Competition with exclusive contracts and market-share discounts*

Giacomo Calzolari

University of Bologna and CEPR

Vincenzo Denicolò

University of Bologna and University of Leicester

April 25th, 2012

Abstract

We analyze firms that compete by means of exclusive contracts and market-share discounts (conditional on the seller's share of the customer's total purchases) with incomplete information about demand. Firms use exclusive contracts and market-share discounts to better extract the buyers' informational rents. In this framework we show that exclusive contracts intensify competition, reducing prices and profits, but market-share discounts produce a double marginalization effect, increasing prices and damaging consumers. We discuss the implications for competition policy.

*An earlier version of this paper was circulated as CEPR discussion paper DP7613. We are grateful to three anonymous referees and Editor Alessandro Lizzeri for their perceptive comments and suggestions. We also thank Richard Gilbert, Doh-Shin Jeon, David Martimort, Volker Nocke, Arsen Palestini, Patrick Rey, Christian Schultz and seminar participants at the University of Mannheim, IMT Lucca, Leicester, Durham, Bicocca, Bologna, Bergamo, Naples, the 2010 EARIE conference (Istanbul), the 2011 IIOC conference (Boston), the XII CEPR/JIE conference (Tel Aviv), the 2011 CSEF-IGIER conference (Capri), and the 2011 NIE conference (Leicester) for useful discussions. E-mail addresses: giacomo.calzolari@unibo.it (Calzolari) and vincenzo.denicolo@unibo.it (Denicolò).

In this paper, we study firms that compete by means of exclusive contracts or market-share discounts (i.e., discounts that depend on the seller's share of the customer's total purchases). These are common but still little explored forms of competition. The literature has focused mainly on incumbents' use of exclusive contracts or market-share discounts to deter new entrants and has paid less attention to competitors deploying similar pricing strategies.

Antitrust authorities view competition by means of exclusive contracts or market-share discounts with suspicion.[1] A prime instance is the case of Intel, which offered electronics manufacturers discounts that depended on their buying some minimum share of their total purchases of microchips from the firm. Some customers, to qualify for the discount, had to choose an exclusive supply contract; that is, the minimum share was set at 100%. In 2009, the European Commission found these contracts to be abusive and levied the largest fine in the history of European competition policy, over a billion euros.

The authorities' concern appears to be that a dominant firm can use exclusivity or market-share discounts to eliminate not only potential, but also actual competitors. (Intel has a sizeable competitor, AMD, which has been active in the market for microchips for many years.) Yet it is not clear that such discounts have this exclusionary effect when equally efficient competitors, too, can make the same sort of offer. In fact, according to the theory of common agency under complete information, competition in simple non-linear pricing leads to the efficient outcome, and this outcome holds also when firms can make their prices conditional on their competitor's volume.[2]

This Chicago-style neutrality result raises the question of why firms do not simply compete in non-linear pricing. The thesis of this paper is that neutrality hinges, crucially, on the hypothesis of complete information about demand. With complete information, non-linear pricing suffices for perfect price discrimination, and rent extraction is limited only by the

[1] For an overview of the competition policy debate, see Ahlborn and Bailey (2008) and Kobayashi (2005).

[2] See O'Brien and Shaffer (1997) and Bernheim and Whinston (1998).

competition among the sellers. This makes more complex contracts redundant. With incomplete information, however, buyers also have informational rents, so that the sellers have an incentive to use exclusive contracts and market-share discounts to extract these rents.

We develop this explanation in a model positing two firms producing horizontally differentiated products, the willingness to pay for which (a one-dimensional parameter) is the buyers' private information. We model exclusive contracts by allowing each firm to submit two different non-linear price schedules, one for buyers who agree to exclusivity, and the other for those who prefer to retain the option to purchase from both producers. Our first result is that exclusive contracts, which are indeed offered in equilibrium, heigthen competition, lowering prices and profits. The intuition is very simple. When firms compete in non-linear pricing, competition is limited by product differentiation. With exclusive contracts, though, firms compete in utility space, where their products effectively become homogeneous. The familiar Bertrand undercutting process then drives exclusive prices towards marginal costs.

When firms are symmetric, for instance, there is always a zero-profit equilibrium in which exclusive prices are equal to marginal cost and non-exclusive prices are exorbitantly high, so that no buyer purchases both products. But this zero-profit equilibrium arises only in the case of very serious failures of coordination between the firms. If buyers value product variety, firms can earn positive profits even in a non-cooperative equilibrium by coordinating their non-exclusive pricing strategies. The key is to lower non-exclusive prices just enough to induce buyers to purchase both products, extracting the buyers' preference for variety.

When buyers are purchasing both products, however, firms have no definite incentive to undercut one another's exclusive prices, which creates scope for further coordination: at this point, exclusive prices too may be raised above marginal cost. In a non-cooperative equilibrium, however, this scope is limited. Exclusive contracts always give buyers an extra option that disciplines firms' non-exclusive pricing. As a result, prices and profits are lower than in the non-linear pricing equilibrium, and buyers' surplus is greater.

We then turn to market-share discounts. The idea that these discounts are simply a weaker form of exclusive contracts is intuitive, widely held, and wrong. Actually, market-share discounts increase prices over the equilibrium with exclusive contracts, reducing efficiency and harming buyers. The reason is that they allow each firm, in effect, to impose a tax on the other's product, creating the same double marginalization as if the products were perfect complements.

Thus, exclusive contracts reduce prices while market-share discounts increase them. But market-share discounts also embrace, as a special case, exclusivity discounts. Therefore, using as a benchmark simple non-linear pricing, market-share discounts have not only the double-marginalization effect but also the pro-competitive effect of exclusive contracts. Compared to the non-linear pricing benchmark, market-share discounts benefit low-demand and damage high-demand buyers. The overall net effect is in principle ambiguous.

These results imply that if economies of scale are relatively unimportant and coordination failures are limited, exclusive contracts should be permitted and market-share discounts prohibited. However, where economies of scale are substantial the stronger firm may actually gain from fiercer competition that drives the other's profit below the level justifying continued operation. And if coordination failures are serious, competition may become disruptive. In these cases, the policy implications are less clear-cut. They are discussed in the concluding section.

The literature. — Most of the literature on exclusive contracts focuses on the situation in which the incumbent contracts with buyers before a potential new entrant comes into the market and ignores the possibility that active firms may compete for exclusivity.[3] To the best of our knowledge, competition for exclusive dealing among active firms has only

[3]See, for instance, Aghion and Bolton (1987), Rasmusen *et al.* (1990), Segal and Whinston (2000) and Fumagalli and Motta (2006). One notable exception is Matthewson and Winter (1984), but they restrict firms to linear pricing. Whinston (2007) provides an excellent survey of the literature.

been analyzed in models of one-stop shopping, which simply posit exclusivity,[4] or on the assumption of complete information.[5] Ours is thus the first paper to study competition in exclusive contracts with incomplete information about demand.

Some papers have analyzed competition in market-share discounts. Majumdar and Shaffer (2009), considering a dominant firm facing a competitive fringe, show that the firm has an incentive to use market-share discounts, with mixed welfare effects. However, their dominant firm effectively faces a monopolistic screening problem, whereas we analyze a problem of duopolistic screening.[6] The paper whose approach most resembles ours is Armstrong (2010), which explores the case of duopoly with multi-dimensional heterogeneity. This allows for more general patterns of demand than those posited here but considerably complicates the analysis. For simplicity of treatment, Armstrong (2010) takes the case of indivisible goods and unit demand, showing that market-share premia emerge in equilibrium. Unfortunately, this makes his specification unsuitable to address the competition policy issues discussed above. Nevertheless, the case of multi-dimensional heterogeneity is an important topic for future research.

Structure of the paper. — The rest of the paper is organized as follows. Section 1 sets up the model and presents the non-linear pricing equilibrium benchmark. Section 2 analyzes exclusive contracts with symmetric firms, and Section 3 deals with market-share discounts. Section 4 extends the analysis to allow for shopping costs. Section 5 briefly considers the case of asymmetric firms. Section 6 sums up and suggests directions for future work. The proofs are set out in the Appendix.

[4] See, for instance, Armstrong and Vickers (2001) and Rochet and Stole (2002).

[5] See O'Brien and Shaffer (1997) and Bernheim and Whinston (1998).

[6] Yehezkel (2008) analyzes the case of an incompletely informed manufacturer selling a product through a retailer, which can supply its own low-quality variant and is fully informed on demand. Choné and Linnemer (2011) model the competition between an incumbent that is incompletely informed and a potential entrant that enters the market once uncertainty on demand has been resolved. In both papers, only one firm faces a screening problem. The literature on market-share discounts with complete information includes Inderst and Shaffer (2010) and Mills (2010).

1 The model

Two firms, denoted by $i = A, B$, supply horizontally differentiated products q_A and q_B at constant unit costs c_A and c_B. We assume that $c_A = c_B$ (this assumption is relaxed in Section 5) and normalize the costs to zero. As long as both firms remain active, one can abstract from fixed costs.

A buyer of type θ who buys q_A units of good A and q_B units of good B obtains a benefit, measured in monetary terms, of $u(q_A, q_B, \theta)$. The one-dimensional parameter θ is the buyer's private information; it is distributed over the interval $[\theta_{\min}, \theta_{\max}]$ according to a distribution function $F(\theta)$ with density $f(\theta)$. We may think of A and B as upstream firms that sell to a downstream firm of unknown type θ, and of u as the downstream firm's gross profit. Alternatively, we could imagine the firms selling directly to a mass of final consumers, with u as the latters' utility function. If the firms are risk neutral, the two models are equivalent.[7]

The function $u(q_A, q_B, \theta)$ is taken to be symmetric, thrice differentiable, strictly concave, and increasing in q_i as long as q_i is not too large. At the same time, we posit that there exists a finite satiation point, so that the full information first-best quantities $q^{fb}(\theta) = \arg\max_q u(q, q, \theta)$ are well defined. Denoting partial derivatives with subscripts, we assume that $u_\theta(q_A, q_B, \theta) \geq 0$ (a normalization) and that $u_{\theta q_i}(q_A, q_B, \theta) \geq 0$ (the sorting condition), with strict inequalities whenever $q_A, q_B > 0$. We also make the mild assumption that, possibly after suitable relabeling of the types, $u_{\theta\theta q}(q_A, q_B, \theta) = 0$.

To simplify the exposition, we assume that in the non-linear pricing equilibrium the market is uncovered. A sufficient condition for this is that $q^{fb}(\theta_{\min}) = 0$ and $q^{fb}(\theta) > 0$ for all $\theta > \theta_{\min}$.[8] Additional regularity assumptions on preferences and the distribution of types, which are needed for technical reasons, will be introduced later.

[7] In practice, exclusive contracts and market-share discounts tend to be used only when buyers are large enough so that firms can monitor their purchases from their competitors.

[8] In fact it suffices that there exists a $\theta \in [\theta_{\min}, \theta_{\max}]$ such that $q^{fb}(\theta) = 0$. If this holds, one can always choose $\theta_{\min}$ as the largest θ for which $q^{fb}(\theta) = 0$ and re-scale the distribution function accordingly.

Our assumptions imply two noteworthy properties of demand. First, the fact that heterogeneity is one-dimensional implies that the demand for the two products is correlated. The demand for a product therefore implies information about that for the other, explaining why firms may want to set prices conditional upon their competitor's volume. Second, the strict concavity of u implies that buyers have a preference for variety, which can be defined as the extra benefit from buying q units of both goods rather than $2q$ units of one: $\ell(q, \theta) \equiv u(q, q, \theta) - u(2q, 0, \theta) > 0$.

