Export Restraints in a Model of Trade with Capital Accumulation*

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Abstract

This paper examines the impact of voluntary export restraints (VERs) in an international duopoly modelled as a differential game. With a Ramsey capital accumulation dynamics, the game admits multiple steady states, and a VER cannot be 'voluntarily' employed by the foreign firm in case of Cournot behaviour in demand substitutes. Hence, the dynamic framework confirms the results of the VERs literature with static interaction in output levels. In the case of price behaviour, the adoption of an export restraint may increase the profits of both firms if products are substitutes and the steady state is "market-driven". However, contrary to the acquired wisdom based upon the static approach, the dynamic analysis also admits an equilibrium outcome, identified by the Ramsey golden rule, where the incentive to adopt a VER is ruled out, irrespective of whether firms are quantity- or price-setters.

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1 Introduction

Voluntary export restraints (VERs) are often considered as coordinating or quasi-collusive devices (see for example Harris 1985, Krishna 1989 and more recently Berry, Levinsohn and Pakes, 1999). Indeed, most of the existing theoretical literature justifies this view only insofar as firms are price setters. Harris (1985) first analyzed VERs in a static duopoly model. He showed that when firms compete *à la* Bertrand on differentiated products, then a VER at the free trade level of imports increases profits of both domestic and foreign firms. This result is consistent with what has been found by Mai and Hwang (1988) in a more general analysis based on a conjectural variation static approach. However, their paper shows that with Cournot competition VER is ineffective, and in more collusive settings (i.e., those with positive quantity conjectures) it hurts the foreign firm and fails to be a 'voluntary' strategic trade policy. In general, the consequences of quantity restrictions are known to depend on whether imports are strategic and/or demand substitutes or complements for domestic products. This, in turn, depends on whether market interaction takes place in outputs or prices. When firms set quantities (prices) and goods are demand substitutes (complements), output restrictions impede the ability of the foreign firm to compete in the domestic market, thereby acting to facilitate collusion and raise prices and profits (see Krishna, 1989).[1] This view is reinforced by Suzumura and Ishikawa (1997), who explore the implications of a voluntary export restraint agreement on profits and welfare in a duopoly model with product differentiation and conjectural variations. They assume that the imposition of a VER makes the domestic firm into a Stackelberg leader, and show that a VER introduced at the free-trade equilibrium level of export is welfare-improving for the importing country if and only if the foreign exporter is forced to comply with the restraint involuntarily.

The main limit of the existing literature on strategic trade policy is its essentially static nature, as Brander (1995) well pointed out. Nevertheless, long-term interactions characterizing international oligopolistic markets are at odds with the one-shot static games generally employed and one may well expect that introducing real time in these models substantially affects firms' behavior.

This paper analyzes VERs in a dynamic setting where oligopolistic firms interact participating in a differential game. Introducing real time in the analysis we are in a position to explicitly account for the fact that firms strategically interact along several dimensions, not only prices or quantities but also capacities, investment (or capital accumulation) and growth, following the literature initiated by Spence (1979). Hence, an important novelty in our paper is that we explicitly model firms' dynamic capital accumulation which is certainly one of the most important strategic decisions firms are confronted with in a dynamic context. Indeed, in

[1] See also Pomfret (1989) for a detailed survey on VERs.

many industries (such as automotive, electronics and aircraft) supply cannot be expanded at will and some form of accumulation of capacity or capital investment is first required. We then deal with the effects of VERs over profits and equilibrium quantities using the Ramsey (1928) model of capital accumulation (i.e., the well known "corn-corn" growth model) and considering both Cournot and Bertrand competition.

When dealing with differential games, several strategies and solution concepts may be applied.[2] The existing literature mainly concentrates on two kind of strategies: the open-loop and the closed-loop ones. In the former case, firms precommit to an investment path over the whole time horizon of the game, and the relevant equilibrium concept is the open-loop Nash equilibrium. In the latter, firms do not precommit on investment paths and their strategies at any instant depend upon all the preceding history of the game, as described by the evolution of state variables and their influence upon the evolution of control variables. In this situation, the information set used by firms in fixing their strategies at any given time is often simplified to be only the current value of the capital stocks at that time. The relevant equilibrium concept is in this (sub-)case the closed-loop no-memory (or Markov Perfect) Nash equilibrium. In order to further simplify the analysis, most of the literature adopts a refinement of the closed-loop Nash equilibrium, which is known as the feedback Nash equilibrium.[3] In the present paper we will not restrict to this refinement and deal with the open-loop and closed loop no-memory solutions. We study a differential game with the interesting property that the open-loop Nash equilibrium is independent of initial conditions and thus represents a feedback or Markov Perfect equilibrium. Building upon this technical property, we will study whether a VER leads to more or less cooperative equilibria and then higher or smaller profits for firms, as compared to the free trade equilibrium.

We show that when firms supply substitute goods and compete on quantities any VER benefits the domestic firm but it hurts the foreign firm which imposes it. It follows that a VER cannot be observed in equilibrium. Although this confirms the analysis in the mentioned static literature, this result is here obtained as one outcome out of the richer set of equilibria that the dynamic setting provides, also including a Ramsey equilibrium driven by intertemporal allocation of capital. Under Bertrand competition, the VER may increase the profits of both firms, as we are used to see in static models of price competition, and exactly for the same reason, namely that provided products are substitutes, a quantity commitment on the part of the foreign firm yields a quasi-Stackelberg price equilibrium, with the foreign firm in the

[2] See Kamien and Schwartz (1981); Başar and Olsder (1982); Mehlmann (1988); Dockner *et al.* (2000).

[3] For a clear exposition of the difference among these equilibrium solutions see Başar and Olsder (1982, pp. 318-327, and chapter 6, in particular Proposition 6.1).

follower's position.[4] However, contrary to what a static setting delivers, we also show that with substitute products and price-setting firms there are equilibria that are specific to the dynamic framework and where VER cannot arise.

These results allow us to draw the following implication, which extends to a dynamic setting a conclusion reached by most of the aforementioned static literature on this topic. As VERs are usually observed in several markets, and their adoption is not justified when firms set output levels, then the viability of VERs as coordinating or quasi-collusive instruments is confined to those cases where firms supply substitute (complement) goods and compete in prices (quantities).

To our knowledge, the only existing contribution on VERs in a differential-game is Dockner and Haug (1991), who analyze VERs in a dynamic oligopoly game with Cournot competition and sticky prices.[5] There are several and fundamental differences between this contribution and our analysis. First, in Dockner and Haug (1991) the dynamics uniquely rests upon price kinetics as a consequence of price stickiness. Hence, they do not explicitly consider dynamic capital accumulation and the relationship between firms' capacity to serve a market and their capital stock. On the contrary, it is clear that this is the main theme of our analysis and, as discussed above, it turns out to be crucial for the results. Second, restricting to a speed of price adjustment which goes to infinity, they show that VER is voluntary as it increases the profits of both domestic and foreign firms. However, this result stems from the fact that, since the price is the state variable, interaction among output levels takes place only through co-state equations, as each firm's first order condition w.r.t. own quantity is independent of the other firms'. In contrast with these results, our analysis shows that in a dynamic context, the capital accumulation process adopted by firms plays a crucial role for the equivalence of trade policies. Finally, Dockner and Haug (1991) restrict the analysis to the Markov Perfect Nash equilibrium thus emphasizing a "closed-loop motive for VER". On the contrary, as discussed above, in our model there is no difference between open-loop and closed-loop solution concepts so that our results are not a consequence of firms' commitment ability and rather follow from having made firms' investment process explicit.

Some of the few exceptions dealing with dynamic differential games in international trade but not explicitly related to VERs include Cheng (1987) who studies the interplay between innovation policies and trade policies in international oligopolistic markets; Driskill and McCafferty (1996) studying strategic trade policy where dynamics is introduced assuming that demand depends on past consumption (see also Driskill and McCafferty, 1989); Dockner and

[4] We also show that, if goods are complements, exactly the opposite considerations apply.

