero). But generally speaking, when there are many competing principals the revelation principle does not hold. As a consequence, TIOLI offers may not suffice to support an equilibrium, unless special assumptions are made, as in Kirchsteiger and Prat [9] and Attar et al. [2]. Constraining principals to make TIOLI offers would result in a different game than ours, with different equilibria.

principal 2 she must still be able to obtain her equilibrium payoff. Since  $b_1^*$  is the only non-null contract offered by principal 1, this implies  $U(\frac{2}{3}) - P_1^* - P_2^* = U(\frac{1}{3}) - P_1^*$ , whence  $P_2^* = \frac{1}{6}$ . The joint equilibrium payoff of the agent and principal 1 is therefore  $\pi_0 + \pi_1 = U(\frac{2}{3}) - \frac{1}{6} - C_1(\frac{1}{3}) = \frac{2}{9}$ . However, bilateral efficiency means that  $\pi_0 + \pi_1$  is maximized along principal 2's equilibrium supply schedule. Since principal 2's supply schedule contains the null contract, we must have  $\pi_0 + \pi_1 \geq V_1(0)$ , or  $\frac{2}{9} \geq \frac{1}{4}$ , which is impossible. A symmetrical argument applies to principal 2; hence, there is no equilibrium in which a principal offers only the efficient contract and the null contract.

The example shows that equilibrium supply schedules must contain some contracts that will never be accepted. How many such contracts are needed to support an equilibrium? Proposition 3 resolves the problem of the minimum cardinality of equilibrium supply schedules showing that at least two principals must submit at least one contract that will not to be accepted.

**Proposition 3** *In any equilibrium,  $|\hat{B}_i| \geq 2$  for all  $i \in N$  and  $|\hat{B}_i| \geq 3$  for at least two principals. Conversely, any equilibrium outcome can be supported by equilibrium supply schedules such that  $|\hat{B}_i| = 2$  for  $N-2$  principals and  $|\hat{B}_i| = 3$  for the remaining two principals.*

The relative simplicity of these equilibrium strategies differentiates our common agency models from others in which the computational complexity of certain equilibria is so burdensome as to make them unappealing (Kirchsteiger and Prat [9]). On the other hand, Proposition 3 means that our trading model does not possess any “natural” equilibrium, i.e., an equilibrium in which each principal makes TIOLI offers (see Kirchsteiger and Prat [9], Attar et al. [2]).

## 6 Conclusions

Common agency theory has focused almost exclusively on truthful strategies and truthful equilibria. The few works that have analyzed non-truthful equilibria have studied specialized frameworks, such as models of vertical relationships, public good provision, and lobbying. Some of these studies have sought to characterize all Nash equilibria of the game (Martimort and Stole [16], Segal and Whinston [23]), while others have looked at alternative notions of equilibrium, such as natural equilibria (Kirchsteiger and Prat [9]), or equilibria that can be rationalized as limits of equilibria of the asymmetric information game (Martimort and Stole [17]).

This paper contributes to the literature by analyzing an abstract model of trading in which  $N$  principals submit price-quantity schedules that describe the trades they offer to the agent, after which the agent optimally chooses how much to trade with each principal. We provide a full characterization of all equilibrium payoffs (it is already known that the equilibrium allocation is efficient), and some insights into the structure of equilibrium supply schedules.

We have sought to characterize all equilibria, but our finding that the set of the principals' equilibrium payoffs is a hyper-rectangle suggests a natural alternative to the truthful equilibrium, namely, the equilibrium that is Pareto-dominant for the principals. These equilibria coincide when  $N = 2$ , but differ when  $N > 2$ . Which equilibrium, then, is most likely to prevail? This is a difficult question, and beyond the scope of this paper. One approach would be to refine the notion of players' rationality until only one equilibrium is left. A number of studies have been conducted along these lines, but no consensus has emerged, so other approaches may also be of interest. For instance, one could address the problem experimentally, or develop the comparative statics properties of different equilibria in order to determine which one best matches the empirical evidence in economic applications. We leave this work for future research.