Firms simultaneously and independently offer a menu of contracts. We distinguish three different games according to the type of contract that the firms may offer. In the benchmark case, firms compete in simple non-linear pricing. That is, the payment to each firm depends only on its own quantity. A strategy for firm i then is a function $P_i(q_i)$ in which q_i is the quantity firm i is willing to supply and $P_i(q_i)$ is the corresponding total payment it asks. In the second case, firms can offer exclusive contracts but not market-share discounts. Here, a strategy for firm i comprises two price schedules, $P_i^E(q_i)$ and $P_i^{NE}(q_i)$. The former applies to exclusive contracts ($q_{-i} = 0$), the latter to non exclusive ones ($q_{-i} > 0$). Finally, in the third case a firm can freely condition its payment request on its competitor's sales volume: $P_i = P_i(q_i, q_{-i})$. This allows for market-share discounts, and includes exclusive contracts as a special case.[9]

Buyer θ first observes the firms' offers and then chooses the contracts that maximize his net payoff $U = u(q_A, q_B, \theta) - P_A(q_A, q_B) - P_B(q_B, q_A)$. (Here $P_i(q_i, q_{-i})$ is used as a general notation that also covers non-linear pricing and exclusive contracts.) Each price schedule P_i must be non decreasing in q_i (a free disposal assumption),[10] and must satisfy $P_i = 0$ when $q_i = 0$. To guarantee that the buyer's maximization problem has a solution, we restrict P_i to

[9] The literal interpretation of market-share discounts is that firm i's payment request depends both on its own quantity, q_i, and on its share of the costumer's purchases, $s_i = \frac{q_i}{q_i + q_{-i}}$. However, with two firms any function $P_i(q_i, s_i)$ can be rewritten as a function of q_i and q_{-i}, and vice versa.

[10] This implies that price schedules must be differentiable almost everywhere.

be upper semi-continuous in q_i and q_{-i}. Each firm i maximizes its total (or expected) profit $\pi_i = \int_{\theta_{\min}}^{\theta_{\max}} P_i[q_i(\theta), q_{-i}(\theta)] f(\theta) d\theta$.

This completes the description of the model. To obtain explicit solutions, at times we adopt a quadratic-uniform specification where the function u is quadratic:

$$u(q_A, q_B, \theta) = \theta(q_A + q_B) - \frac{1-\gamma}{2}(q_A^2 + q_B^2) - \gamma q_A q_B, \tag{1}$$

and θ is uniformly distributed over the interval $[0,1]$. The parameter γ captures the degree of substitutability among the products: it ranges from $\frac{1}{2}$ (perfect substitutes) to $-\infty$ (perfect complements); when $\gamma = 0$ the goods are independent. The factor $\frac{1-\gamma}{2}$ in the middle term in (1) prevents changes in γ from affecting the size of the market.[11]

The non-linear pricing equilibrium. — Our benchmark is simple non-linear pricing. This case has been analyzed in a framework similar to ours by Martimort and Stole (2009), who show that efficiency is achieved only in the limiting case of perfect substitutes. In general, marginal prices exceed marginal costs, equilibrium quantities are distorted downward, and some low-demand buyers are inefficiently excluded. However, competition preserves the property of no distortion at the top, so in equilibrium type $\theta_{\max}$ buys the efficient quantities $q^{fb}(\theta_{\max})$. Because buyers obtain informational rents, firms' profits are lower than their marginal contributions to social welfare.

These properties are easily demonstrated in the quadratic-uniform case, where the non-linear pricing equilibrium can be calculated explicitly. In equilibrium, both firms offer the price schedule

$$P^*(q) = \alpha q - \frac{\alpha}{2} q^2 \qquad \text{for } 0 \le q \le 1, \tag{2}$$

where $\alpha = \frac{1}{4}\left[3(1-\gamma) - \sqrt{1 - 2\gamma + 9\gamma^2}\right] \ge 0$ is a decreasing function of γ that vanishes when $\gamma = \frac{1}{2}$. The equilibrium price schedules are flat at $q^{fb}(1) = 1$, and there are no fixed fees. The equilibrium net utility is $U^*(\theta) = \frac{(\theta-\alpha)^2}{1-\alpha}$. By the envelope theorem, equilibrium

[11] As argued by Shubik and Levitan (1980), this rules out spurious effects in the comparative statics analysis.

quantities satisfy $\frac{dU^*(\theta)}{d\theta} = 2q^*(\theta)$, and are $q^*(\theta) = \frac{\theta-\alpha}{1-\alpha}$. In the limiting case of perfect substitutes ($\gamma = \frac{1}{2}$), marginal prices equal marginal costs, and the equilibrium reproduces the efficient solution: $q^*(\theta) = q^{fb}(\theta) = \theta$. As the degree of substitutability γ decreases, competition becomes less intense, prices increase, and quantities decrease. With independent goods ($\gamma = 0$), the monopoly solution returns. Equilibrium quantities are smallest in the case of perfect complements ($\gamma = -\infty$), when owing to the problem of Cournot complements firms price above the monopoly level.

2 Exclusive contracts

We now assume that firms can offer exclusive contracts, but not market-share discounts. This captures the case in which firms observe whether or not the buyer purchases from their competitor, but not the exact quantity purchased and so cannot condition their payment request on it.[12] A strategy for firm i then comprises two price schedules, $P_i^E(q_i)$ which applies when the buyer agrees to exclusivity ($q_{-i} = 0$); and $P_i^{NE}(q_i)$ when he buys also from the rival ($q_{-i} > 0$). Each firm can unilaterally enforce exclusive dealing by charging exorbitant non-exclusive prices. However, whether buyers ultimately purchase only one product or both will be an equilibrium property of the model.

2.1 The incentive to offer exclusive contracts

To show that firms have an incentive to offer exclusive contracts, first consider the case in which the goods are substitutes. Let $P^*(q)$ be a symmetric non-linear pricing equilibrium. We shall show that if one firm sticks to $P^*(q)$, the other has a profitable deviation. The deviation is to offer exorbitantly high non-exclusive prices, and exclusive prices $P^E(q) = P^*(q) - \varepsilon$, where $\varepsilon > 0$ is arbitrarily small.[13] With this strategy, the deviating firm unilaterally

[12] Alternatively, competition policy may permit exclusive contracts but prohibit market-share discounts – a rare course today but one that may be optimal in our model.

[13] In fact this is not the best response to $P^*(q)$. The best response is to set $P^E(q) = P^*(q) - \varepsilon$ and raise the non-exclusive prices by adding a positive fixed fee, but the fixed fee should be low enough so that the

enforces exclusive dealing, and then captures the entire market by slightly undercutting its rival. Because the goods are substitutes, demand will then be higher than in the non-linear pricing equilibrium. But prices have declined only slightly and still exceed marginal costs, so the deviating firm's profits must increase.[14]

Firms also have an incentive to offer exclusive contracts even when the goods are complements.[15] By offering exclusive contracts a firm can increase its sales to low-demand buyers without attracting high-demand ones, and hence without raising their informational rents. This is possible because high-demand buyers have a greater relative preference for variety than low types, and hence have more to lose by exclusive contracts.[16]

2.2 Coordination failures

The foregoing implies that in equilibrium firms offer exclusive contracts, and that, as a result, the equilibrium outcome changes. In fact, the equilibrium outcome with exclusive contracts is not unique, in that firms face coordination problems that generally have multiple solutions. Until Section 5, we focus on symmetric equilibria, which resolves some but not all of this multiplicity.

The equilibrium in which coordination failures are most serious is that in which both firms offer only exclusive contracts (or, equivalently, charge exorbitant non-exclusive prices). This means that firms effectively compete in utility space, so that exclusive prices must be equal to the marginal cost. Both firms make zero profits, and neither has a profitable

highest demand buyers still purchase both products.

[14]Under complete information, by contrast, exclusive contracts would be irrelevant (O'Brien and Shaffer, 1997; Bernheim and Whinston, 1998). The reason is that under complete information two-part tariffs suffice for perfect price discrimination, so in equilibrium firms charge a marginal price equal to the marginal cost and extract their profit, equal to their marginal contributions to social surplus, through fixed fees. This makes it clear that the strategy of undercutting cannot be profitable even if exclusive contracts are permitted.

[15]The incentive vanishes only in the limiting cases of perfect substitutes (in which case non-linear prices equal the marginal cost leaving no scope whatever for undercutting) and perfect complements (in which case exclusive contracts are totally unattractive).

[16]For details, see the working paper version of this paper.

deviation available. However, this equilibrium is Pareto dominated. Both firms can obtain positive profits and buyers larger surpluses, but this requires some coordination among the firms. Specifically, they must lower their non-exclusive prices, and raise their exclusive prices, in coordinated fashion. The first move induces buyers to purchase both products, allowing firms to extract the buyers' preference for variety; the second reduces the intensity of competition. We consider each form of coordination in turn.

2.3 Extracting the buyers' preference for variety

When both firms offer competitive exclusive prices $P^E(q) = 0$, consumers have the option of buying $q^E(\theta) = \arg\max_q u(q, 0, \theta)$ from either. This option guarantees a type-dependent "reservation utility" $U^E(\theta) = \max_q u(q, 0, \theta)$. Now, suppose that firms lower their non-exclusive prices just enough to get buyers to purchase both products. That is, they introduce the non-exclusive price schedule $\bar{P}^{NE}(q)$ implicitly defined by the condition:[17]

$$\max_q \left[u(q, q, \theta) - 2\bar{P}^{NE}(q)\right] = U^E(\theta) \tag{3}$$

(with a small tie-breaking discount if necessary), so that buyers choose positive quantities of both products, $\bar{q}(\theta) = \arg\max_q \left[u(q, q, \theta) - 2\bar{P}^{NE}(q)\right]$. Because they value product variety, the non-exclusive prices $\bar{P}^{NE}(q)$ are strictly positive, allowing firms to make positive profits. For example, when u is separable as follows:

$$u(q_A, q_B, \theta) = \theta(q_A + q_B) + \omega(q_A, q_B), \tag{4}$$

so that the preference for variety $\ell(q) = u(q, q, \theta) - u(2q, 0, \theta)$ does not depend directly on θ, one immediately gets $\bar{P}^{NE}(q) = \frac{1}{2}\ell(q)$.[18]

We now show that firms can indeed extract the buyers' preference for variety in a non-cooperative equilibrium. We focus on equilibria where all buyers are induced to purchase both

[17] Relaxing symmetry, condition (3) would pin down only the sum of the non-exclusive prices. Asymmetric equilibria are discussed in Section 5.

[18] When the preference for variety $\ell(q, \theta)$ does depend on θ, the expression for $\bar{P}^{NE}(q)$ is more complicated as one must suitably map θ into q by inverting the function $\bar{q}(\theta)$ (see the proof of Lemma 1).

products.[19] To proceed, we impose two regularity conditions. Consider a symmetric non-linear pricing equilibrium $P^*(q)$. We already know that the non-linear pricing equilibrium quantities $q^*(\theta)$ vanish when θ is low (i.e., some low-demand buyers are inefficiently excluded) and coincide with the efficient quantities at $\theta = \theta_{\max}$ (no distortion at the top). On the other hand, it can be shown that the function $\bar{q}(\theta)$ is strictly increasing and satisfies $0 < \bar{q}(\theta) < q^{fb}(\theta)$ for all $\theta > \theta_{\min}$ (see Lemma 1 in the Appendix). This implies that the curve $\bar{q}(\theta)$ must intersect $q^*(\theta)$ from above at least once. The first regularity condition is:

R1. *The curves $\bar{q}(\theta)$ and $q^*(\theta)$ intersect only once.*

The intersection point $\hat{\theta}$ and $\hat{q}$ can then be defined by the condition $\hat{q} = q^*(\hat{\theta}) = \bar{q}(\hat{\theta})$. Next, observe that with non-linear pricing, the residual demand for a firm's product can be described in terms of an indirect utility function, which accounts for any potential benefit to the buyer from optimally trading with the firm's competitor (Martimort and Stole, 2009). The indirect utility function is $v^*(q,\theta) = \max_x\{u(q,x,\theta) - P^*(x)\}$. Our second regularity condition is:

[19] The preference for variety might also be extracted only from a subset of buyers. For example, assume a degenerate distribution with just three types: $\theta_{\min}$, θ_0 and $\theta_{\max}$. (In fact $\theta_{\min}$ is a dummy buyer whose demand is always nil.) Consider a candidate equilibrium in which both firms offer $P^E(q) = 0$ and a flat non-exclusive price schedule: $P^{NE}(q) = \breve{P}$ for any $q > 0$. Let the fixed fee $\breve{P}$ be such that type $\theta_{\max}$ is perfectly indifferent between a zero-price exclusive contract and his most preferred combination of non-exclusive contracts:

$$2\breve{P} = u\left[q^{fb}(\theta_{\max}), q^{fb}(\theta_{\max}), \theta_{\max}\right] - u\left[q^E(\theta_{\max}), 0, \theta_{\max}\right].$$

A sufficient condition for these pricing strategies to be an equilibrium, irrespective of the proportion of buyers of each type, is:

$$u\left[q^{fb}(\theta_0), q^{fb}(\theta_0), \theta_0\right] - \breve{P} \leq u\left[q^E(\theta_0), 0, \theta_0\right].$$

This inequality, which is always satisfied if θ_0 is low enough relative to $\theta_{\max}$, guarantees that there is no scope for offering a contract that is acceptable to type θ_0 and profitable. In equilibrium, type $\theta_{\max}$ now buys the efficient quantities, but type θ_0 opts for an exclusive contract. Firms make positive profits by extracting the preference for variety of type $\theta_{\max}$ only. Like those in which firms completely fail to coordinate, however, equilibria of this sort are Pareto dominated.