[5] This model is due to Simaan and Takayama (1978). It has been extended by Fershtman and Kamien (1987, 1990) and Tsutsui and Mino (1990).

Haug (1990) employ a model similar to the one in their paper discussed above and study the equivalence between a tariffs and quotas which we also analyze in Calzolari and Lambertini (2006) studying firms' capital accumulation; Kemp, Shimomura and Van Long (2001) analyze a tariff-setting game between governments where national welfare depends also on the accumulated stocks of goods; Finally, Miravete (2003) studies time consistency of trade policies with learning-by-doing effects.[6]

The paper is organized as follows. The general setting is laid out in section 2. Section 3 is devoted to the analysis of Cournot competition, whilst Bertrand behavior is discussed in section 4. We then have a general discussion in Section 5 where we contrast our results with those that may arise in a static setting. Concluding remarks are in section 6.

2 The setup

As in the previous literature on this topic, we consider a duopoly market supplied by a domestic producer (firm D) and a foreign rival (firm F).

The model is built in continuous time. The market exists over $t \in [0, \infty)$. Let $q_i(t)$ define the quantity sold by firm i, $i = D, F$, at time t. The inverse demand function of firm i at time t is:

$$p_i(t) = a - q_i(t) - sq_j(t), \tag{1}$$

with $i = D, F$, $i \neq j$. When $s \in (0, 1]$, products are substitutes, while they are complements when $s \in [-1, 0)$, and independent with $s = 0$. The substitutability parameter $s \in [-1, 1]$ ensures that quantities are never negative.[7]

In order to produce, firms must accumulate capacity or physical capital $k_i(t)$ over time, with initial conditions $k_i(t) = k_{0i}$ for $t = 0$. Following Ramsey (1928), we assume that the capital accumulation process takes place according to the following dynamic equation:

$$\frac{dk_i(t)}{dt} = f(k_i(t)) - q_i(t) - \delta k_i(t), \tag{2}$$

where $f(k_i(t))$ denotes the output produced by firm i at time t, with $\partial f(k_i(t))/\partial k_i(t) > 0$ and $\partial^2 f(k_i(t))/\partial k_i^2(t) < 0$. That is, capital accumulates (or decumulates) as a result of intertemporal relocation of unsold output $f(k_i(t)) - q_i(t)$. This can be interpreted in two ways. The

[6] Although not dealing with a differential game the paper by Herguera, Kujal and Petrakis (2000) is interesting because it studies the effects of VERs on the long run choice of quality in a vertical product differentiation model with Cournot competition.

[7] This formulation of market demand functions with product differentiation dates back to Bowley (1924) and is commonly used in the industrial organization literature since Dixit (1979) and Singh and Vives (1984).

first consists in viewing this setup as a corn-corn model, where unsold output is reintroduced in the production process. The second consists in thinking of a two-sector economy where there exists an industry producing the capital input which can be traded against the final good at a price equal to one (see Cellini and Lambertini, 1998, 2006). The control variable is $q_i(t)$, while the state variable is $k_i(t)$.

Concerning instantaneous variable costs, we assume that unit production cost is constant and equal across firms. For the sake of simplicity, and without further loss of generality, we also assume it to be nil. Accordingly, firm i's instantaneous profits are $\pi_i(t) = p_i(t)\, q_i(t)$, and output $q_i(t)$ must be chosen so as to maximize the discounted flow of profits:

$$J_i(t) = \int_0^\infty \pi_i(t)\, e^{-\rho t} dt \tag{3}$$

under the dynamic constraint (2). The discount rate $\rho \geq 0$ is constant and equal across firms.

3 Cournot competition

Here, we examine quantity competition. Under the capital accumulation rule (2), the Hamiltonian function of firm i is the following:

$$\begin{aligned} \mathcal{H}_i(t) &= e^{-\rho t} \{ q_i(t) [a - q_i(t) - s q_j(t)] + \\ &\quad + \lambda_{ii}(t) [f(k_i(t)) - q_i(t) - \delta k_i(t)] + \\ &\quad + \lambda_{ij}(t) [f(k_j(t)) - q_j(t) - \delta k_j(t)] \}, \end{aligned} \tag{4}$$

where $\lambda_{ij}(t) = \mu_{ij}(t) e^{\rho t}$, and $\mu_{ij}(t)$ is the co-state variable associated by firm i to state $k_j(t)$; $k_i(0) \equiv k_{i0}$ defines the initial condition for firm i. However, since firm i's instantaneous payoff is independent of firm j's state, and the dynamics of firm i's state is independent of the state and control of the rival, w.l.o.g. we may pose $\lambda_{ij}(t) = 0$ and reformulate firm i's Hamiltonian as follows:

$$\mathcal{H}_i(t) = e^{-\rho t} \{ q_i(t) [a - q_i(t) - s q_j(t)] + \lambda_{ii}(t) [f(k_i(t)) - q_i(t) - \delta k_i(t)] \}. \tag{5}$$

On the basis of (5), we can prove:

Lemma 1 *The open-loop Nash equilibrium of the Cournot-Ramsey game is subgame perfect.*

Proof. See the appendix. ■

The game we are investigating belongs to the class of the so-called *state-redundant games*, as defined in Mehlmann (1988, Ch. 4). Such games are characterized by the property that, after replacement of co-state variables by the solutions of the corresponding co-state equations, the first order conditions on controls are independent of the state variables, and in particular their initial values. As a result, the open-loop equilibrium produced by these games is Markov-perfect, where Markov-perfect is an alternative but equivalent way of labelling subgame perfection and strong time consistency. In identifying Markov-perfect open-loop solutions of differential games, the existing literature has largely focussed upon the so-called *linear state* games (see Dockner *et al.*, 2000, section 7.2), where the Hamiltonian function is linear in the state variables. This, however, is not the only class of games ensuring state redundancy. In fact, the present model implies state redundancy by virtue of the additive separability of the Hamiltonian function w.r.t. states and controls. Moreover, (i) state equations are decoupled, as $dk_i(t)/dt$ is independent of $k_j(t)$, and (ii) states appear in state equations but not in the profit functions. These properties imply that

$$\frac{\partial q_i(t)}{\partial k_j(t)} = \frac{\partial q_i(t)}{\partial k_i(t)} = 0. \tag{6}$$

From the first order condition on $q_i(t)$, we obtain the best reply function of firm i :

$$q_i^{br}(t) = \frac{a - sq_j(t) - \lambda_{ii}(t)}{2}. \tag{7}$$

The co-state equation of firm i writes as follows:

$$-\frac{\partial \mathcal{H}_i(t)}{\partial k_i(t)} = \frac{\partial \mu_{ii}(t)}{\partial t} \Rightarrow \frac{\partial \lambda_{ii}(t)}{\partial t} = [\rho + \delta - f'(k_i(t))]\,\lambda_{ii}(t). \tag{8}$$

The best reply function (7) can be differentiated w.r.t. time to yield:

$$\frac{dq_i(t)}{dt} = -\frac{s \cdot dq_j(t)/dt + d\lambda_{ii}(t)/dt}{2}. \tag{9}$$

Then, using

$$\lambda_{ii}(t) = a - 2q_i(t) - sq_j(t) \tag{10}$$

from (7), together with (8), we obtain:

$$\frac{dq_i(t)}{dt} = -\frac{1}{2}\left\{s \cdot dq_j(t)/dt + [a - 2q_i(t) - sq_j(t)]\,[\rho + \delta - f'(k_i(t))]\right\} \tag{11}$$

which, invoking symmetry, can be rearranged to yield:

$$\frac{dq(t)}{dt} = \frac{1}{2+s}\,[a - (2+s)\,q(t)]\,[f'(k(t)) - \rho - \delta]$$