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## Appendix

**Proof of Proposition 1. Necessity.** Let  $\langle \hat{\beta}(\mathbf{B}), \hat{B}_1, \hat{B}_2, \dots, \hat{B}_N \rangle$  be an equilibrium. Necessity of  $\hat{\pi}_0 + \hat{\pi}_1 + \hat{\pi}_2 + \dots + \hat{\pi}_N = S^*$  follows from Lemma 2. Assumptions  $U'(X) > 0$  and  $\lim_{x_i \rightarrow 0} C'_i(x_i) = 0$  imply

$$\max_{\substack{x_i \\ (x_j, P_j) \in \hat{B}_j, \\ j \neq i}} \left[ U(X_{-i} + x_i) - \sum_{j \neq i} P_j - C_i(x_i) \right] > \max_{\substack{(x_j, P_j) \in \hat{B}_j, \\ j \neq i}} \left[ U(X_{-i}) - \sum_{j \neq i} P_j \right]$$

for all  $i \in N$ . By (8), the right-hand side of this inequality is  $\hat{\pi}_0$ , and by (9) the left-hand side is  $\hat{\pi}_0 + \hat{\pi}_i$ . The inequality thus becomes  $\hat{\pi}_0 + \hat{\pi}_i > \hat{\pi}_0$  whence  $\hat{\pi}_i > 0$  for all  $i \in N$ . It remains to show that  $\hat{\pi}_i \leq \bar{\pi}_i$ . This can be proved by *reductio ad absurdum*. Suppose to the contrary that for some principal  $i$  we have  $\hat{\pi}_i > \bar{\pi}_i$ , or equivalently  $\hat{P}_i^* > \bar{P}_i^*$ . We obtain a contradiction as follows

$$\begin{aligned} \hat{\pi}_0 + \hat{\pi}_{v_i} &= \max_{\substack{x_{v_i} \\ (x_j, P_j) \in \hat{B}_j, \\ j \neq v_i}} \left[ U(X_{-v_i} + x_{v_i}) - \sum_{j \neq v_i} P_j - C_{v_i}(x_{v_i}) \right] \\ &\geq V_{v_i}(X_{-\{i, v_i\}}^*) - \sum_{j \neq i, v_i} \hat{P}_j^* \\ &= V_{v_i}(X_{-v_i}^*) - \sum_{j \neq i, v_i} \hat{P}_j^* - \bar{P}_i^* \\ &> U(X^*) - C_{v_i}(x_{v_i}^*) - \sum_{j \neq v_i} \hat{P}_j^* \\ &= \hat{\pi}_0 + \hat{\pi}_{v_i} \end{aligned}$$

The first line is (9), which by Lemma 1 must hold; the second line follows because the null contract of principal  $i$  and the efficient contracts of principals  $j \neq i, v_i$  must belong to their respective equilibrium supply schedules. The next equality follows from the definition of  $\bar{P}_i^*$  (Eq. (13)), and the subsequent strict inequality

follows from the definition of the functions  $V_i(y)$  and the assumption  $\hat{P}_i^* > \bar{P}_i^*$ . Finally, the last equality, which establishes the contradiction, follows from the fact that  $\hat{\pi}_0 + \hat{\pi}_1 + \hat{\pi}_2 + \dots + \hat{\pi}_N = S^*$ .

*Sufficiency.* The proof is constructive: for any vector of payoffs  $(\pi'_0, \pi'_1, \pi'_2, \dots, \pi'_N)$  that satisfies  $0 < \pi'_i \leq \bar{\pi}_i$  for all  $i \in N$  and  $\pi'_0 + \pi'_1 + \pi'_2 + \dots + \pi'_N = S^*$ , we exhibit an equilibrium that yields such payoffs.