R2. *The function*

$$\frac{v_q^*\left[\bar{q}(\theta),\theta\right]}{v_{\theta q}^*\left[\bar{q}(\theta),\theta\right]}f(\theta)-\left[1-F(\theta)\right] \tag{5}$$

is non-decreasing in θ *for* $\theta \leq \hat{\theta}$.

This condition guarantees that the non-linear pricing equilibrium is "stable" in a sense that will be clarified below. One can verify that both **R1** and **R2** are easily satisfied in the quadratic-uniform model.

Proposition 1 *Let* $P^*(q)$ *be a symmetric non-linear pricing equilibrium such that* ***R1*** *and* ***R2*** *hold.*[20] *Then there exists a symmetric equilibrium where both firms offer both the exclusive price schedule* $P^E(q)=0$ *and the non-exclusive price schedule*

$$P^{NE}(q)=\begin{cases}\bar{P}^{NE}(q) & \text{for } 0\leq q\leq \hat{q}\\ \\ P^*(q)-P^*(\hat{q})+\bar{P}^{NE}(\hat{q}) & \text{for } \hat{q}\leq q\leq q^{fb}(\theta_{\max}),\end{cases}$$

where $\bar{P}^{NE}(q)$ *is the solution to functional equation (3).*

Equilibrium quantities are $\bar{q}(\theta) > q^*(\theta)$ for $\theta_{\min} \leq \theta < \hat{\theta}$ and $q^*(\theta)$ for $\hat{\theta} \leq \theta \leq \theta_{\max}$. Low-demand types buy both products, but obtain the same payoff as if they had opted for exclusive contracts. High-demand types purchase the same quantities as in the non-linear pricing equilibrium. Non-exclusive prices are lower than in the non-linear pricing equilibrium,[21] and are sufficiently low that buyers prefer to purchase both goods even though the firms offer cheaper exclusive contracts.

For some intuitive insight into Proposition 1, notice first of all that setting $P_i^E(q)=0$ is always a best response to $P_{-i}^E(q)=0$. This follows from the fact that with exclusive contracts

[20] If there were multiple non-linear pricing equilibria, for each there would exist a corresponding equilibrium with exclusive contracts.

[21] Since equilibrium quantities are greater with exclusive contracts, marginal prices must be lower. In other words, whenever the price schedules are differentiable we must have $\frac{dP^{NE}(q)}{dq} \leq \frac{dP^*(q)}{dq}$, with a strict inequality for $q<\hat{q}$. Since $P(0)=0$, it follows that $P^{NE}(q) < P^*(q)$ for $q>0$.

firms compete in utility space, where their products are effectively homogeneous.[22] In fact, in our baseline model there can be no equilibrium with exclusive prices above marginal costs where some buyers accept exclusive contracts: the standard Bertrand logic would then imply that firms try to undercut each other, driving exclusive prices to cost.

Now suppose that both firms offer $P^E(q) = 0$, and consider what the non-exclusive prices should be. Given the exclusive prices, buyers have a strictly positive and type-dependent reservation utility $U^E(\theta)$. In this case, the strategic choice of non-exclusive prices is similar to a standard game of non-linear pricing, except that now there is a type-dependent "participation constraint." That is, firms can extract positive rents only if buyers obtain at least $U^E(\theta)$.

Condition **R1** guarantees that the reservation utility is weakly convex in the sense of Maggi and Rodriguez-Clare (1995) and Jullien (2000). This implies that the participation constraint is binding for low-demand, but not high-demand buyers. The equilibrium non-exclusive price schedules must then comprise two branches: a lower branch for buyers who obtain exactly the reservation utility $U^E(\theta)$, and an upper branch for those who obtain strictly more.

By construction, the lower branch must be $\bar{P}^{NE}(q)$. Firms here make positive profits by extracting the value of product variety. To see why firms have no incentive to deviate, note that a unilateral price increase would violate the participation constraint, inducing buyers to opt for a zero-profit exclusive contract. In the case of a unilateral price cut, the participation constraint is no longer a concern. But when a firm prices at $\bar{P}^{NE}(q)$, this is below the equilibrium price in the absence of the participation constraint, i.e. $P^*(q)$. If the non-linear pricing equilibrium is "stable," a firm's unconstrained best response should then be to price above the rival, not below. Condition **R2** guarantees such stability.[23]

[22] The argument is familiar from models of one-stop shopping where exclusivity is postulated exogenously: see, for instance, Armstrong and Vickers (2001) and Rochet and Stole (2002).

[23] Stability would actually require that function (5) be decreasing not only when $q(\theta) = \bar{q}(\theta)$, but also for any $q(\theta) \geq q^*(\theta)$. However, the weaker condition **R2** suffices for our purposes. In a model of linear pricing,

Consider now the upper branch of the non-exclusive price schedule. The key idea is that if the participation constraint does not bind for $\theta > \hat{\theta}$, the equilibrium for high-demand types must be independent of whatever happens to low-demand buyers – except possibly for a constant transfer that ensures that the participation constraint actually does not bind. We shall refer to this property as "type consistency." The intuitive reason why type consistency must hold is that if there were a profitable deviation from the upper branch of the non-exclusive price schedule, this would also be a profitable deviation (*modulo* a constant term) in the non-linear pricing game without the participation constraint. But this contradicts the assumption that $P^*(q)$ is an equilibrium of that game.

This argument implies that the equilibrium quantities for high-demand buyers, who obtain more than their reservation utility $U^E(\theta)$, must be the same as in the non-linear pricing equilibrium. Consequently, the upper branch of the non-exclusive price schedules can differ from $P^*(q)$ only by a constant term. It is easy to see that the constant must be negative – a fixed subsidy. The reason is that when the constant is nil firms earn more on the upper than on the lower branch, and accordingly have an incentive to bribe buyers into the upper branch with a subsidy. The incentive is eliminated only when the non-exclusive price schedule is continuous.

In conclusion, let us note that when the products are functionally independent the exclusive contract equilibrium does not reproduce the monopoly solution. This is because even where the products are functionally independent, the demands for them are correlated. When firms can make their prices conditional on their rival's sales volume, this correlation by itself creates a strategic link between the two markets.

2.4 Reducing the intensity of competition

We have seen that in a non-cooperative equilibrium firms can coordinate their non-exclusive prices so as to extract the buyers' preference for variety. If they succeed, however, a new

the stability condition would be (with obvious notation) $\frac{dp_i}{dp_{-i}} < 1$. This condition is routinely assumed and typically requires only mild restrictions on demand.

opportunity for coordination arises. Since buyers do not to choose exclusive contracts, firms have no definite incentive to undercut one another's exclusive prices. This opens up the possibility that there may exist equilibria with supra-competitive exclusive prices. Multiple equilibria arise because while the exclusive contracts are not accepted, they do affect the equilibrium outcome. The less aggressively firms bid for exclusivity, the lower the reservation utility $U^E(\theta)$, and hence the greater the payments firms can obtain for non-exclusive contracts.[24]

What degree of coordination among firms is consistent with a non-cooperative equilibrium? In other words, supposing that firms agree to request positive payments in their exclusive contracts, when is the agreement sustainable? To answer, assume that both firms offer an exclusive price schedule $P^E(q)$ such that $q^E(\theta) = \arg\max_q \left[u(q,0,\theta) - P^E(q)\right] > 0$. This guarantees to buyers a type-dependent "reservation utility" $U^E(\theta) = \max_q \left[u(q,0,\theta) - P^E(q)\right]$. (Both $q^E(\theta)$ and $U^E(\theta)$ now depend on $P^E(q)$, but for ease of notation this is not indicated.) Again let $\bar{P}^{NE}(q)$ be the non-exclusive price schedule that generates the same net utility as $P^E(q)$. This is still given by functional equation (3), where now, however, $U^E(\theta)$ and hence $\bar{P}^{NE}(q)$ depend on $P^E(q)$. The exclusive prices $P^E(q)$ are *sustainable* if for all $\theta > \theta_{\min}$

$$P^E\left[q^E(\theta)\right] \leq \bar{P}^{NE}\left[\bar{q}(\theta)\right]. \quad (6)$$

The economic interpretation of condition (6) is that if both firms offer the price schedules $P^E(q)$ and $\bar{P}^{NE}(q)$, the buyer θ is indifferent between $\bar{q}(\theta)$ units of both goods and $q^E(\theta)$ units of one good. An arbitrarily small discount would then induce buyers to opt for exclusivity. The exclusive price schedule $P^E(q)$ is sustainable if this move is not profitable. For example, when u is separable as in (4), from the envelope theorem one immediately gets $\bar{q}(\theta) = \frac{1}{2}q^E(\theta)$,

[24] Similarly, under complete information firms can submit an entire price schedule even if only one contract will be accepted. This also typically results in multiple equilibria. Uncertainty mitigates or even fully solves the indeterminacy problem. With non-linear pricing, for instance, the equilibrium price schedules are fully specified, as each contract is chosen by some buyers. However, the problem re-emerges with exclusive contracts, since firms offer two price schedules, and hence many contracts will inevitably not be accepted.

and hence $\bar{P}^{NE}(q) = \frac{1}{2}\left[\ell(q) + P^{E}(2q)\right]$. In view of (6), then, an exclusive price schedule is sustainable if and only if $P^{E}(q) \leq \ell\left(\frac{q}{2}\right)$.

Proposition 2 *Let $P^*(q)$ be a symmetric non-linear pricing equilibrium and $P^E(q)$ any sustainable exclusive price schedule such that **R1** and **R2** hold. When firms compete with exclusive contracts, there exists a symmetric equilibrium in which both firms offer both the exclusive price schedule $P^E(q)$ and the non-exclusive price schedule*

$$P^{NE}(q) = \begin{cases} \bar{P}^{NE}(q) & \text{for } 0 \leq q \leq \hat{q} \\ \\ P^*(q) - P^*(\hat{q}) + \bar{P}^{NE}(\hat{q}) & \text{for } \hat{q} \leq q \leq q^{fb}(\theta_{\max}), \end{cases}$$

where $\bar{P}^{NE}(q)$ is the solution to functional equation (3) for the given schedule $P^E(q)$.

As long as they are sustainable, higher exclusive prices are clearly associated with less intense competition: first, the equilibrium non-exclusive prices depend positively on $P^E(q)$; second, both $\hat{\theta}$ and $\hat{q}$ decrease as the exclusive prices $P^E(q)$ increase; third, the equilibrium quantities $\bar{q}(\theta)$ decrease for $\theta_{\min} \leq \theta < \hat{\theta}$. Equilibrium quantities are independent of $P^E(q)$ for $\theta > \hat{\theta}$, but these buyers obtain a subsidy that is larger, the higher $P^E(q)$.

Example. — When the utility function is separable as in (4), one can parametrize the degree of competition by introducing a variable, μ, ranging from 0 to 1. The equilibrium exclusive price schedule then is $P^E(q) = \mu\ell\left(\frac{q}{2}\right)$, and the lower branch of the corresponding non-exclusive price schedule is $\bar{P}^{NE}(q) = \frac{1}{2}(1+\mu)\ell(q)$.