Imposing $dq(t)/dt = 0$ and solving, we obtain the following set of solutions:

$$f'(k) = \rho + \delta \tag{12}$$

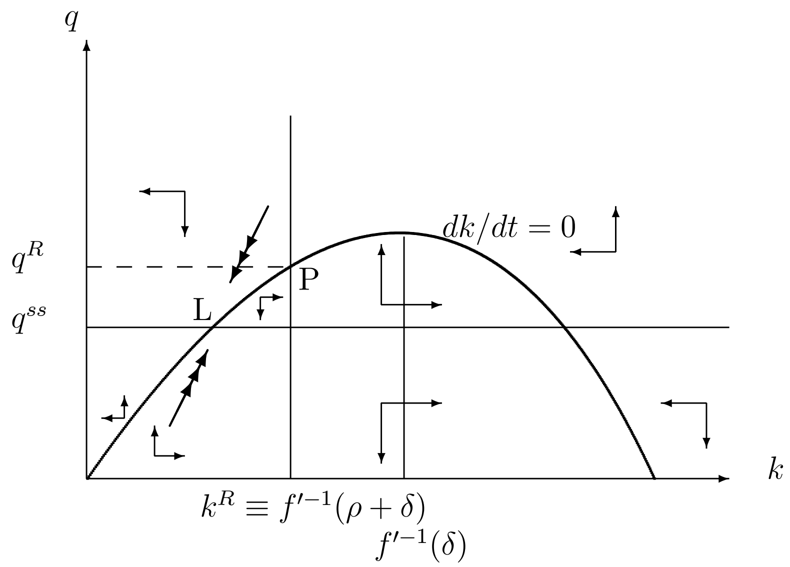
or

$$q = q^{ss} \equiv \frac{a}{2+s}\ , \tag{13}$$

where q^{ss} is the "demand-driven" equilibrium (observationally equivalent to the static Cournot equilibrium), while $f'(k) = \rho + \delta$ is the Ramsey "golden rule" equilibrium dictated by intertemporal capital accumulation alone. This entails a capital endowment equal to k^R and quantity $q^R = f\left(k^R\right) - \delta k^R$, with R standing for Ramsey.

The phase diagram illustrating the dynamics of the system is in figure 1, where the locus $dk/dt = 0$ as well as the behavior of k, depicted by horizontal arrows, derive from (2). Steady states are identified by the intersections between loci.

Figure 1: The Cournot-Ramsey phase diagram



It is worth noting that the situation illustrated in figure 1 is only one out of several possible configurations, due to the fact that the position of the vertical line $f'(k) = \rho + \delta$ is independent of demand parameters, while the horizontal loci q^{ss} shifts upwards (downwards) as a increases (decreases). Here, we confine to the case where the horizontal locus q^{ss} intersects the locus $dk/dt = 0$ in the region where the latter is increasing in k, to the left of the Ramsey equilibrium $f'(k) = \rho + \delta$. Steady state points are identified as L and P. Intersections to the right of $k = f^{'-1}(\delta)$ are clearly inefficient and therefore can be disregarded. Stability analysis reveals that $\{L\ ,\ P\}$ are, alternatively, a saddle point and an unstable focus. In particular, in the case depicted in figure 1, L is a saddle point (with the associated saddle path indicated in figure) while P is an unstable focus. If, on the contrary, parameter a is large enough to drive L to the right of P, then P becomes a saddle point while L becomes an unstable focus.[8]

The foregoing discussion can be summarized as follows:

Lemma 2 *Under free trade, for all $\{a, s\}$ such that $a/(2+s) \leq q^R$, the system reaches a steady state at $q^{ss} = a/(2+s)$, which is a saddle.*

In correspondence of either $q^{ss} = a/(2+s)$ or $q^R = f(k^R) - \delta k^R$, firms' steady state profits under free trade are, respectively:

$$\pi^{ss} = \frac{a^2}{(2+s)^2} \,;\, \pi^R = \left[a - (1+s)q^R\right] q^R. \tag{14}$$

Now we shall take into consideration the alternative setting where firm F adopts an export restraint $\overline{q}_F$ (which, for instance but not necessarily, can be fixed at the free trade level).[9] The issue can be quickly dealt with by observing how the best reply of firm D modifies in the presence of an export restraint. Suppose the Cournot equilibrium prevails (i.e., it is a saddle point) under free trade. Then, from (7), we can write:

$$q_D^{br}(t) = \frac{a - s\overline{q}_F - \lambda_{DD}(t)}{2}\,, \tag{15}$$

where $\overline{q}_F \leq a/(2+s)$. It is immediate to verify that

$$\frac{dq_D(t)}{dt} = -\frac{1}{2}\frac{d\lambda_{DD}(t)}{dt} = -\frac{1}{2}\left[\rho + \delta - f'(k_D(t))\right]\lambda_{DD}(t)\,, \tag{16}$$

where, from (15), $\lambda_{DD}(t) = a - 2q_D(t) - s\overline{q}_F$. This entails that the optimal quantity offered by the domestic firm in steady state, as a reaction to a VER, may be either the Cournot best reply to the VER, or the Ramsey output, depending on which one identifies a saddle point. While in the free trade setting the imposition of symmetry entails that both firms converge either to the demand-driven or to the Ramsey equilibrium, here the adoption of a VER amounts to abandoning symmetry, with the domestic firm being in steady state at either $f'(k^R) = \rho + \delta$ or $q_D^{ss} \equiv (a - s\overline{q}_F)/2$. Define the optimal domestic choice as q_D^{VER}. Then, $q_D^{VER} = q_D^{ss}$ if $(a - s\overline{q}_F)/2 < q^R$, and $q_D^{VER} = q^R$ otherwise.[10] An interesting limit case may arise, where $\overline{q}_F$ is

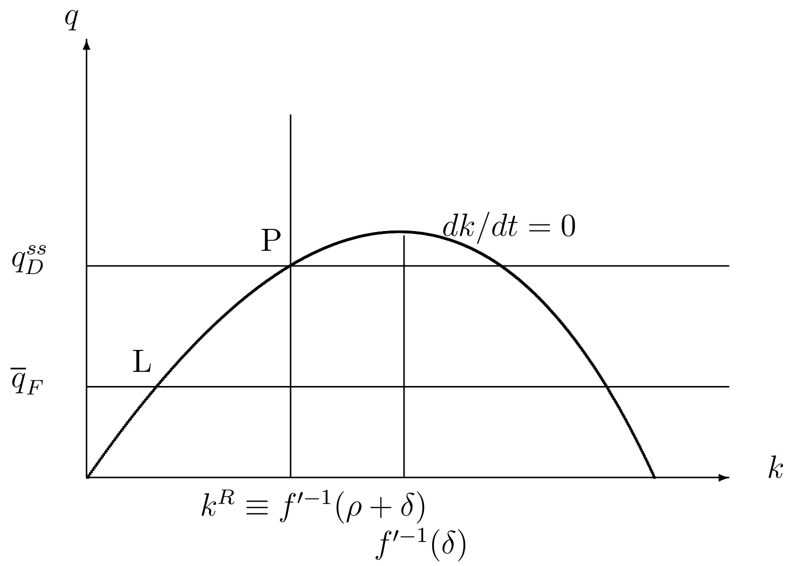
[8]The stability analysis is omitted for the sake of brevity. See Cellini and Lambertini (1998) for details.

[9]Since we focus upon the steady state analysis, we assume that $q_F = \overline{q}_F$. In general, during the adjustment process towards the steady state, it can be the case that $q_F < \overline{q}_F$, but excess capacity cannot be observed in the subgame perfect steady state equilibrium.

[10]In fact, recall that the smallest output level between q_D^{ss} and q^Ralways defines a saddle point, while the larger always defines an unstable focus.

sufficiently lower than $a/(2+s)$ and consequently q_D becomes sufficiently large to coincide with the Ramsey equilibrium. This situation is illustrated in figure 2 (the horizontal and vertical arrows describing the dynamics of $\{k, q\}$ are omitted).