Let  $P_i'^* = C_i(x_i^*) + \pi'_i \leq \bar{P}_i^*$  and  $b_i'^* = (x_i^*, P_i'^*)$  for all  $i \in N$ . The players' strategies are defined as follows. First, for  $i = 3, \dots, N$  we assume that principal  $i$  offers only the efficient contract  $b_i'^*$  and the null contract  $b^0$ , i.e.,

$$B_i' = \{b_i'^*, b^0\} \quad i = 3, \dots, N.$$

Second, we assume that principals 1 and 2 offer also another contract, which is defined as follows. Let  $\tilde{x}'$  be the minimum output level that satisfies the following conditions:

$$V_i(x') - U(x') \leq \pi'_i \quad \forall i \in N \quad (\text{A1})$$

and

$$2U(x') - U(2x') \geq \pi'_0. \quad (\text{A2})$$

By assumption,  $\pi'_i > 0$  for all  $i \in N$ . Assumptions 2-4 then guarantee that the set of values of  $x'$  satisfying (A1) and (A2) is non-empty; hence  $\tilde{x}'$  exists. Then, the extra contract is defined as

$$\tilde{b}' = (\tilde{x}', U(\tilde{x}') - \pi'_0),$$

(in fact, any  $x \geq \tilde{x}'$  would do) and we posit

$$B_1' = \{b_1'^*, b^0, \tilde{b}'_1\}$$

and

$$B_2' = \{b_2'^*, b^0, \tilde{b}'_2\},$$

where  $\tilde{b}'_1 = \tilde{b}'_2 = \tilde{b}'$ . Finally, we assume that  $\beta'(\mathbf{B})$  is a function that satisfies (6) for all  $\mathbf{B} \neq \mathbf{B}'$  and such that  $\beta'(\mathbf{B}') = \mathbf{b}'^*$ .

Given Lemma 1, to show that  $\langle \beta'(\mathbf{B}), B'_1, B'_2, \dots, B'_N \rangle$  is an equilibrium we must prove that  $\langle \beta'(\mathbf{B}), B'_1, B'_2, \dots, B'_N \rangle$  satisfies (6), (8), and (9). Consider (6) first, i.e.

$$\mathbf{b}^{l*} \in \arg \max_{\beta \in \times_{i=1}^N B'_i} \pi_0(\beta).$$

First, we show that the agent cannot earn more than  $\pi'_0$  by choosing  $b^0$  for  $i \in \Omega \subset N$  and  $b_i^{l*}$  for  $i \in N - \Omega$ . We already know (footnote 13) that  $P_i^{l*} < U(X^*) - U(X_{-i}^*)$ . By the concavity of  $U(X)$ , this implies  $P_i^{l*} < U(y) - U(y - x_i^*)$  for all  $x_i^* \leq y \leq X^*$  and hence

$$\sum_{i \in \Omega} P_i^{l*} < U(X^*) - U(X_{-\Omega}^*)$$

for all  $\Omega \subset N$ , which proves the claim. Second, we show that the agent cannot earn more than  $\pi'_0$  by choosing  $\tilde{b}'_1, \tilde{b}'_2$ , or both. It suffices to show that if the agent selects  $\tilde{b}'_1$  (resp.,  $\tilde{b}'_2$ ), she will then select  $b^0$  for  $i \in N - \{1\}$  (resp.,  $i \in N - \{2\}$ ). Condition (A1) implies

$$U(\tilde{x}' + x_i^*) - U(\tilde{x}') \leq C_i(x_i^*) + \pi'_i = P_i^{l*} \quad \forall i \in N$$

which means that if the agent accepts  $\tilde{b}'_1$ , she cannot gain more than  $\pi'_0$  by accepting the efficient contract of any other principal  $i$  and  $N - 2$  null contracts. Condition (A2) guarantees that if the agent accepts  $\tilde{b}'_1$ , she prefers to accept the remaining  $N - 1$  null contracts rather than  $\tilde{b}'_2$  and  $N - 2$  null contracts. Reasoning as above, the concavity of  $U(X)$  implies that if the agent accepts  $\tilde{b}'_1$ , she prefers to accept the remaining  $N - 1$  null contracts rather than any other combination of contracts. The same is true if the agent accepts  $\tilde{b}'_2$ , confirming that (6) holds.

Individual excludability is obvious, since the agent can always pick up  $\tilde{b}'_1$  (or  $\tilde{b}'_2$ ) and  $N - 1$  null contracts.