With this parametrization, the greater μ, the less intense competition. The most competitive equilibrium (i.e., that of Proposition 1) recurs when $\mu = 0$. The upper bound $\mu = 1$ corresponds to the most "cooperative" equilibrium, where profits are maximized and buyers' surplus is minimum. For any degree of competition, however, the equilibrium has the same structure: low-demand types purchase the quantities $\bar{q}(\theta)$, which now depend on μ, and high-demand types $q^*(\theta)$.

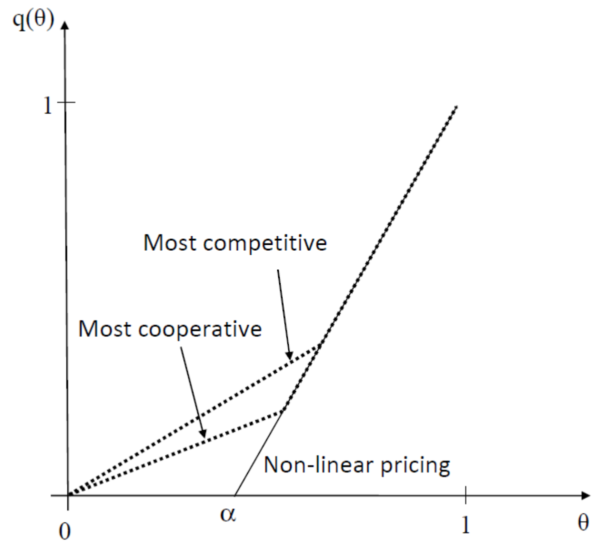

Figure 1: Equilibrium quantities with exclusive contracts ($\gamma = \frac{1}{4}$).

In the quadratic-uniform model, the equilibrium price schedules can be calculated explicitly using (2) and the formula for the value of product variety: $\ell(q) = (1-2\gamma)\, q^2$. We have $P^E(q) = \frac{1}{4}\mu(1-2\gamma)q^2$ and $\bar{P}^{NE}(q) = \frac{1}{2}(1+\mu)(1-2\gamma)q^2$. The non-exclusive price schedules are first convex (meaning that buyers pay quantity premia) and then concave (so firms offer quantity discounts eventually, as under non-linear pricing). The switching point is $\hat{q} = \frac{\alpha}{\alpha+(1-2\gamma)(1+\mu)}$ and $\hat{\theta} = [1+(1+\mu)(1-2\gamma)]\,\hat{q}$, and the fixed subsidy granted to high-demand buyers is $\frac{\alpha}{2}\hat{q}$. Equilibrium quantities are (see Figure 1)

$$q(\theta) = \begin{cases} \dfrac{\theta}{1+(1+\mu)(1-2\gamma)} & \text{for } 0 \le \theta \le \hat{\theta} \\ \\ \dfrac{\theta-\alpha}{1-\alpha} & \text{for } \hat{\theta} \le \theta \le 1. \end{cases} \tag{7}$$

Welfare comparison. — Although there are multiple equilibria, for the Pareto undominated equilibria of Propositions 1 and 2 the comparison with the non-linear pricing equilibrium is largely unambiguous. As noted, non-exclusive prices are always lower than under non-linear pricing, and hence buyers' surplus is strictly greater. Equilibrium quantities are larger and are everywhere closer to the first best. In particular, low-demand buyers ($\theta < \hat{\theta}$) increase their purchases, and the market is covered. High-demand buyers ($\theta \geq \hat{\theta}$) purchase the same quantities as in the non-linear pricing equilibrium (but they too are better off as they now obtain a fixed subsidy). Since the social surplus u (i.e., the sum of buyers' surplus and firms' profits) is concave in q, it follows that exclusive contracts increase the social surplus.[25] Yet even in the most competitive equilibrium (i.e., when $P^E(q) = 0$) the efficient solution is not attained, for the non-exclusive prices are still supra-competitive.

The effect of exclusive contracts on profits is harder to determine in general. The fact that they lower prices is evidently bad for profits when the goods are substitutes, but where they are complements the economic intuition is ambiguous, as the equilibrium prices with non-linear pricing are higher than monopoly prices due to the problem of Cournot complements. In the quadratic-uniform model, however, algebraic calculations show that exclusive contracts always lower profits. To summarize:

Corollary 1 *Exclusive contracts intensify the competition between firms. For any sustainable exclusive price schedule, they lower prices, increase buyers' surplus and social welfare, and (in the quadratic-uniform model) decrease profits.*[26]

3 Market-share discounts

Up to this point we have assumed that a firm can make its prices conditional on whether or not the customer buys from a competitor, but not on the quantity bought. Now we

[25] When buyers are downstream firms and not consumers, the social surplus does not coincide with the standard notion of social welfare (unless downstream firms fully capture consumers surplus). However, lower upstream prices will generally translate into lower downstream prices, so final consumers presumably also benefit from exclusive contracts.

[26] When firms fail to coordinate their pricing strategies, exclusive contracts intensify competition even more strongly. However, to the extent that exclusive contracts are accepted in equilibrium, competition becomes disruptive, and the effect on buyers' surplus and social welfare may be ambiguous. For example, in the equilibrium where only exclusive contracts are offered low-demand types benefit from exclusive contracts, but high-demand ones are harmed.

consider the case in which that quantity is observable, permitting market-share discounts. In this case, each firm i can request a payment $P_i(q_i, q_{-i})$ that depends not only on its own quantity, q_i, but also on its rival's, q_{-i}.[27]

When P_i may depend freely on q_{-i}, it might seem redundant to allow firms to submit a separate exclusive price schedule. However, the schedule $P_{-i}(q_{-i}, q_i)$ must satisfy the condition $P_{-i}(0, q_i) = 0$. If $P_{-i}(q_{-i}, q_i)$ is separable and the component that depends on q_i is strictly positive, as proves to be the case in equilibrium, $P_{-i}(q_{-i}, q_i)$ will be discontinuous at $q_{-i} = 0$. One must then allow firm i to respond discontinuously at $q_{-i} = 0$, thus effectively offering a separate exclusive price schedule $P_i^E(q_i)$. For notational convenience, we therefore assume that firms may submit two different price schedules, $P_i^E(q_i)$ for $q_{-i} = 0$ and $P_i^{NE}(q_i, q_{-i})$ for $q_{-i} > 0$.

We focus on equilibria in which firms coordinate pricing strategies so as to extract the buyers' preference for variety. Let us define

$$u^+(q_A, q_B, \theta) = u\left(\min\{q_A, q_B\}, \min\{q_A, q_B\}, \theta\right). \tag{8}$$

Intuitively, $u^+(q_A, q_B, \theta)$ treats the goods as perfect complements, but with symmetric prices it generates the same total demand as the original function $u(q_A, q_B, \theta)$. Let $P^+(q)$ be a symmetric non-linear pricing equilibrium of the hypothetical game in which the buyers' benefit is $u^+(q_A, q_B, \theta)$ rather than $u(q_A, q_B, \theta)$, and let $q^+(\theta)$ denote the corresponding equilibrium quantities. We have $q^+(\theta) \leq q^*(\theta)$ for $\theta < \theta_{\max}$ and $q^+(\theta_{\max}) = q^*(\theta_{\max})$, so $q^+(\theta_{\max})$ must increase with θ more steeply than $q^*(\theta_{\max})$. This implies that the regularity condition **R1** becomes even milder:

R′1. *The curves $\bar{q}(\theta)$ and $q^+(\theta)$ intersect only once.*

Let θ^+ and q^+ be implicitly defined by $q^+ = q^+(\theta^+) = \bar{q}(\theta^+)$.

[27] In practice, firms may have only imperfect observation of the customer's total purchases. For example, they may only be able to tell whether theis share is above or below some critical threshold. In such cases, the ability to make P_i conditional on q_{-i} is correspondingly limited. How this might affect our results is an interesting issue for future work.

Proposition 3 *Let $P^{+}(q)$ be a symmetric non-linear pricing equilibrium of the game where the utility function is $u^{+}(q_A, q_B, \theta)$, and let $P^{E}(q)$ be any sustainable exclusive price schedule such that **R′1** and **R2** hold. When firms compete with market-share discounts, there exists a symmetric equilibrium in which both firms offer both the exclusive price schedule $P^{E}(q)$ and the non-exclusive price schedule*

$$P^{NE}(q_i, q_{-i}) = \begin{cases} \frac{1}{2}\bar{P}^{NE}(q_i) + \frac{1}{2}\bar{P}^{NE}(q_{-i}) & \text{for } 0 \le q_i, q_{-i} \le q^{+} \\ \\ \frac{1}{2}P^{+}(q_i) + \frac{1}{2}P^{+}(q_{-i}) - P^{+}(q^{+}) + \bar{P}^{NE}(q^{+}) & \text{for } q^{+} \le q_i, q_{-i} \le q^{fb}(\theta_{\max}), \end{cases}$$

where $\bar{P}^{NE}(q)$ is given by (3) for the given schedule $P^{E}(q)$.

These equilibria have several properties practically in common with the exclusive contracts equilibria. As before, consumers can opt for exclusive contracts, obtaining a type-dependent reservation utility $U^{E}(\theta)$ that depends on the exclusive prices $P^{E}(q)$. The existence of multiple equilibria has the same intuitive explanation as under exclusive contracts. Firms now offer many more contracts that are not accepted (i.e., all those with $q_i \neq q_{-i}$),[28] but the upper bound on exclusive prices is still set by (6). The non-exclusive price schedule again has two branches, depending on whether buyers obtain exactly their reservation utility or more. The lower branch is specified by the condition that low-demand buyers purchase both products but obtain exactly $U^{E}(\theta)$.[29] Thus, for low types equilibrium quantities are the same as under exclusive contracts. However, high-demand buyers now purchase the same quantities as in a hypothetical non-linear pricing equilibrium where the goods are perfect complements. This means that the new equilibrium quantities chosen by high-demand buyers are less than $q^{*}(\theta)$ – unless, of course, the goods *are* perfect complements. Another

[28] The non-exclusive price schedule in Proposition 3 actually specifies only some of these contracts. When $0 \le q_i \le q^{+}$ and $q^{+} \le q_{-i} \le 1$, or vice versa, one can imagine that prices are arbitrarily large.

[29] Within the limits of the total payment that a firm can request, it may elect to charge a positive price for its own product or for its rival's. The equal division of the total payment into a part attributable to q_i and a part to q_{-i} postulated in Proposition 3 preserves symmetry but is irrelevant to the payoff.

interesting property of the equilibria is that each firm charges the same average and marginal price on its own product as on its rival's.

These two properties are in fact closely related. With market-share discounts, each firm can effectively tax the other's product. This creates an externality similar to that arising when the goods are perfect complements. In both cases, the total price paid by the buyer is the sum of the prices charged by two separate firms, each of which, acting non-cooperatively, does not fully internalize the demand repercussions of raising its own prices. This produces an effect of Cournot complements for strategic reasons, irrespective of the goods' functional complementarity or substitutability.

In particular, starting from the non-linear pricing equilibrium, firm i always has an incentive to charge a small positive price on good $-i$, since this negatively affects only the revenue of firm $-i$. Ultimately, both firms must charge the same marginal price for each good, as they have exactly the same incentive to increase each. (With positive marginal costs, firms equalize the price-cost margins, so each firm charges a higher marginal price on its own product, which alone entails a positive marginal cost, than on its rival's).