Figure 2: Steady state equilibrium under a VER



The foregoing discussion proves the following result, which holds if the (static) Cournot equilibrium obtains with free trade:

Lemma 3 *Suppose $s \in (0,1]$ and the demand-driven equilibrium q^{ss} prevails under free trade (i.e. $q^R > q^{ss}$).*

Irrespective of whether the VER $\overline{q}_F \leq q^{ss}$ leads firms to the demand-driven equilibrium or to the Ramsey equilibrium, we have

$$\pi_D(\overline{q}_F) \geq \pi^{ss} \geq \pi_F(\overline{q}_F).$$

where the above chain of inequalities is strict if and only if $\overline{q}_F < q^{ss}$.

Proof. See the appendix. ■

Yet, under free trade, the Ramsey equilibrium may be reached, whereby firms choose capacity and sales in relation to intertemporal parameters only. Here, the following holds:

Lemma 4 *Suppose $s \in (0,1]$ and the Ramsey equilibrium q^R prevails at the free trade equilibrium.*

Two situations may arise, where the export restraint is either $\overline{q}_F = q^R$ or any $\overline{q}_F < q^R$. In the first case, $q_D = \overline{q}_F = q^R$. Therefore, the profits associated with the VER are observationally the same as under free trade. In the second case, firm D's best reply is either $q_D^{ss} = (a - s\overline{q}_F)/2$ or $q_D = q^R$. Therefore, in general:

$$\pi_D(\overline{q}_F) \geq \pi^R \geq \pi_F(\overline{q}_F).$$

Proof. See the appendix. ■

Lemmata 2-4 produce the following result:

Proposition 1 *A voluntary export restraint cannot be observed at equilibrium in the Cournot case, for all $s \in (0,1]$.*

Of course, the VER becomes desirable if $s \in [-1, 0)$ and $\overline{q}_F \neq q^R$. In such a case, the Cournot game with complement goods exhibits increasing best replies, i.e., $\partial q_i^{br}/\partial q_j > 0$, and therefore the VER is a profitable restriction for both firms. Otherwise, if $\overline{q}_F = q^R$ because the free trade equilibrium is the Ramsey golden rule and firm F sticks to it, the best reply to the VER is $q_D = \overline{q}_F = q^R$ and the VER is ineffective. Summing up, notwithstanding the fact that the dynamic game yields a richer set of equilibria as compared to the static game, the Cournot-Ramsey game leads to the same qualitative conclusions we are accustomed with in static settings.

4 Bertrand competition

With price competition, from (1) the demand function firm i faces at time t is:

$$q_i(t) = \frac{a}{1+s} - \frac{p_i(t)}{1-s^2} + \frac{sp_j(t)}{1-s^2} \quad (17)$$

where $p_i(t)$ and $p_j(t)$ are respectively the price set by firms i and j, respectively.

The Hamiltonian of firm i is the following:

$$\begin{aligned} \mathcal{H}_i(t) &= e^{-\rho t}\left\{\left[\frac{a}{1+s} - \frac{p_i(t)}{1-s^2} + \frac{sp_j(t)}{1-s^2}\right]p_i(t) + \right. \\ &\lambda_{ii}(t)\left[f(k_i(t)) - \frac{a}{1+s} + \frac{p_i(t)}{1-s^2} - \frac{sp_j(t)}{1-s^2} - \delta k_i(t)\right] + \\ &\left.\lambda_{ij}(t)\left[f(k_j(t)) - \frac{a}{1+s} + \frac{p_j(t)}{1-s^2} - \frac{sp_i(t)}{1-s^2} - \delta k_j(t)\right]\right\}, \end{aligned} \quad (18)$$

where $\lambda_{ij}(t) = \mu_{ij}(t)e^{\rho t}$, and $\mu_{ij}(t)$ is the co-state variable associated by firm i to state $k_j(t)$.

Although in this case firm j's control enters explicitly the accumulation dynamics of firm i, we can prove:

Lemma 5 *The co-state variable $\lambda_{ij}(t)$ is redundant, that is, w.l.o.g. we can set $\lambda_{ij}(t) = 0$ at all $t \in [0, \infty)$.*

Proof. See the appendix. ■

On these grounds, we can rewrite the Hamiltonian of firm i as follows:

$$\begin{aligned} \mathcal{H}_i(t) &= e^{-\rho t}\left\{\left[\frac{a}{1+s} - \frac{p_i(t)}{1-s^2} + \frac{sp_j(t)}{1-s^2}\right]p_i(t) + \right. \\ & \left. \lambda_{ii}(t)\left[f(k_i(t)) - \frac{a}{1+s} + \frac{p_i(t)}{1-s^2} - \frac{sp_j(t)}{1-s^2} - \delta k_i(t)\right]\right\} \end{aligned} \tag{19}$$

and then prove:

Lemma 6 *The open-loop Nash equilibrium of the Bertrand-Ramsey game is subgame perfect.*

Proof. See the appendix. ■

Moving on to the solution of the open-loop problem based on (19), from the first order condition on $p_i(t)$ (see the Appendix) we obtain the best reply function of firm i :

$$p_i^{br}(t) = \frac{a(1-s) + sp_j(t) + \lambda_{ii}(t)}{2}. \tag{20}$$

Function (20) can be differentiated w.r.t. time to yield:

$$\frac{dp_i(t)}{dt} = \frac{1}{2}\left[\frac{dp_j(t)}{dt}s + \frac{d\lambda_{ii}(t)}{dt}\right]. \tag{21}$$

Then, using

$$\lambda_{ii}(t) = 2p_i(t) - a(1-s) - sp_j(t), \tag{22}$$

from (20) and the co-state equation of firm i which writes as in (8), we obtain:

$$\frac{dp_i(t)}{dt} = \frac{1}{2}\left\{\frac{dp_j(t)}{dt}s + [2p_i(t) - a(1-s) - sp_j(t)]\left[\rho + \delta - f'(k_i(t))\right]\right\} \tag{23}$$

Invoking symmetry, this can be rearranged to yield:

$$\frac{dp(t)}{dt} = \frac{1}{2-s}\left[(2-s)\,p(t) - a(1-s)\right]\left[\rho + \delta - f'(k(t))\right]$$

Imposing $dp(t)/dt = 0$ and solving, we obtain the Ramsey equilibrium:

$$f'(k^R) = \rho + \delta \tag{24}$$

with quantity $q^R = f(k^R) - \delta k^R$ and, substituting $p = a\,(1-s)\,/\,(2-s)$ into (1), the demand-driven solution:

$$q^{ss} = \frac{a}{(1+s)\,(2-s)}. \tag{25}$$

The phase diagram illustrating the dynamics of the system is as in figure 1, and Lemma 2 applies qualitatively unmodified, although of course Bertrand behavior entails a larger output and a lower price in steady state, as compared to Cournot, for all s.

With free trade at q^{ss}, the instantaneous profit each firm obtains then is

$$\pi^{ss} = \frac{a^2\,(1-s)}{(2-s)^2\,(1+s)}. \tag{26}$$

Now let us turn to the case where firm F adopts an export restraint $\overline{q}_F$. Suppose the demand-driven equilibrium prevails under free trade. Then, replace $q_F = \overline{q}_F$ into the inverse demand for firm F and substitute back into the Hamiltonian of firm D. The best reply function of firm D now becomes

$$p_D^{br}(t) = \frac{a - s\overline{q}_F + \lambda_{DD}(t)}{2}. \tag{27}$$

Differentiating this best reply w.r.t. time yields:

$$\frac{dp_D(t)}{dt} = \frac{1}{2}\frac{d\lambda_{DD}(t)}{dt}. \tag{28}$$

Then, using

$$\lambda_{DD}(t) = 2p_D(t) - a + s\overline{q}_F \tag{29}$$

and (8), we obtain:

$$\frac{dp_D(t)}{dt} = \frac{1}{2}\left[2p_D(t) - a + s\overline{q}_F\right]\left[\rho + \delta - f'(k_D(t))\right] \tag{30}$$

Imposing $dp_D(t)/dt = 0$ and solving, we obtain the Ramsey and the demand-driven equilibria:

$$\begin{array}{c} f'(k^R) = \rho + \delta \\ q_D^{VER} = \dfrac{a - s\overline{q}_F}{2}. \end{array} \tag{31}$$

Note that q_D^{VER} coincides with the expression obtained in the Cournot model. This is due to the fact that, once firm F sets her quantity restriction, firm D becomes a monopolist on the residual demand function. Therefore, D must be indifferent between maximizing profits w.r.t.

a price or a quantity. What determines the desirability of the VER is the slope of reaction functions and the sign of parameter s.