To complete the proof, it remains to show that  $\langle \beta'(\mathbf{B}), B'_1, B'_2, \dots, B'_N \rangle$  satisfies bilateral efficiency. This requires that for every principal  $i$ , within the set of allocations that are feasible given  $\mathbf{B}'_{-i}$ , no allocation can provide the agent and principal  $i$  a joint payoff greater than  $\pi'_0 + \pi'_i$ . Note first of all that if the coalition formed

by the agent and principal  $i$  picks up  $\tilde{b}'_1$  or  $\tilde{b}'_2$ , by conditions (A1) and (A2) and the concavity of  $U(X)$  such a coalition would choose the null contracts of all the remaining principals, getting  $V_i(\tilde{x}') - [U(\tilde{x}') - \pi'_0]$ , which by (A1) cannot be greater than  $\pi'_0 + \pi'_i$ . Therefore, let us suppose that the coalition formed by the agent and principal  $i$  does not pick up  $\tilde{b}'_1$  nor  $\tilde{b}'_2$ . The only way in which this coalition can earn more than  $\pi'_0 + \pi'_i$  given  $\mathbf{B}'_{-i}$  is by selecting the null contract of the principals in some set  $\Omega \subset N - \{i\}$ . Thus, suppose to the contrary that for some  $\Omega \subset N - \{i\}$ ,

$$V_i\left(X^*_{-\Omega-\{i\}}\right) - \sum_{j \in N-\Omega-\{i\}} P_j^{'*} > \pi'_0 + \pi'_i$$

i.e., that the coalition formed by the agent and principal  $i$  can profitably crowd out a group of principals  $\Omega$  selecting the efficient contracts of the others. Using the fact that  $\pi'_0 + \pi'_i = V_i(X^*_{-i}) - \sum_{j \neq i} P_j^{'*}$ , this inequality can be rewritten as

$$\sum_{s \in \Omega} P_s^{'*} > V_i(X^*_{-i}) - V_i\left(X^*_{-\Omega-\{i\}}\right) \quad (\text{A3})$$

Next we show that if this inequality holds, the coalition formed by the agent and principal  $i$  can earn more than  $\pi'_0 + \pi'_i$  by excluding only one principal, say  $k \in \Omega$ . If  $\Omega$  is a singleton, the proof is complete, so let us suppose that  $|\Omega| \geq 2$  and partition  $\Omega$  into two non-empty sets  $\Omega_1$  and  $\Omega_2$ , such that  $\Omega_1 \cap \Omega_2 = \emptyset$  and  $\Omega_1 \cup \Omega_2 = \Omega$ . We need to show that the coalition formed by the agent and principal  $i$  can profitably crowd out only the principals in  $\Omega_1$  or only those in  $\Omega_2$ , selecting the efficient contracts of the others. Suppose this is not the case; then we have

$$\sum_{s \in \Omega_1} P_s^{'*} \leq V_i(X^*_{-i}) - V_i\left(X^*_{-\Omega_1-\{i\}}\right) \quad (\text{A4})$$

and

$$\sum_{s \in \Omega_2} P_s^{'*} \leq V_i(X^*_{-i}) - V_i\left(X^*_{-\Omega_2-\{i\}}\right). \quad (\text{A5})$$

Adding (A4) and (A5) and using (A3) we get

$$V_i(X^*_{-i}) + V_i\left(X^*_{-\Omega-\{i\}}\right) > V_i\left(X^*_{-\Omega_1-\{i\}}\right) + V_i\left(X^*_{-\Omega_2-\{i\}}\right) \quad (\text{A6})$$

Since  $V_i(\cdot)$  is strictly concave, for any  $a > b, c > d$  such that  $a + d = b + c$  we must have  $V(a) + V(d) < V(b) + V(c)$ , but this contradicts (A6), since  $X_{-i}^* + X_{-\Omega-\{i\}}^* = X_{-\Omega_1-\{i\}}^* + X_{-\Omega_2-\{i\}}^*$ . This contradiction proves that the coalition formed by the agent and principal  $i$  can earn more than  $\pi'_0 + \pi'_i$  by excluding only the principals in  $\Omega_1$  or only those in  $\Omega_2$  while selecting the efficient contracts of all the remaining principals. Since  $N$  is finite, by repeated application of this argument we conclude that the coalition formed by the agent and principal  $i$  can earn more than  $\pi'_0 + \pi'_i$  by excluding only one principal, say  $k$ , and selecting the efficient contracts of the remaining  $N - 2$  principals, i.e.,  $P_k'^* > V_i(X_{-i}^*) - V_i(X_{-\{i,k\}}^*)$ .