The quadratic-uniform case. — Again using the variable μ to parametrize the degree of competition, exclusive prices are $P^E(q) = \frac{1}{4}\mu(1-2\gamma)q^2$ and non-exclusive prices are

$$P^{NE}(q_i, q_{-i}) = \begin{cases} \frac{1}{4}(1+\mu)(1-2\gamma)\left(q_i^2 + q_{-i}^2\right) & \text{for } 0 \leq q_i, q_{-i} \leq q^+ \\ \\ \frac{1}{3}\left(q_i + q_{-i}\right) - \frac{1}{6}\left(q_i^2 + q_{-i}^2\right) - \frac{q^+}{3} & \text{for } q^+ \leq q_i, q_{-i} \leq 1, \end{cases}$$

where $q^+ = \frac{2}{5-6\gamma+3\mu(1-2\gamma)}$ and $\theta^+ = \frac{2}{3} + \frac{q^+}{3}$. Equilibrium quantities are (see Figure 2)

$$q(\theta) = \begin{cases} \dfrac{\theta}{1+(1+\mu)(1-2\gamma)} & \text{for } 0 \leq \theta \leq \theta^+ \\ \\ 3\theta - 2 \quad \text{for } \theta^+ \leq \theta \leq 1. \end{cases}$$

Welfare comparison. — The equilibria with market-share discounts may be compared either to the non-linear pricing equilibrium or to the exclusive contracts equilibria. Let us start

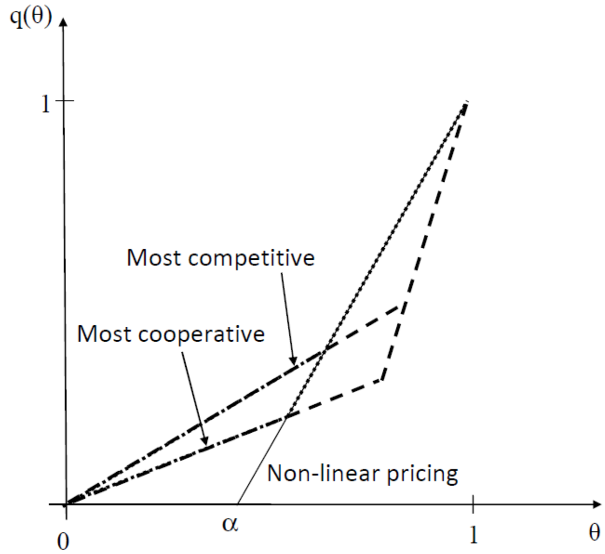

Figure 2: Equilibrium quantities with market-share discounts ($\gamma = \frac{1}{4}$).

with the latter. For any given sustainable exclusive price schedule, the equilibrium quantities of low-demand buyers do not change, but those of high-demand buyers decrease. This follows from the double-marginalization effect and the fact that the non-linear pricing equilibrium prices are highest, and quantities smallest, when the goods are perfect complements. Thus, low-demand buyers are not affected by market-share discounts, but high-demand buyers are harmed. Based on these observations, it is simple to trace the effects of market-share discounts:

Corollary 2 *If exclusive contracts are permitted, allowing firms to offer market-share discounts as well increases prices and reduces quantities, buyers' surplus, and social welfare.*

However, taking the non-linear pricing equilibrium as benchmark, market-share discounts have not only a double-marginalization effect, but also the pro-competitive effects of exclusive contracts discussed above. Accordingly, the comparison is ambiguous. Low-demand buyers, who would be excluded in the non-linear pricing equilibrium, clearly benefit from market-share discounts. The same is true of intermediate types whose quantities increase, or who are more than compensated for the lower quantities by the subsidy they receive.

However, high-demand buyers might lose out with market-share discounts. Even taking the most competitive equilibrium, which makes the comparison more favorable to market-share discounts, some high-demand buyers necessarily lose if the goods are sufficiently close substitutes (in the quadratic-uniform model, the condition is $\gamma > \frac{1}{5}$).[30]

4 Shopping costs

We have shown that exclusive contracts reduce prices and market-share discounts increase them. However, while contracts involving market-share discounts are always accepted in equilibrium, until now we have found that exclusive dealing contracts are accepted only if there are coordination failures among the firms. We now consider two extensions of the model in which, even when firms coordinate, exclusive contracts are accepted but have the same qualitative effects as in the baseline model. To shorten the exposition, we take the quadratic-uniform specification, but the results hold more generally.

We begin here by assuming that buyers face shopping costs, i.e. any extra cost or inconvenience entailed in dealing with two firms rather than one. For final consumers, shopping costs may include the time, trouble and expense of visiting two shops rather than one. For downstream firms, they could include the cost of designing the final product so that both intermediate goods can be used. Following Armstrong and Vickers (2010) and Chen and Rey (2012), we model shopping costs as a fixed cost $z > 0$ borne by the buyers who purchase both products.

Non-linear pricing. — Like exclusivity discounts, shopping costs make it more attractive for

[30]In the quadratic-uniform model, the ambiguity disappears by comparing buyers' *expected* surplus and welfare and focusing on the most competitive equilibrium ($\mu = 0$). That is, market-share discounts increase buyers' surplus and social welfare vis-à-vis the non-linear pricing equilibrium, and decrease expected profits. When $\mu > 0$, however, even the *ex ante* comparison is ambiguous. When μ and γ are sufficiently large, market-share discounts reduce buyers' surplus and social welfare and increase profits vis-à-vis the non-linear pricing equilibrium (see the working paper version of this paper for details).

buyers to purchase one product only. It is the contention of this paper that this should intensify competition, reducing prices and profits, even if exclusive contracts are not permitted. This is confirmed by Proposition 4 below.

To state the proposition, let us define $\tilde{\theta}_z$ as the type for whom the gain from purchasing both products is just equal to the shopping cost z; that is:

$$u\left[q^*(\tilde{\theta}_z), q^*(\tilde{\theta}_z), \tilde{\theta}_z\right] - u\left[q^*(\tilde{\theta}_z), 0, \tilde{\theta}_z\right] = z,$$

where $q^*(\theta) = \frac{\theta-\alpha}{1-\alpha}$ is the equilibrium quantity in the non-linear pricing game without shopping costs. Clearly, $\tilde{\theta}_z$ is an increasing function of z. Assume that z is small enough that $\tilde{\theta}_z \leq \theta_{\max}$.[31]

Proposition 4 *In the uniform-quadratic model with shopping costs, there exists a non-linear-pricing symmetric equilibrium where both firms offer the price schedule*

$$P(q) = \max\left\{0, P^*(q) - P^*\left[q^*(\tilde{\theta}_z)\right]\right\},$$

where $P^*(q) = \alpha q - \frac{\alpha}{2}q^2$.

Shopping costs shift the equilibrium price schedule down by a fixed amount $P^*\left[q^*(\tilde{\theta}_z)\right]$. As a result, prices initially reflect the marginal cost, but then at $q^*(\tilde{\theta}_z)$ the price schedule has a kink, and the price-cost margin starts to increase. The upper part of the price schedule applies to those buyers who keep on purchasing both products and who therefore take the same quantities as in the absence of shopping costs, $q^*(\theta)$. Low-demand buyers ($\theta \leq \tilde{\theta}_z$), however, now purchase only one product. The quantity purchased is $q^E(\theta) = \arg\max_q u(q, 0, \theta)$, which is strictly increasing in θ, when $\theta \leq \hat{\theta}_z$, where $\hat{\theta}_z$ is implicitly defined by the condition $q^E(\hat{\theta}_z) = q^*(\tilde{\theta}_z)$. For $\hat{\theta}_z \leq \theta \leq \tilde{\theta}_z$, by contrast, all buyers purchase the largest quantity offered at zero price, i.e. $q^*(\tilde{\theta}_z)$. Naturally, given the kink in the price schedule, the equilibrium exhibits bunching.

[31] If this inequality is reversed, shopping costs are so high that the model effectively becomes a one-stop shopping model where the price-cost margin vanishes.

Intuitively, for buyers to be willing trade with both firms, they must get a benefit $u(q, q, \theta) - 2P(q)$ that covers the shopping cost, z. This "participation constraint" cannot be satisfied for low-demand buyers, so they only purchase one product and the competition for them effectively becomes a competition for exclusive deals, which must drive prices to marginal costs. For high-demand buyers, by contrast, the participation constraint is satisfied; the result then follows from the property of type consistency discussed above. Compared to the case with no shopping costs, these buyers obtain a fixed subsidy, as they now have a more attractive "outside option," namely to buy $q^*(\tilde{\theta}_z)$ at zero price. The total subsidy may even be greater than the shopping cost, in which case buyers actually gain from shopping costs. Firms, by contrast, are harmed as competition intensifies, confirming that under nonlinear pricing, shopping costs (and the consequent threat of exclusivity) increase the degree of competition.

Exclusive contracts. — We now show that competition becomes fiercer still if firms can also offer exclusive contracts. (The equilibria can be simply adapted to account for market-share discounts, just as we showed in Section 3.)

Let $\hat{\theta}$ and $\hat{q}$ be implicitly defined by the condition $\hat{q} = q^*(\hat{\theta}) = \frac{1}{2}q^E(\hat{\theta})$, as in Section 2, and now let q^0 be implicitly defined by the condition $\ell(q^0) = z$, where $\ell(q) = (1 - 2\gamma)\, q^2$. Clearly, q^0 is an increasing function of z. Assume that z is small enough so that $q^0 \leq \hat{q}$.[32] We then have:

Proposition 5 *When firms compete with exclusive contracts in the presence of shopping costs z, for any $\mu \in [0, 1]$ there exists a symmetric equilibrium where both firms offer the exclusive price schedule*

$$P^E(q) = \max\left\{0, \mu\left[\ell\left(\frac{q}{2}\right) - z\right]\right\}$$

[32] If $q^0 > \hat{q}$, exclusive contracts are irrelevant, and the equilibrium is still given by Proposition 4.

and the non-exclusive price schedule

$$P^{NE}(q) = \begin{cases} 0 \quad \textit{for } 0 \leq q \leq q^0 \\ \frac{1}{2}(1+\mu)\left[\ell\left(q\right)-z\right] \quad \textit{for } q^0 \leq q \leq \hat{q} \\ P^*(q) - \frac{1}{2}(1+\mu)\left[\ell\left(\hat{q}\right)-z\right] \quad \textit{for } \hat{q} \leq q \leq q^{fb}(\theta_{\max}), \end{cases}$$

where $\ell(q) = (1-2\gamma)\,q^2$ *and* $P^*(q) = \alpha q - \frac{\alpha}{2}q^2$.

Contrasting these equilibria with the non-linear pricing equilibrium of Proposition 4, it is clear that exclusive contracts have the same qualitative effects with as without shopping costs. For any value of μ, they intensify the competition between the firms, decreasing prices and profits, and increasing buyers' surplus and social welfare. Compared to the case without shopping costs, the main difference is that the preference for variety is now $\ell\left(q,\theta\right)-z$ and so is positive only if θ is large enough. This implies that exclusive contracts are now accepted in equilibrium by those buyers who effectively do not have any preference for variety, even if firms coordinate their pricing strategies.[33]

5 Asymmetry of firms

So far we have always posited symmetric firms in symmetric equilibria. Now we briefly examine the case of a dominant firm and a smaller competitor. A simple way to introduce firm heterogeneity is to assume that firms have different marginal costs. Let A be the more efficient firm, normalize its marginal cost c_A to zero, and denote $c_B = c \geq 0$. Thus, the parameter c captures the degree of asymmetry between the firms.[34]

[33] These are the types $\theta \leq \theta^0$, where θ^0 is implicitly defined by $q^E(\theta^0) = 2q^0$. Notice that if θ is low enough that $q^E(\theta) < q^0$, the buyer could pick up a non-exclusive contract as well, since quantities up to q^0 are offered at competitive prices even without an exclusivity clause. However, when $q^0 < q^E(\theta) \leq 2q^0$ buyers must necessarily choose an exclusive contract.

[34] The parameter c may also capture demand asymmetries. For example, one could add a linear term $(\alpha_A q_A + \alpha_B q_B)$ to the function u, where $\alpha_A - \alpha_B$ can be interpreted as an index of vertical product differentiation. Setting $c_A - \alpha_A = 0$ and $c = c_B - \alpha_B \geq 0$ one returns to the formulation used in the text.

Many of the qualitative effects of exclusive contracts remain similar to the case of symmetric firms. For example, when firms offer only exclusive contracts and do not coordinate pricing strategies at all, competition in utility space drives prices to marginal cost: $P_A^E(q) = P_B^E(q) = cq$. The more efficient firm now captures the entire market by slightly undercutting its rival and makes positive profits. If c is low, the less efficient firm is excluded inefficiently, since it supplies a differentiated product for which there would be positive demand in the absence of exclusive clauses.