When $f'(k) = \rho + \delta$ holds at equilibrium, then firm D chooses $q^R = f(k^R) - \delta k^R$. On this basis, we can prove the following:

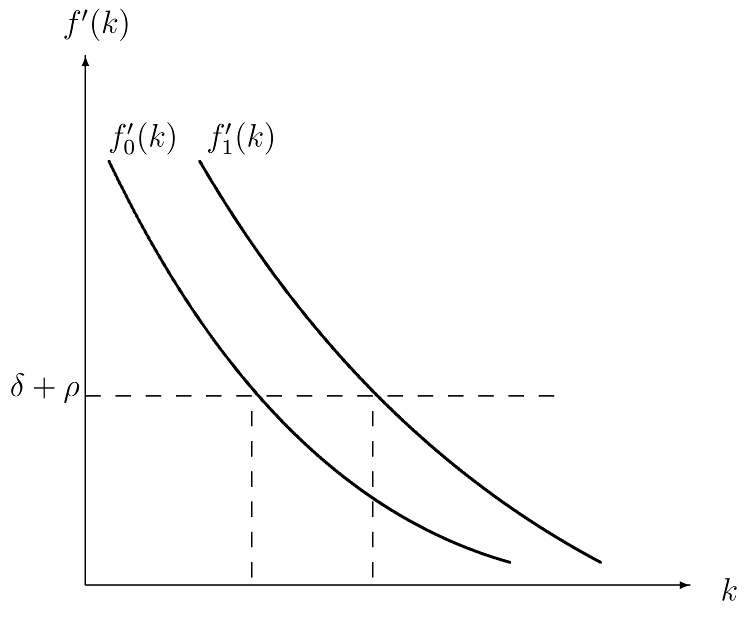
Proposition 2 *Under Bertrand competition,*

1. *there exists an export restraint* $\overline{q}_F \leq q^{ss}$, *that benefits both firms as long as the steady state is driven by demand conditions only,* $q^{ss} \leq q^R$ *and* $s \in (0,1]$. *Firm* F *benefits more than firm* D *for any* s.

2. *If instead* $q^{ss} > q^R$ *or* $s \in [-1,0)$, *then the steady state profits of firm* F *under any VER are lower than under free trade, and therefore the VER cannot be part of a subgame perfect equilibrium.*

Proof. See the Appendix ■

Figure 3 : The effect of a change in the productivity of capital



Proposition 2 shows that when firms compete in prices, then VERs may indeed be *voluntary* and serve as coordinating or quasi-collusive instruments. *Ceteris paribus*, this depends on the slope of technology. To see this, consider as given the set of demand and intertemporal parameters $\{a, s, \delta, \rho\}$. If so, the capital level associated with the Ramsey equilibrium increases as $f'(k)$ increases. Accordingly, the same holds for the corresponding sales level q^R. Hence, the production possibility set wherein the VER is adopted in equilibrium is directly related to the marginal productivity of capital. This phenomenon is illustrated in figure 3, where we consider a technical progress increasing the marginal productivity of capital from $f_0'(k)$ to $f_1'(k)$.

5 Discussion: dynamic vs. static analysis

We are now interested in a general appraisal of our previous analysis by contrasting our results also with those obtained in literature with static modellization.

When the saddlepoint equilibrium in our analysis is the one driven by demand parameters, then as one would expect, the feedback steady state equilibrium of the dynamic game closely replicates the subgame perfect equilibrium of the corresponding static versions of Cournot and Bertrand games. In such cases, the conclusions drawn from either the static literature or its dynamic reinterpretation indeed coincide as to the desirability of a VER at equilibrium. As in Krishna (1989), VERs are desirable if firms compete on quantities (prices) and goods are demand substitutes (complements), because output restrictions make the foreign firm a passive player thus acting to facilitate collusion and raise prices and profits of all firms. Furthermore, VERs are irrelevant for the adopting country with Cournot competition or even suboptimal with more collusive firms' behavior and, on the contrary, they are desirable with more competitive behavior, as discussed by Mai and Hwang (1988) in a static context with conjectural variations.

However, considering a dynamic accumulation process, as we do in our analysis, allows us to reach some new interesting results. In fact, we have shown the existence of an additional equilibrium consisting in the Ramsey golden rule, which, by definition, cannot arise in the static setup or in any dynamic environment with no real investment. In particular, the golden rule whereby $f'(k^R) = \rho + \delta$ is independent of demand conditions, which implies that this equilibrium appears unmodified under both our Bertrand and Cournot competition models. It is worth noticing that the presence of multiple equilibria (the demand driven and the Ramsey equilibrium) our analysis is a consequence of making the accumulation process explicit so that firms are not obliged to sell exactly what they produce. Indeed, firms in our model can realistically accumulate capacity by selling less than what they produce or de-cumulate it by producing less than what they sell. It is precisely this feature of production and accumulation that does not constrain the game to a Cournot or Bertrand-like static interaction so that the Ramsey equilibrium ultimately emerges.

An interesting result that we have obtained in the previous analysis is that in the case of quantity-competing firm for substitute goods, VERs are suboptimal not only as a result of the demand-driven (quasi-static) equilibrium but *also* considering the additional Ramsey equilibrium driven by intertemporal allocation of capital.

Showing that two types of steady state equilibria may emerge, our analysis with capital accumulation can also lead to other interesting conclusions. To illustrate this point consider the case of demand substitutes, i.e., $s \in (0, 1]$, where the Cournot output, $a/(2+s)$, is lower than the Bertrand output, $a/[(1+s)(2-s)]$. Suppose the market dimension a is sufficiently low to ensure that the quasi-static (or demand-driven) Cournot and Bertrand outcomes prevail at the free trade equilibrium in the two alternative strategic settings. Let us now examine the effects of an increase in a. For intermediate values of a, we obtain:

$$\frac{a}{(1+s)(2-s)} \geq q^R > \frac{a}{2+s} \tag{32}$$

which entails that there exists a parameter region where the demand-driven equilibrium obtains only in the Cournot game. Hence, contrary to the acquired wisdom based upon the static approach, the dynamic analysis admits equilibrium outcomes where the incentive to adopt a VER is ruled out, irrespective of Cournot or Bertrand competition. This produces the following Corollary to Propositions 1-2:

Corollary 1 *Contrary to the static models or dynamic models with no capital accumulation, the dynamic game with investment admits equilibrium outcomes where the incentive to adopt a VER is ruled out, irrespective of whether firms are quantity- or price-setters.*

Further discussing the role of capital accumulation in our results it may be of interest to extend our model and consider a more general production function where output is obtained combining a durable input such as capital as in the current set-up, together with a non-durable input which cannot be stored, e.g. labor $l(t)$ available with a fixed endowment at any date t. Using, for example, a Cobb-Douglas production function $f(k_i(t), l(t)) = zk_i(t)^{\alpha} l(t)^{1-\alpha}$ with $z > 0$, one can show that in the limiting case where capital's share α goes to zero, the Ramsey "golden rule" equilibrium vanishes. This is natural and intuitive because this equilibrium is the one dictated by intertemporal capital accumulation alone. On the contrary, the "demand-driven" equilibrium survives being observationally equivalent to the static Cournot equilibrium. Clearly then, whenever a durable input that can be accumulated, such as capital, plays also a minor role in the production process, our analysis holds unaffected.[11]

Finally, we conclude this section by contrasting our results with those in Dockner and Haug (1991) who provide a "closed loop motive" for VERs with quantity-setting firms. Their result

[11] We thank the Editor Peter Ireland for suggesting this extension.

only apparently contrasts with the static literature because, as the authors notice, the closed-loop equilibrium of their limit model (with instantaneous price adjustment) corresponds to the equilibrium of a static model with negative conjectural variations where VERs are optimal, as shown by Mai and Hwang (1988) in a purely static context.[12] As the title of their paper emphasizes, the authors attribute this result to the commitment power available to firms in the closed loop solution concept. Our previous analysis shows that in our model the closed-loop and open-loop solution concepts coincide so that none of our results is a consequence of different commitment power arising in open and closed loop strategies.