The next step in the proof is to show that if this is true of principal  $i$ , it must also be true of any principal  $s \leq i$ . In other words, we must show that inequality

$$P_k'^* > V_i(X_{-i}^*) - V_i(X_{-\{i,k\}}^*)$$

implies

$$P_k'^* > V_s(X_{-s}^*) - V_s(X_{-\{s,k\}}^*),$$

for all  $s \leq i$ . A sufficient condition for this is

$$V_s(X_{-\{s,k\}}^*) + C_s(x_s^*) \geq V_i(X_{-\{i,k\}}^*) + C_i(x_i^*).$$

This inequality can be rewritten as

$$\begin{aligned} & \max_{x_s} \left\{ U(X_{-k}^*) + \int_0^{x_s - x_s^*} [U'(X_{-k}^* + \xi) - C'_s(x_s^* + \xi)] d\xi \right\} \\ & \geq \max_{x_i} \left\{ U(X_{-k}^*) + \int_0^{x_i - x_i^*} [U'(X_{-k}^* + \xi) - C'_i(x_i^* + \xi)] d\xi \right\}, \end{aligned}$$

which, by Assumption 5, must hold. This implies that if the coalition formed by the agent and principal  $i$  can profitably crowd out a group of principals  $\Omega$  selecting the efficient contracts of the others, we must have  $P_k'^* > V_{v_k}(X_{-v_k}^*) - V_{v_k}(X_{-\{v_k,k\}}^*) = \bar{P}_k^*$  for some principal  $k$ , which is impossible since  $\pi'_i \leq \bar{\pi}_i$  for all  $i \in N$ . Therefore,

the coalition formed by the agent and principal  $i$  cannot get more than  $\pi'_0 + \pi'_i$ , thus bilateral efficiency holds. ■

**Proof of Proposition 2.** When  $N = 2$ , Pareto dominance coincides with coalition proofness, so the first part of the proposition follows from the result that coalition-proof equilibria are truthful (see Bernheim and Whinston [4]). Turning to the general case, note that since  $V_i(X_{-i}^*) = S^* + \sum_{j \neq i} C_j(x_j^*)$ ,  $\bar{\pi}_i$  can be rewritten as

$$\begin{aligned} \bar{\pi}_i &= S^* - \left[ V_{v_i}(X_{-\{i, v_i\}}^*) - \sum_{j \neq i, v_i} C_j(x_j^*) \right] \\ &\geq S^* - \max_{\substack{x_j \\ j \neq i}} \left[ U(X_{-i}) - \sum_{j \neq i} C_j(x_j) \right] \\ &= S^* - S_{-i}^* \end{aligned}$$

where the inequality is strict when  $N > 2$ . This means that the maximum payoff a principal can get exceeds his contribution to maximized social surplus. Since in any truthful equilibrium  $\pi_i = S^* - S_{-i}^*$ , the minimum rent equilibrium is not truthful when  $N > 2$ . ■

**Proof of Proposition 3.** Sufficiency has already been proved in the course of the proof of Proposition 1, so it only remains to prove necessity. Recall that individual excludability means that the agent must earn her equilibrium payoff if she excludes any one principal  $i$ . However, if she excluded principal  $i$  while continuing to accept the efficient contracts of the remaining principals, by inequality  $P_i^* < U(X^*) - U(X_{-i}^*)$  she would earn less than her equilibrium rent. This implies that there must be an additional contract of some principal  $j \neq i$  that the agent would accept in place of principal  $j$ 's efficient contract if the agent refused to trade with  $i$ . Let  $k$  be the principal who submits such an additional contract. By individual excludability, the agent must continue to earn her equilibrium rent if she does not trade with principal  $k$ . Therefore, by the same argument as above, at least one other principal must also submit a contract that will not be accepted. ■

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