As under symmetry, however, firms can coordinate their non-exclusive prices so as to extract buyers' preference for variety. Firms might also coordinate their exclusive prices, but for brevity we consider only equilibria in which exclusive prices remain $P_A^E(q) = P_B^E(q) = cq$ (i.e., the counterpart of the equilibrium of Proposition 1). This guarantees that buyers will have a type-dependent "reservation utility" $U^E(\theta) = \max_{q_A} \left[u(q_A, 0, \theta) - cq_A \right]$. In equilibrium, buyers are again divided into two groups: low-demand buyers get exactly their reservation utility, high-demand buyers strictly more. Similarly, each non-exclusive price schedule will consist of two branches. To match the reservation utility, prices in the lower branches must be lower than in the non-linear pricing equilibrium. By type consistency, the upper branches must be the same as in the non-linear pricing equilibrium, except for a fixed subsidy.

Overall, prices are therefore lower and quantities greater than in the non-linear pricing equilibrium. Thus, exclusive contracts intensify competition and improve efficiency. In particular, with non-linear pricing the no-distortion-at-the-top property is preserved even when firms are asymmetric. This immediately implies that in the absence of economies of scale, the less efficient firm is excluded only when this is socially efficient.[35] The same property must then hold when exclusive contracts are offered, since high-demand buyers purchase the same quantities as in the non-linear pricing equilibrium.

[35] That is, firm B is excluded only if c is greater than the marginal social value of product B when the efficient quantity of product A is being purchased, $u_{q_B}(q^{fb}(\theta_{\max}), 0, \theta_{\max})$.

The most significant difference is that when firms are asymmetric one can no longer invoke symmetry to pin down the lower branches of the non-exclusive price schedules. But under symmetry, the symmetric equilibrium luckily has two other nice distinguishing properties: continuous equilibrium quantities and maximization of the preference for variety extracted from low-demand buyers.

These properties can serve as selection criteria in the case of asymmetry. To illustrate, consider the case of independent goods in the quadratic-uniform model (i.e. $\gamma = 0$).[36] When $c = 0$, the condition that low-demand buyers must obtain exactly $U^E(\theta)$ (which is now $\frac{\theta^2}{2}$) implies $\bar{q}_A(\theta) + \bar{q}_B(\theta) = \theta$; symmetry then implies $\bar{q}_A(\theta) = \bar{q}_B(\theta) = \frac{\theta}{2}$, yielding the equilibrium characterized in Proposition 1. However, there is in fact a continuum of asymmetric equilibria, with $\bar{q}_A(\theta) = \lambda\theta$ and $\bar{q}_B(\theta) = (1-\lambda)\theta$ for $\lambda \in [0, 1]$. In these equilibria the low-demand buyers' preference for variety is divided unevenly between the firms. But in fact, the more asymmetric the division, the lower the preference for variety that can be extracted. In the limiting cases $\lambda = 0$ or $\lambda = 1$, for instance, low-demand buyers enjoy no product variety so there is nothing to be extracted and firms earn no profit on these buyers. Moreover, in asymmetric equilibria individual quantities are discontinuous at $\theta = \hat{\theta}$ (where $\hat{\theta}$ is now implicitly defined by the condition that $\bar{q}_A(\hat{\theta}) + \bar{q}_B(\hat{\theta}) = 2q^*(\hat{\theta})$); only the total quantity is everywhere continuous. In the symmetric equilibrium, by contrast, individual quantities are continuous, and the preference for variety extracted from low-demand buyers is greatest (it turns out that these two properties are in fact equivalent).

Now consider the case $c > 0$. Postulating $c < \frac{1}{2}$ (otherwise firm B will not be active), the non-linear pricing equilibrium price schedules are $P_A^*(q_A) = \frac{1}{2}q_A - \frac{1}{4}q_A^2$ and $P_B^*(q_B) = \frac{1}{2}(q_B + c) - \frac{1}{4}q_B^2$, and the corresponding quantities are $q_A^*(\theta) = 2\theta - 1$ and $q_B^*(\theta) = 2\theta -$

[36] This example is particularly simple, since with non-linear pricing there are effectively two separate monopolies. With exclusive contracts, however, the correlation in the demand for the two products creates a strategic link between the two markets, which implies that firms no longer behave like two separate monopolies.

$1-c$. With exclusive contracts, still assuming that firms charge competitive exclusive prices $P_A^E(q) = P_B^E(q) = cq$, buyers have a reservation utility of $U^E(\theta) = \frac{(\theta-c)^2}{2}$, which low-demand buyers must obtain exactly. As in the symmetric case, however, this condition determines only the sum of equilibrium quantities, i.e. $\bar{q}_A(\theta) + \bar{q}_B(\theta) = \theta - c$.

Since we can no longer invoke symmetry to pin down a unique equilibrium, it seems natural to consider the equilibrium in which quantities are continuous and the preference for variety extracted from low-demand buyers (net of production costs) is greatest. Either of these conditions implies $\bar{q}_A(\theta) - \bar{q}_B(\theta) = c$. The corresponding equilibrium quantities are $\bar{q}_A(\theta) = \frac{\theta}{2}$ and $\bar{q}_B = \frac{\theta}{2} - c$. However, these quantities are now chosen only by types $\theta \in [2c, \frac{2}{3}]$. For $\theta \in [c, 2c]$, buyers choose the exclusive contracts offered by firm A, buying $q_A^E(\theta) = \theta - c$ units of product A only and so obtaining their reservation utility $U^E(\theta) = \frac{(\theta-c)^2}{2}$. (For $\theta > \frac{2}{3}$ buyers purchase the same quantities as in the non-linear pricing equilibrium, and for $\theta < c$ buyers are excluded as their reservation utility vanishes.) As in the baseline model, competition is more intense with exclusive contracts, even if those contracts now are accepted by some buyers.

6 Conclusion

We have modeled an industry in which firms compete in exclusive contracts and market-share discounts with incomplete information about demand. We have found that exclusive contracts lower prices and market-share discounts raise them. The intuititive reason for this is that with exclusive contracts firms compete in utility space, where the degree of competition is not lessened by product differentiation, while market-share discounts allow firms to tax one another's product, creating the same double-marginalization effect as in the case of products that are perfect complements. Therefore, there is a clear sense in which exclusive contracts are pro-competitive and market-share discounts anti-competitive.

However, the normative implications of this finding are less clear. With economies of scale, fiercer competition may lower the weaker firm's profit below the level that would justify continued operation, so the stronger firm may actually gain from exclusive contracts, which can thus serve as a market foreclosure mechanism. And the exclusion of the weaker firm may be socially inefficient, since its gross profits are typically lower than its marginal contribution to social welfare when buyers obtain informational rents.

Further, competition with exclusive contracts can be disruptive. When firms do not coordinate their pricing strategies, buyers benefit from low prices but lose product variety. This effect may be compounded with that of scale economies: dominant firms with exclusionary intents may reject coordination, preferring to enforce a bad equilibrium where only exclusive contracts are offered. If this is so, then exclusive contracts might harm both the non-dominant firm and buyers.

However, a more benign interpretation is also possible. That is, firms may simply be caught in a prisoner's dilemma, each one having its own unilateral incentive to offer exclusive contracts even if both would gain if they were prohibited; any coordination failures may be involuntary. The fact that the same equilibria are open to different interpretations greatly complicates the specification of any practical legal standard.

One significant limitation of the analysis is the assumption that heterogeneity is one-dimensional. This restricts the possible demand patterns in several ways. For example, it implies that low-demand buyers are more inclined than high-demand buyers to abandon either firm's product and excludes Hotelling-style product differentiation. Relaxing these assumptions is an important task for future work. Multi-dimensional heterogeneity certainly complicates the analysis, but it might also reduce the number of possible equilibria.

Appendix

This Appendix provides the proofs of Propositions 1-3. Propositions 4 and 5 are proved in a separate Annex. We begin with the following

Lemma 1 *For any sustainable $P^E(q)$, the function $\bar{q}(\theta)$ is strictly increasing and satisfies $0 < \bar{q}(\theta) < q^{fb}(\theta)$ for all $\theta > \theta_{\min}$.*

Proof. For any sustainable $P^E(q)$, by the sorting condition the function $u(q,0,\theta) - P^E(q)$ has strictly increasing differences in q and θ. This implies that $q^E(\theta) = \arg\max_q \left[u(q,0,\theta) - P^E(q)\right]$ is strictly increasing in θ (and hence almost everywhere differentiable). Next, using the envelope theorem (3) can be rewritten as

$$u_\theta\left[\bar{q}(\theta), \bar{q}(\theta), \theta\right] \equiv u_\theta\left[q^E(\theta), 0, \theta\right]. \tag{A1}$$

Given $P^E(q)$ and hence $q^E(\theta)$, this identity implicitly defines the function $\bar{q}(\theta)$. Totally differentiate (A1) to get

$$2u_{\theta q}\left[\bar{q}(\theta), \bar{q}(\theta), \theta\right]\frac{d\bar{q}}{d\theta} + u_{\theta\theta}\left[\bar{q}(\theta), \bar{q}(\theta), \theta\right] \equiv u_{\theta q}\left[q^E(\theta), 0, \theta\right]\frac{dq^E}{d\theta} + u_{\theta\theta}\left[q^E(\theta), 0, \theta\right].$$

We have $u_{\theta\theta}\left[\bar{q}(\theta), \bar{q}(\theta), \theta\right] = u_{\theta\theta}\left[q^E(\theta), 0, \theta\right]$ by the assumption that $u_{\theta\theta q} = 0$ (in fact, the weaker assumption $u_{\theta\theta}\left[\bar{q}(\theta), \bar{q}(\theta), \theta\right] \leq u_{\theta\theta}\left[q^E(\theta), 0, \theta\right]$ would suffice). Since $\frac{dq^E}{d\theta} > 0$ and $u_{\theta q} > 0$, it follows that

$$\frac{d\bar{q}}{d\theta} \equiv \frac{1}{2}\frac{u_{\theta q}\left[q^E(\theta), 0, \theta\right]}{u_{\theta q}\left[\bar{q}(\theta), \bar{q}(\theta), \theta\right]}\frac{dq^E}{d\theta} > 0.$$

From (A1), the sorting condition $u_{\theta q} > 0$ and the fact that $q^E(\theta) > 0$ for $\theta > \theta_{\min}$, it follows immediately that $\bar{q}(\theta) > 0$ for $\theta > \theta_{\min}$. It remains to show that $\bar{q}(\theta) < q^{fb}(\theta)$. Denote by $\bar{\theta}(q)$ the inverse of $\bar{q}(\theta)$ and by $\theta^E(q)$ the inverse of $q^E(\theta)$. Also denote, with a slight abuse of notation, $q^E(q) \equiv q^E\left[\bar{\theta}(q)\right]$. We then have:

$$\bar{P}^{NE}(q) = \frac{1}{2}\left\{u\left[q, q, \bar{\theta}(q)\right] - u\left[q^E(q), 0, \theta^E(q^E(q))\right] + P^E\left[q^E(q)\right]\right\}.$$

It is then immediate to verify that $\frac{d\bar{P}^{NE}(q)}{dq} > 0$, which implies that $\bar{q}(\theta) < q^{fb}(\theta)$. ■

Proposition 1 is a special case of Proposition 2 in which $P^E(q) = 0$. The proof is accordingly contained in the proof of Proposition 2.

Proof of Proposition 2. We must show that if firm $-i$ offers a sustainable exclusive schedule $P^E(q)$ and the corresponding equilibrium non-exclusive price schedule $P^{NE}(q)$, it is a best response for firm i to offer the same price schedules $\left\{P^E(q), P^{NE}(q)\right\}$.