6 Conclusions

In this paper, we have analyzed the effects of voluntary export restraints in a continuous time differential game where we have explicitly introduced the firms' accumulation dynamics. Using the Ramsey (1928) accumulation dynamics, we have proved that the open-loop Nash equilibrium is a degenerate feedback one, so that firms play subgame perfect and strongly time consistent strategies. This holds irrespective of whether one considers Cournot or Bertrand competition. We have shown that with substitute goods and quantity-setting firms, any VER hurts the firm employing this policy. Hence, contrary to the conclusions reached by Dockner and Haug (1991), VERs are not 'voluntarily' employed by Cournot firms. The opposite holds when Cournot firms supply complements. Therefore, the above analysis suggests that the empirical observation of VERs corresponds to their use as either coordinating or quasi-collusive devices in markets where firms have increasing reaction functions, which applies to either price competition with substitutes or quantity competition with complements. This is confirmed by the analysis of price competition, where the adoption of an export restraint increases the profits of the foreign firm, provided that the market-driven equilibrium prevails and goods are substitutes. If instead the domestic firm is at the Ramsey equilibrium or goods are complements, the VER will not be adopted by the foreign rival. This outcome contrasts with the result in the static literature, establishing that VERs emerge only if firms compete on prices.

Appendix

Proof of Lemma 1. Firm i's first order condition concerning the control variable is:

$$\frac{\partial \mathcal{H}_i(t)}{\partial q_i(t)} = a - 2q_i(t) - sq_j(t) - \lambda_{ii}(t) = 0 \, . \tag{33}$$

[12] Moreover, restricting the analysis to the case of instantaneous price adjustment prevents Dockner and Haug from producing a general assessment of the feasibility of VERs for the general case where prices are sticky.

Now look at the co-state equation of firm i, for the closed-loop solution of the game:

$$-\frac{\partial \mathcal{H}_i(t)}{\partial k_i(t)} - \frac{\partial \mathcal{H}_i(t)}{\partial q_j(t)} \frac{\partial q_j(t)}{\partial k_i(t)} = \frac{\partial \mu_{ii}(t)}{\partial t} \tag{34}$$

where

$$\frac{\partial q_j(t)}{\partial k_i(t)} = 0 \tag{35}$$

as it appears from a quick inspection of best replies obtained from (33):

$$q_i^{br}(t) = \frac{a - sq_j(t) - \lambda_{ii}(t)}{2} \,. \tag{36}$$

To guarantee that $q_i^{br}(t)$ indeed identifies a feedback (Markovian) solution, we have to verify that $\lambda_{ii}(t)$ is independent of state variables at any time t during the game. If so, then co-states and controls are also independent of initial states, and controls depend only on time. Therefore, in such a case, controls qualify as feedback ones. Following Leitmann and Schmitendorf (1978; see also Mehlmann, 1988, p. 134), examine the open-loop version of the adjoint equation for $\lambda_{ii}(t)$:

$$-\frac{\partial \mathcal{H}_i(t)}{\partial k_i(t)} = \frac{\partial \mu_{ii}(t)}{\partial t} \Rightarrow \frac{\partial \lambda_{ii}(t)}{\partial t} = [\rho + \delta - f'(k_i(t))]\, \lambda_{ii}(t) \,. \tag{37}$$

This is a differential equation in separable variables, which admits a solution where $\lambda_{ii}(t)$ is independent of the vector of initial states at all $t \in [0, \infty)$, $q_i^{br}(t)$ is in fact a degenerate feedback control which depends only on time.[13] ■

Proof of Lemma 3. Suppose there is substitutability between the output of firm D and that of firm F, i.e. $s \in (0, 1]$.

Now, recall that we are considering the case $q^R > q^{ss}$ and, as stated in the text, with the VER $\overline{q}_F$ the equilibrium quantity-pair is $(q_D^{ss}, \overline{q}_F)$ if $q_D^{ss} < q^R$ with $q_D^{ss} = (a - s\overline{q}_F)/2$ or $(q^R, \overline{q}_F)$ otherwise, that is in general $\left(\min\{q^R, q_D^{ss}\}, \overline{q}_F\right)$.

Consider now the cut-off value of $\overline{q}_F$ that makes the best reply quantity q_D^{ss} of firm D equal to q^R, i.e. $\tilde{q}_F \equiv \frac{a-2q^R}{s} (< q^{ss})$ implicitly defined by $(a - s\overline{q}_F)/2 = q^R$. Hence, for any $\tilde{q}_F \leq \overline{q}_F (\leq q^{ss})$ the equilibrium quantity pair is $(q_D^{ss}, \overline{q}_F)$ and for any $\overline{q}_F \leq \tilde{q}_F$ it is $(q^R, \overline{q}_F)$. Also note that $q_D^{ss} > q^R$ requires $\overline{q}_F < q^{ss}$ because if it were $\overline{q}_F = q^{ss}$, we would clearly have $q_D^{ss} = q^{ss}$ and then necessarily $q^R > q_D^{ss} = q^{ss}$. We have then two possible cases.

[13] The solution attainable through the Bellman equation of the problem cannot be reached as $f(k_i(t))$ is implicit and concave so that the Bellman equation cannot be solved analytically. However, this does not imply that the open-loop solution is not a degenerate feedback one. Several examples exist in the literature on differential games where degenerate feedback solutions obtain under open-loop rules, in games which are not linear-quadratic and where the Bellman equation is not analytically solvable (a survey of such games is in Dockner *et al.*, 2000, Ch. 5).

Case 1. Assume $\tilde{q}_F \leq \overline{q}_F \leq q^{ss}$ so that the demand-driven equilibrium prevails and the following inequalities immediately follow

$$q^R > q_D^{ss} \geq q^{ss} \geq \overline{q}_F\,.$$

Now consider firms' equilibrium profits, with and without VER: $\pi^{ss} = a^2/\left(2+s\right)^2$, $\pi_D\left(\overline{q}_F\right) = \left[a - q_D^{ss} - s\overline{q}_F\right] q_D^{ss}$ and $\pi_F\left(\overline{q}_F\right) = \left[a - \overline{q}_F - sq_D^{ss}\right]\overline{q}_F$. Using the previous inequalities on quantities and noticing that firm D is on its own best reply function, then we immediately obtain $\pi_D\left(\overline{q}_F\right) \geq \pi^{ss} \geq \pi_F\left(\overline{q}_F\right)$ which clearly become strict inequality if and only if $\overline{q}_F < q^{ss}$ as in any standard static Cournot model.

Case 2. Assume $\overline{q}_F \leq \tilde{q}_F$ so that the Ramsey equilibrium prevails with the VER. Being $\overline{q}_F < q^{ss}$ we have

$$q_D^{ss} > q^R > q^{ss} > \overline{q}_F.$$

Consider first the profits when $\overline{q}_F = \tilde{q}_F$,

$$\begin{aligned}\pi_D\left(\tilde{q}_F\right) &= \left[a - q^R - s\tilde{q}_F\right] q^R = \left[a - q_D^{ss} - s\tilde{q}_F\right] q_D^{ss} > \pi^{ss},\\ \pi_F\left(\tilde{q}_F\right) &= \left[a - \tilde{q}_F - sq^R\right]\tilde{q}_F = \left[a - \tilde{q}_F - sq_D^{ss}\right]\tilde{q}_F < \pi^{ss},\end{aligned} \tag{38}$$

where the inequalities follow from what stated in case 1. For any any smaller $\overline{q}_F < \tilde{q}_F$, profits are

$$\begin{aligned}\pi_D\left(\tilde{q}_F\right) &= \left[a - q^R - s\overline{q}_F\right] q^R > \left[a - q^R - s\tilde{q}_F\right] q^R > \pi^{ss},\\ \pi_F\left(\tilde{q}_F\right) &= \left[a - \overline{q}_F - sq^R\right]\overline{q}_F < \left[a - \tilde{q}_F - sq^R\right]\tilde{q}_F < \pi^{ss}.\end{aligned} \tag{39}$$

Inequality

$$\left[a - q^R - s\overline{q}_F\right] q^R > \left[a - q^R - s\tilde{q}_F\right] q^R$$

in the first row of (39) is immediate because $\overline{q}_F < \tilde{q}_F$. In the second row of (39), inequality

$$\left[a - \overline{q}_F - sq^R\right]\overline{q}_F < \left[a - \tilde{q}_F - sq^R\right]\tilde{q}_F$$

is due to the fact that the function $\left[a - q_F - sq^R\right] q_F$ is increasing in q_F if and only if $q_F \leq \frac{a-sq^R}{2}$ which is always the case here because we are considering $q_F < \tilde{q}_F$ with $\tilde{q}_F = \frac{a-2q^R}{s}$ and $\frac{a-2q^R}{s} \leq \frac{a-sq^R}{2}$ as long as $q^R \geq q^{ss}$.

Hence, inequalities (38) and (39) prove that in this case 2 we have $\pi_D\left(\overline{q}_F\right) > \pi^{ss} > \pi_F\left(\overline{q}_F\right)$.

Proof of Lemma 4. Consider first the case with $\overline{q}_F = q^R$. Then, only the Ramsey equilibrium can prevail. In fact note that $q_D^{ss} = \left(a - sq^R\right)/2 > q^{ss} > q^R$ because for the Ramsey equilibrium to prevail with free trade it must be $q^{ss} > q^R$. But then we have that the pair $(q_D^{ss}, \overline{q}_F = q^R)$ is an unstable focus. Hence, with $\overline{q}_F = q^R$ we have $\pi_D\left(\overline{q}_F\right) = \pi^R = \pi_F\left(\overline{q}_F\right)$.

Consider now $\overline{q}_F < q^R$. If the Ramsey equilibrium $(q^R, \overline{q}_F)$ prevails also with VER we immediately have $\pi_D(\overline{q}_F) > \pi^R > \pi_F(\overline{q}_F)$. If on the contrary the demand-driven equilibrium $(q_D^{ss}, \overline{q}_F)$ prevails with VER we have $q_D^{ss} > q^{ss} > q^R > \overline{q}_F$ so that $\pi_D(\overline{q}_F) > \pi^R > \pi_F(\overline{q}_F)$. Hence, in general, for any $\overline{q}_F \leq q^R$ we have $\pi_D(\overline{q}_F) \geq \pi^R \geq \pi_F(\overline{q}_F)$.

Proof of Lemma 5. The first order condition on the control of firm i is:

$$\frac{\partial \mathcal{H}_i(t)}{\partial p_i(t)} = \frac{a(1-s) - 2p_i(t) + sp_j(t) + \lambda_{ii}(t) - s\lambda_{ij}(t)}{1-s^2} = 0 \tag{40}$$

while the co-state equations are:

$$-\frac{\partial \mathcal{H}_i(t)}{\partial k_i(t)} - \frac{\partial \mathcal{H}_i(t)}{\partial p_j(t)}\frac{\partial p_j(t)}{\partial k_i(t)} = \frac{\partial \mu_{ii}(t)}{\partial t} \tag{41}$$

$$-\frac{\partial \mathcal{H}_i(t)}{\partial k_j(t)} - \frac{\partial \mathcal{H}_i(t)}{\partial p_j(t)}\frac{\partial p_j(t)}{\partial k_j(t)} = \frac{\partial \mu_{ij}(t)}{\partial t} \tag{42}$$

>From the equivalent of (40) for $p_j(t)$, we obtain the best reply function of firm j :

$$p_j^{br}(t) = \frac{a(1-s) + sp_i(t) + \lambda_{jj}(t) - s\lambda_{ji}(t)}{2} \tag{43}$$

Now, note that $\partial p_j(t)/\partial k_j(t) = 0$ on the basis of (43). Accordingly, we may rewrite the co-state equation (42) as

$$\frac{\partial \lambda_{ij}(t)}{\partial t} = [\rho + \delta - f'(k_j(t))]\,\lambda_{ij}(t)\,. \tag{44}$$

Given that it is a separable differential equation, it admits the solution $\lambda_{ij}(t) = 0$ at any t during the game. ■

Proof of Lemma 6. The necessary conditions are:

$$\frac{\partial \mathcal{H}_i(t)}{\partial p_i(t)} = \frac{a(1-s) - 2p_i(t) + sp_j(t) + \lambda_{ii}(t)}{1-s^2} = 0 \tag{45}$$

$$-\frac{\partial \mathcal{H}_i(t)}{\partial k_i(t)} - \frac{\partial \mathcal{H}_i(t)}{\partial p_j(t)}\frac{\partial p_j(t)}{\partial k_i(t)} = \frac{\partial \mu_{ii}(t)}{\partial t} \tag{46}$$

where $\partial p_i(t)/\partial k_j(t) = 0$ on the basis of the best reply (see eq. (43)). Accordingly, the co-state equation simplifies as follows:

$$\frac{\partial \lambda_{ii}(t)}{\partial t} = [\rho + \delta - f'(k_i(t))]\,\lambda_{ii}(t)\,. \tag{47}$$

As in the Cournot-Ramsey game (see the proof of Lemma 1), also here the above equation admits a solution for $\lambda_{ii}(t)$ at a generic time t, which is independent of the vector of initial conditions. This proves the Lemma. ■

Proof of Proposition 2. To begin with, we have to compare the profits firms F and D obtain when they respectively use the VER $\overline{q}_F$ and the implied "market-driven" equilibrium quantity q_D^{VER}, against the profits they obtain with no quantity restrictions adopted by firm F.

Calculating firm F's profit with the VER

$$\Pi_F^{VER} = \frac{\overline{q}_F \left[a(2-s) - \overline{q}_F(2-s^2)\right]}{2}, \tag{48}$$

the comparison of instantaneous profits for firm F reveals that $\Pi_F^{VER} - \Pi^{ss} = 0$ at

$$\overline{q}_{F1,2} = \frac{a\left[(1+s)(2-s)^2 \pm \sqrt{(1+s)^2(2-s)^4 - 8(2-s^2)(1-s^2)}\right]}{2(2-s^2)(1+s)(2-s)} \tag{49}$$

Similarly, for firm D, we have

$$\Pi_D^{VER} = \frac{(a - s\overline{q}_F)^2}{4} \tag{50}$$

and then $\Pi_D^{VER} - \Pi^{ss} = 0$ at

$$\overline{q}_{F3,4} = \frac{a\left[(1+s)(2-s)^2 \pm 2(s-2)\sqrt{(1-s^2)}\right]}{s(1+s)(2-s)^2} \tag{51}$$

The expression for $\overline{q}_{Fi}$, $i = 1,2$ is real if and only if $s \in (0,1]$. Given the sign of the coefficients of $\overline{q}_F$ in $\Pi_F^{VER} - \Pi^{ss}$, this reality condition proves that the VER cannot be adopted when goods are complements.