First consider the non-exclusive price schedule. Given its competitor's strategy, firm i behaves like a monopolist facing a buyer with a suitably defined indirect utility function and reservation utility. The indirect utility function is the maximum utility that buyer θ can obtain by purchasing q_i and then trading optimally with firm $-i$. If $q_i = 0$, the buyer must choose an exclusive contract from firm $-i$. By construction, the optimal choice is $q^E(\theta)$, which gives a net reservation utility of $U^E(\theta)$. However, if $q_i > 0$ the indirect utility function is (henceforth omitting the index i)

$$v(q,\theta) = \max_x \left[u(q,x,\theta) - P^{NE}(x)\right].$$

By construction, $v(q,\theta)$ is continuous and almost everywhere differentiable. At any point where the derivative exists, by the envelope theorem we have $v_\theta(q,\theta) = u_\theta(q,\tilde{x}(q,\theta),\theta) \geq 0$, where $\tilde{x}(q,\theta) = \arg\max_x \left[u(q,x,\theta) - P^{NE}(x)\right]$.

Obviously, firm i's profits vanish when $q_i = 0$. In maximizing its profits, firm i can therefore proceed as if the buyer had the utility function $v(q,\theta)$ and a type-dependent reservation utility $U^E(\theta)$. This maximization problem, which we call problem $\mathcal{P}\left[v(q,\theta), U^E(\theta)\right]$, can be stated as follows:

$$\begin{aligned} \max_{P(q)} \pi_i &= \int_{\theta_{\min}}^{\theta_{\max}} P[q(\theta)] f(\theta) d\theta \\ \text{s. t. } q(\theta) &= \arg\max_q \left[v(q,\theta) - P(q)\right] \\ \max_q \left[v(q,\theta) - P(q)\right] &\geq U^E(\theta). \end{aligned}$$

We can restate this optimal control problem using $q(\theta)$ as our control variable and $U(\theta) = \max_q \left[v(q,\theta) - P(q)\right]$ as the corresponding state variable. The problem is then to maximize $\int_{\theta_{\min}}^{\theta_{\max}} \left[v(q(\theta),\theta) - U(\theta)\right] f(\theta) d\theta$ subject to $U(\theta) \geq U^E(\theta)$.

To proceed, following Armstrong *et al.* (1994) we can simply guess the solution and use sufficiency arguments to show that we are right. Our guess is

$$q^{NE}(\theta) = \begin{cases} \bar{q}(\theta) & \text{if } \theta_{\min} \leq \theta \leq \hat{\theta} \\ q^*(\theta) & \text{if } \hat{\theta} \leq \theta \leq \theta_{\max}. \end{cases}$$

with the associated utility $U^{NE}(\theta) = v\left[q^{NE}(\theta), \theta\right] - P^{NE}\left[q^{NE}(\theta)\right]$. Theorem 1 in Seierstad and Sydsaeter (1987 p. 317) implies that a sufficient condition for a maximum is that there exists a continuous and piecewise differentiable function $\xi(\theta)$ such that $\xi'(\theta) \geq 0$ and $\xi'(\theta)\left[U(\theta) - U^E(\theta)\right] \equiv 0$, and such that $q^{NE}(\theta)$ and $U^{NE}(\theta)$ maximize the Lagrangean

$$\mathcal{L} = \int_{\theta_{\min}}^{\theta_{\max}} \left\{ \left[v(q(\theta), \theta) - U(\theta) \right] f(\theta) + \xi'(\theta) \left[U(\theta) - U^E(\theta) \right] \right\} d\theta$$

over all functions $q(\theta)$ and $U(\theta)$. Integrating the second and fourth term inside curly brackets by parts and normalizing $\xi(\theta_{\max})$ to zero, we get

$$\mathcal{L} = \int_{\theta_{\min}}^{\theta_{\max}} \left\{ v(q(\theta), \theta) f(\theta) - \left[1 - F(\theta)\right] v_\theta(q(\theta), \theta) - \xi(\theta) v_\theta(q(\theta), \theta) - \xi'(\theta) U^E(\theta) \right\} d\theta.$$

Consider the following function $\xi(\theta)$:

$$\tilde{\xi}(\theta) = \begin{cases} \frac{v_q[\bar{q}(\theta), \theta]}{v_{\theta q}[\bar{q}(\theta), \theta]} f(\theta) - \left[1 - F(\theta)\right] & \text{if } \theta_{\min} \leq \theta \leq \hat{\theta} \\ 0 & \text{if } \hat{\theta} \leq \theta \leq \theta_{\max}. \end{cases}$$

By the regularity condition **R2**, we have $\tilde{\xi}'(\theta) \geq 0$. Setting $\xi(\theta) = \tilde{\xi}(\theta)$, we can rewrite $\mathcal{L}$ as $\mathcal{L} = \mathcal{A} + \mathcal{B}$, where:

$$\begin{aligned} \mathcal{A} &= \int_{\theta_{\min}}^{\hat{\theta}} \left\{ v(q(\theta), \theta) f(\theta) - \left[1 - F(\theta)\right] v_\theta(q(\theta), \theta) - \tilde{\xi}(\theta) v_\theta(q(\theta), \theta) - \tilde{\xi}'(\theta) U^E(\theta) \right\} d\theta \\ \mathcal{B} &= \int_{\hat{\theta}}^{\theta_{\max}} \left\{ v(q(\theta), \theta) f(\theta) - \left[1 - F(\theta)\right] v_\theta(q(\theta), \theta) \right\} d\theta. \end{aligned}$$

To prove that our guess is correct, we must show: (i) that $\bar{q}(\theta)$ maximizes $\mathcal{A}$; (ii) that $q^*(\theta)$ maximizes $\mathcal{B}$; and (iii) that $\tilde{\xi}(\theta)$ is everywhere continuous.

(i) When $\xi(\theta) = \tilde{\xi}(\theta)$, $\mathcal{A}$ becomes

$$\int\limits_{\theta_{\min}}^{\hat{\theta}} \left\{ \left[v(q(\theta),\theta) - \frac{v_q\left[\bar{q}(\theta),\theta\right]}{v_{\theta q}\left[\bar{q}(\theta),\theta\right]} v_\theta(q(\theta),\theta) \right] f(\theta) - \tilde{\xi}'(\theta) U^E(\theta) \right\} d\theta.$$

It is immediate to verify that $\bar{q}(\theta)$ maximizes $\mathcal{A}$ pointwise noting that the derivative of the term inside square brackets with respect to q vanishes at $q = \bar{q}(\theta)$.

(ii) By definition, the non-linear pricing equilibrium quantities $q^*(\theta)$ and the associated equilibrium price schedule $P^*(q)$ maximize a firm's profit given the pricing strategy of its rival in the non-linear pricing gane. That is, they solve the problem

$$\begin{aligned} \max_{P(q)} \pi_i &= \int\limits_{\theta_{\min}}^{\theta_{\max}} P[q(\theta)] f(\theta) d\theta \\ \text{s. t. } q(\theta) &= \arg\max_q \left[v^*(q,\theta) - P(q) \right] \\ \max_q \left[v^*(q,\theta) - P(q) \right] &\geq v^*(0,\theta). \end{aligned}$$

where

$$v^*(q,\theta) = \max_x \{ u(q,x,\theta) - P^*(x) \}$$

is the indirect utility function in the non-linear pricing game. Once again, we can restate this optimal control problem using $U(\theta)$ as our state variable and $q(\theta)$ as the corresponding control variable. In this case the firm's objective becomes to maximize

$$\int\limits_{\theta_{\min}}^{\theta_{\max}} \left[v^*(q(\theta),\theta) - U(\theta) \right] f(\theta) d\theta$$

subject to $U(\theta) \geq v^*(0,\theta)$. As in Martimort and Stole (2009), this latter constraint is never binding and so can be neglected. Integrating by parts, the objective becomes

$$\int\limits_{\theta_{\min}}^{\theta_{\max}} \left\{ v^*(q(\theta),\theta) f(\theta) - \left[1 - F(\theta)\right] v^*_\theta(q(\theta),\theta) \right\} d\theta.$$

By construction, $q^*(\theta)$ maximizes this function. In fact, even if the solution may involve ironing and bunching, $q^*(\theta)$ must maximize pointwise any function $\int\limits_{\tilde{\theta}}^{\theta_{\max}} \left\{ v^*(q,\theta) f(\theta) - \left[1 - F(\theta)\right] v^*_\theta(q,\theta) \right\} d\theta$

such that $\frac{dq^*(\theta)}{d\theta} > 0$ at $\theta = \tilde{\theta}$. Since $q^*(\theta)$ intersects $\bar{q}(\theta)$ from below at $\theta = \hat{\theta}$ and $\bar{q}(\theta)$ is strictly increasing (by Lemma 1), it follows that $q^*(\theta)$ is strictly increasing at $\theta = \hat{\theta}$. This implies that $q^*(\theta)$ maximizes $\mathcal{B} \int_{\hat{\theta}}^{\theta_{\max}} \{v^*(q(\theta), \theta) f(\theta) - [1 - F(\theta)]\, v^*_\theta(q(\theta), \theta)\}\, d\theta$ pointwise with respect to $q(\theta)$.

(iii) Since by construction $\bar{q}(\hat{\theta}) = q^*(\hat{\theta})$, it follows that $\xi(\theta)$ is everywhere continuous.

This shows that $P^{NE}(q)$ is a best response to $\{P^E(q), P^{NE}(q)\}$.

To complete the proof, it remains to show that $P^E(q)$ is a best response to $\{P^E(q), P^{NE}(q)\}$. Consider any possible deviation $\tilde{P}^E(q)$. If $\tilde{P}^E(q) > P^E(q)$ for some q, then buyers will simply disregard those contracts and the deviating firm will not earn any extra profit. If instead $\tilde{P}^E(q) < P^E(q)$ for some q, then some types of buyer will accept the deviating firm's offers. However, since $P^E(q)$ is sustainable the deviating firm's profit is lower than the equilibrium profit. This finally shows that $\{P^E(q), P^{NE}(q)\}$ is a best response to $\{P^E(q), P^{NE}(q)\}$.■

Proof of Proposition 3. The proof is similar to that of Proposition 2. In fact, for the lower part of the non-exclusive price schedule, which is unchanged, it is identical. Accordingly, only the upper part requires proof here.

Given its competitor's strategy, with market-share discounts firm i behaves like a multi-product monopolist that supplies both products and faces a buyer whose utility function is the difference between the gross utility u and firm $-i$'s non-exclusive price schedule:

$$v(q_i, q_{-i}, \theta) = u(q_i, q_{-i}, \theta) - P^{NE}_{-i}(q_i, q_{-i}).$$

Proceeding as before, we must show that the quantities $q(\theta) = q^+(\theta)$ maximize $\mathcal{B}$, where now

$$\mathcal{B} = \int_{\theta^+}^{\theta_{\max}} \{v[q(\theta), q(\theta), \theta] f(\theta) - [1 - F(\theta)]\, v_\theta[q(\theta), q(\theta), \theta]\}\, d\theta.$$

Integrating by parts and substituting the expression for the indirect utility function, $\mathcal{B}$ can be rewritten as

$$\mathcal{B}= \int_{\theta^+}^{\theta_{\max}} \left\{ u(q_i, q_{-i}, \theta) f(\theta) - P_{-i}^{NE}(q_i, q_{-i}) f(\theta) - [1 - F(\theta)]\, u_\theta(q_i, q_{-i}, \theta) \right\} d\theta.$$

By symmetry, the solution must be symmetric. Thus, it suffices to show that $q^+(\theta)$ maximizes

$$\tilde{\mathcal{B}} = \int_{\theta^+}^{\theta_{\max}} \left\{ u(q, q, \theta) f(\theta) - P^{NE}(q, q) f(\theta) - [1 - F(\theta)]\, u_\theta(q, q, \theta) \right\} d\theta.$$

To show that $q^+(\theta)$ maximizes $\tilde{\mathcal{B}}$, recall that by definition $q^+(\theta)$ is the equilibrium quantity in a hypothetical non-linear pricing game in which the utility function is $u^+(q_A, q_B, \theta)$. Consider the indirect utility function in such a hypothetical game, i.e.:

$$v^+(q, \theta) = \max_x \{u^+(q, x, \theta) - P^+(x)\}.$$

Given the strict complementarity of the goods, the solution must involve $x = q_i$. This implies

$$v^+(q, \theta) = u^+(q, q, \theta) - P^+(q).$$

The non-linear pricing equilibrium quantity $q^+(\theta)$ must then be a solution of the following program

$$\max_{q(\theta), U^+(\theta)} \int_{\theta_{\min}}^{\theta_{\max}} \left[v^+(q(\theta), \theta) - U(\theta) \right] f(\theta) d\theta$$

subject to $U^+(\theta) \geq v^+(0, \theta)$ where

$$U^+(\theta) = \max_q \left[v^+(q, \theta) - P^+(q) \right].$$

As before, the participation constraint is not binding. Integrating by parts and substituting the expression for $v^+(q, \theta)$, the objective function becomes

$$\int_{\theta_{\min}}^{\theta_{\max}} \left\{ u(q, q, \theta) f(\theta) - P^+(q) f(\theta) - [1 - F(\theta)]\, u_\theta(q, q, \theta) \right\} d\theta.$$

But if $q^+(\theta)$ maximizes this function, it must also maximize $\tilde{\mathcal{B}}$, since in the upper branch $P^{NE}(q, q)$ equals $P^+(q)$ plus a term that does not depend on q (the possibility of ironing and bunching is dealt with as in the proof of Proposition 2). ■

References

[1] Aghion, P. and P. Bolton (1987), "Contracts as a barrier to entry," *American Economic Review* 77, 388-401.