Now observe that $\Pi_F^{VER} - \Pi^{ss} > 0$ for all $\overline{q}_F \in (\overline{q}_{F2}, \overline{q}_{F1})$, while $\Pi_D^{VER} - \Pi^{ss} > 0$ for all $\overline{q}_F$ outside the interval $[\overline{q}_{F4}, \overline{q}_{F3}]$. It is a matter of tedious algebra to check that $\overline{q}_{F3} > \overline{q}_{F1} > \overline{q}_{F4} > q^{ss} > \overline{q}_{F2}$ for all positive values of s. Therefore, any $\overline{q}_F \in (\overline{q}_{F2}, q^{ss})$ benefits both firms. Moreover, it is also immediate to verify that $\arg\max \Pi_F^{VER} = a(2-s) / [2(2-s^2)] \equiv \overline{q}_F^*$, with $\overline{q}_F^* \in (\overline{q}_{F2}, q^{ss})$.

This entails that Π_F^{VER} increases as $\overline{q}_F$ decreases from q^{ss} to $\overline{q}_F^*$, which, in turn, suffices to prove that $\Pi_F^{VER} > \Pi_D^{VER}$ for all $\overline{q}_F \in (\overline{q}_F^*, q^{ss})$ and $s \in (0,1]$.

Now examine the case where $q_D = q^R$, because either (i) $q^{ss} > q^R$, or (ii) $\overline{q}_F^*$ is larger than the value of $\overline{q}_F$ that drives the domestic firm to the Ramsey equilibrium. Case (i) describes a situation where the free trade equilibrium is the symmetric Ramsey golden rule. Case (ii) captures the situation where the output expansion carried out by the domestic firm as a reaction to the VER is sufficiently large to generate the Ramsey output for firm D only. In the first case, firm F has no incentive to adopt a VER because firm D cannot expand output beyond the Ramsey level. In the second case, firm D reaches the Ramsey output starting from the "market-driven" solution. Any further reduction by firm F is unprofitable. ■

References

[1] Başar, T. and G.J. Olsder (1982, 1995²), *Dynamic Noncooperative Game Theory*, San Diego, Academic Press.

[2] Berry, S. J. Levinsohn and A. Pakes (1999), "Voluntary Export Restraints on Automobiles: Evaluating a Trade Policy", *American Economic Review*, **89**, 400-30.

[3] Bowley, A.L. (1924), *The Mathematical Groundwork of Economics*, Oxford, Oxford University Press.

[4] Brander, J. (1995) "Strategic Trade Policy", in Grossman, G.M. and K. Rogoff (eds.), *Handbook of International Economics*, vol. 3, Amsterdam, North-Holland.

[5] Calzolari, G. and L. Lambertini (2006), "Tariffs vs Quotas in a Trade Model with Capital Accumulation", *Review of International Economics*, **16**, 632-44.

[6] Cellini, R. and L. Lambertini (1998), "A Dynamic Model of Differentiated Oligopoly with Capital Accumulation", *Journal of Economic Theory*, **83**, 145-55.

[7] Cellini, R. and L. Lambertini (2006), "Capital Accumulation, Mergers, and the Ramsey Golden Rule", in M. Quincampoix, T. Vincent and S. Jørgensen (eds), *Advances in Dynamic Game Theory and Applications*, Annals of the International Society of Dynamic Games, vol. 8, forthcoming.

[8] Cheng, L. (1987), "Optimal Trade and Technology Policies: Dynamic Linkages", *International Economic Review*, **28**, 757-76.

[9] Dixit, A.K. (1979), "A Model of Duopoly Suggesting a Theory of Entry Barriers", *Bell Journal of Economics*, **10**, 20-32.

[10] Dockner, E.J. and A.A. Haug (1990), "Tariffs and Quotas under Dynamic Duopolistic Competition", *Journal of International Economics*, **29**, 147-59.

[11] Dockner, E.J. and A.A. Haug (1991), "The Closed Loop Motive for Voluntary Export Restraints", *Canadian Journal of Economics*, **3**, 679-85.

[12] Dockner, E.J, S. Jørgensen, N. Van Long and G. Sorger (2000), *Differential Games in Economics and Management Science*, Cambridge, Cambridge University Press.

[13] Driskill, R. and S. McCafferty (1989), "Dynamic Duopoly with Output Adjustment Costs in International Markets: Taking the Conjecture out of Conjectural Variations", in Feenstra, R.C. (ed.), *Trade Policies for International Competitiveness*, NBER Conference Report series, Chicago, University of Chicago Press, 125-37.

[14] Driskill, R. and S. McCafferty (1996), "Industrial Policy and Duopolistic Trade with Dynamic Demand", *Review of Industrial Organization*, **11**, 355-73.

[15] Fershtman, C. and M.I. Kamien (1987), "Dynamic Duopolistic Competition with Sticky Prices", *Econometrica*, **55**, 1151-64.

[16] Fershtman, C. and M.I. Kamien (1990), "Turnpike Properties in a Finite-Horizon Differential Game: Dynamic Duopoly with Sticky Prices", *International Economic Review*, **31**, 49-60.

[17] Harris, R. (1985), "Why Voluntary Export Restraints Are 'Voluntary"', *Canadian Journal of Economics*, **18**, 799-809.

[18] Herguera, I., P. Kujal and E. Petrakis (2000), "Quantity Restrictions and Endogenous Quality Choice", *International Journal of Industrial Organization*, **18**, 1259-78.

[19] Kamien, M.I. and N.L. Schwartz (1981), *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management*, Amsterdam, North-Holland.

[20] Kemp, M.C. K. Shimomura and N. Van Long, (2001), "A Differential Game Model of Tariff War", *Japan and the World Economy,* **13**, 3279-298.

[21] Krishna, K. (1989), "Trade Restrictions as Facilitating Practices", *Journal of International Economics*, **26**, 251-70.

[22] Leitmann, G. and W.E. Schmitendorf (1978), "Profit Maximization through Advertising: A Nonzero Sum Differential Game Approach", *IEEE Transactions on Automatic Control*, **23**, 646-50.

[23] Mai, C. and H. Hwang (1988), "Why Voluntary Export Restraints Are Voluntary: An Extension", *Canadian Journal of Economics*, **21**, 877-82.

[24] Mehlmann, A. (1988), *Applied Differential Games*, New York, Plenum Press.

[25] Miravete, E. J. (2003), "Time–Consistent Protection with Learning by Doing", *European Economic Review*, **47**, 761-790.

[26] Pomfret, R. (1989), "The Economics of Voluntary Export Restraints Agreements", *Journal of Economic Surveys*, **3**, 199-211.

[27] Ramsey, F.P. (1928), "A Mathematical Theory of Saving", *Economic Journal*, **38**, 543-59. Reprinted in Stiglitz, J.E. and H. Uzawa (1969, eds.), *Readings in the Modern Theory of Economic Growth*, Cambridge, MA, MIT Press.

[28] Simaan, M. and T. Takayama (1978), "Game Theory Applied to Dynamic Duopoly Problems with Production Constraints", *Automatica*, **14**, 161-66.

[29] Singh, N. and X. Vives (1984), "Price and Quantity Competition in a Differentiated Duopoly", *RAND Journal of Economics*, **15**, 546-54.

[30] Spence, A. M. (1979), "Investment Strategy and Growth in a New Market", *Bell Journal of Economics*, **10**, 1-19.

[31] Suzumura, K. and J. Ishikawa (1997), "Voluntary Export Restraints and Economic Welfare", *Japanese Economic Review*, 48, 176-86.

[32] Tsutsui, S. and K. Mino (1990), "Nonlinear Strategies in Dynamic Duopolistic Competition with Sticky Prices", *Journal of Economic Theory*, **52**, 136-61.