[2] Ahlborn, C. and D. Bailey (2008), "Discounts, rebates and selective pricing by dominant firms: a trans-Atlantic comparison," *European Competition Journal* 2, 101–143.

[3] Armstrong, M. (2010), "Bundling revisited: substitute products and inter-firm discounts," MPRA Paper No. 26782.

[4] Armstrong, M., Cowan S. and J. Vickers (1994), "Nonlinear pricing and price cap regulation," *Journal of Public Economics* 58, 33-56.

[5] Armstrong, M. and J. Vickers (2001), "Competitive price discrimination," *Rand Journal of Economics* 32, 579-605.

[6] Armstrong, M. and J. Vickers (2010), "Competitive nonlinear pricing and bundling," *Review of Economics Studies* 77, 30-60.

[7] Bernheim, D. and M. Whinston (1998), "Exclusive dealing," *Journal of Political Economy* 106, 64-103.

[8] Chen, Z. and P. Rey (2012), "Loss leading, exploitation, and retailer seller power," *American Economic Review*, forthcoming.

[9] Chone, P. and L. Linnemer (2011), "Leaving the door ajar: non-linear pricing by a dominant firm," CREST working paper.

[10] Fumagalli, C. and M. Motta (2006), "Exclusive dealing and entry when buyers compete," *American Economic Review* 96, 785-795.

[11] Inderst, R. and G. Shaffer (2010), "Market-share contracts as facilitating practices," *Rand Journal of Economics* 41, 709-729.

[12] Jullien, B. (2000), "Participation constraints in adverse selection models," *Journal of Economic Theory* 93, 1-47.

[13] Kobayashi, B. (2005), "The economics of loyalty discounts and antitrust law in the United States," *Competition Policy International*, 1, 115-148.

[14] Maggi, G. and A. Rodriguez-Clare (1995), "On countervailing incentives," *Journal of Economic Theory* 66, 238-263.

[15] Majumdar, A. and G. Shaffer, (2009), "Market-share contracts with asymmetric information," *Journal of Economics and Management Strategy* 18, 393-421.

[16] Martimort, D. and L. Stole (2009), "Market participation in delegated and intrinsic common-agency games," *Rand Journal of Economics* 40, 1, 78-102.

[17] Matthewson, F. and R. Winter (1984), "The competitive effect of vertical agreements: Comment," *American Economic Review* 77, 1057-1062.

[18] Mills, D. (2010), "Inducing downstream selling effort with market-share discounts," *International Journal of the Economics of Business* 17, 129-139.

[19] O'Brien, D., and G. Shaffer (1997), "Nonlinear supply contracts, exclusive dealing, and equilibrium market foreclosure," *Journal of Economics and Management Strategy* 6, 755-785.

[20] Rasmusen, E., Ramseyer, J. and J. Wiley (1991), "Naked exclusion," *American Economic Review* 81, 1137-1145.

[21] Rochet, J-C. and L. Stole (2002), "Nonlinear pricing with random participation," *Review of Economic Studies* 69, 277-311.

[22] Segal, I. and M. Whinston (2000a), "Naked exclusion: Comment," *American Economic Review* 90, 296-309.

[23] Seierstad, A. and K. Sydsæter (1987), Optimal Control Theory with Economic Applications. Amsterdam: North Holland.

[24] Shubik, M. and R. Levitan (1980), *Market structure and behavior*, Cambridge, MA: Harvard University Press.

[25] Yehezkel, Y. (2008), "Retailers' choice of product variety and exclusive dealing under asymmetric information," *Rand Journal of Economics* 39, 115-143

[26] Whiston, M. (2008), *Lectures on antitrust economics*, Cambridge, MA: Mit Press.

Annex
(not for publication)

This Annex contains the proofs of Propositions 4 and 5.

Proof of Proposition 4. Given its rival's strategy, firm i behaves like a monopolist facing a buyer with a suitably defined indirect utility function and reservation utility. The reservation utility is obtained when $q_i = 0$. In this case, the buyer does not pay the shopping cost and so obtains

$$v(0,\theta) = \max_x \left[u(0,x,\theta) - \max\left\{0, P^*(x) - P^*\left[q^*(\tilde{\theta}_z)\right]\right\}\right].$$

The indirect utility function is the maximum utility buyer θ can obtain by purchasing $q_i > 0$ and then trading optimally with firm $-i$. We must distinguish between two cases. If the buyer prefers not to buy from firm $-i$, his indirect utility function is simply $v(q,\theta) = u(q,0,\theta)$. Notice that this can never exceed $v(0,\theta)$ if $q \leq q^*(\tilde{\theta}_z)$. If instead the buyer purchases a positive quantity of good $-i$, his indirect utility function is

$$v(q,\theta) = \max_{x>0} \left[u(q,x,\theta) - \max\left\{0, P^*(x) - P^*\left[q^*(\tilde{\theta}_z)\right]\right\} - z\right].$$

For future reference, we notice that

$$v(q^*(\tilde{\theta}_z), \tilde{\theta}_z) = u\left[q^*(\tilde{\theta}_z), q^*(\tilde{\theta}_z), \tilde{\theta}_z\right] - z$$

since $q^*(\tilde{\theta}_z)$ is offered at zero price. By definition, $u(q^*(\tilde{\theta}_z), 0, \tilde{\theta}_z) = u\left[q^*(\tilde{\theta}_z), q^*(\tilde{\theta}_z), \tilde{\theta}_z\right] - z$ which implies

$$u(q^*(\tilde{\theta}_z), 0, \tilde{\theta}_z) = v(q^*(\tilde{\theta}_z), \tilde{\theta}_z).$$

In other words, at $q_i = q^*(\tilde{\theta}_z)$ and $\theta = \tilde{\theta}_z$, the buyer is exactly indifferent between purchasing $q^*(\tilde{\theta}_z)$ of good $-i$ and not purchasing good $-i$ at all. Clearly, then, any buyer $\theta < \tilde{\theta}_z$ would strictly prefer not to buy product $-i$. In other words, $v\left[q^*(\tilde{\theta}_z), \theta\right] = u(q^*(\tilde{\theta}_z), 0, \theta)$ for

$\theta < \tilde{\theta}_z$.Next consider the case $q_i = q^E(\theta) = \arg\max_q u(q,0,\theta)$. Recall that by definition $q^E(\hat{\theta}_z) = q^*(\tilde{\theta}_z)$. This means that no buyer $\theta < \hat{\theta}_z$ would purchase a positive quantity of product $-i$ when $q_i = q^E(\theta)$. That is, $v\left[q^E(\theta),\theta\right] = u(q^E(\theta),0,\theta)$ for $\theta < \hat{\theta}_z$.

Obviously, firm i's profits vanish when $q_i = 0$. In maximizing its profits, firm i can therefore proceed as if the buyer had the utility function $v(q,\theta)$ and a type-dependent reservation utility $v(0,\theta)$. This maximization problem can be stated as follows:

$$\begin{aligned} \max_{P(q)} \pi_i &= \int_0^1 P[q(\theta)]d\theta \\ \text{s. t. } q(\theta) &= \arg\max_q [v(q,\theta) - P(q)] \\ \max_q [v(q,\theta) - P(q)] &\geq v(0,\theta). \end{aligned}$$

We can restate this optimal control problem using $q(\theta)$ as our control variable and $U(\theta) = \max_q [v(q,\theta) - P(q)]$ as the corresponding state variable. The problem then becomes to maximize $\int_0^1 [v(q(\theta),\theta) - U(\theta)]\, d\theta$ subject to $U(\theta) \geq v(0,\theta)$.

As in the proofs of Proposition 2 and 3, we simply guess the solution and use sufficiency arguments to show that we have guessed correctly. Our guess now is

$$\tilde{q}(\theta) = \begin{cases} q^E(\theta) = \arg\max_q u(q,0,\theta) & \text{for} 0 \leq \theta \leq \hat{\theta}_z \\ q^*(\tilde{\theta}_z) \text{ for } \hat{\theta}_z \leq \theta \leq \tilde{\theta}_z \\ q^*(\theta) \quad \text{for } \tilde{\theta}_z \leq \theta \leq 1, \end{cases}$$

where $\hat{\theta}_z$ is implicitly defined by the condition $q^E(\hat{\theta}_z) = q^*(\tilde{\theta}_z)$, with the associated utility $\tilde{U}(\theta) = v[\tilde{q}(\theta),\theta] - \max\left\{P^*[\tilde{q}(\theta)] - P^*\left[q^*(\tilde{\theta}_z)\right],0\right\}$. Theorem 1 in Seierstad and Sydsaeter (1987 p. 317) implies that a sufficient condition for this to be a maximum is that there exists a continuous and piecewise differentiable function $\xi(\theta)$ such that $\xi'(\theta) \geq 0$ and $\xi'(\theta)[U(\theta) - v(0,\theta)] \equiv 0$, and such that $\tilde{q}(\theta)$ and $\tilde{U}(\theta)$ maximize the Lagrangean

$$\mathcal{L} = \int_0^1 \{[v(q(\theta),\theta) - U(\theta)] + \xi'(\theta)[U(\theta) - v(0,\theta)]\}\, d\theta.$$

Integrating by parts and normalizing $\xi(\theta_{\max})$ to zero, we get

$$\mathcal{L} = \int_0^1 \left[v(q(\theta), \theta) - (1-\theta) v_\theta(q(\theta), \theta) - \xi(\theta) v_\theta(q(\theta), \theta) - \xi'(\theta) v(0, \theta) \right] d\theta.$$

Now consider the following function $\xi(\theta)$:

$$\tilde{\xi}(\theta) = \begin{cases} -(1-\theta) \quad \text{for } 0 \leq \theta \leq \hat{\theta}_z \\ \frac{v_q(q^*(\tilde{\theta}_z),\theta)}{v_{q\theta}(q^*(\tilde{\theta}_z),\theta)} - (1-\theta) \quad \text{for } \hat{\theta}_z \leq \theta \leq \tilde{\theta}_z \\ 0 \quad \text{for } \tilde{\theta}_z \leq \theta \leq 1. \end{cases}$$

Since $v\left[q^*(\tilde{\theta}_z), \theta\right] = u(q^*(\tilde{\theta}_z), 0, \theta)$ when $\hat{\theta}_z \leq \theta \leq \tilde{\theta}$, we have $v_q(q^*(\tilde{\theta}_z), \theta) = \theta - (1-\gamma) q^*(\tilde{\theta}_z)$ and $v_{q\theta}(q^*(\tilde{\theta}_z), \theta) = 1$. Clearly, this implies that $\tilde{\xi}'(\theta) \geq 0$ everywhere. By setting $\xi(\theta) = \tilde{\xi}(\theta)$, we can rewrite $\mathcal{L}$ as $\mathcal{L} = \mathcal{A} + \mathcal{B} + \mathcal{C}$, where:

$$\begin{aligned} \mathcal{A} &= \int_0^{\hat{\theta}_z} \left[v(q(\theta), \theta) - (1-\theta) v_\theta(q(\theta), \theta) - \tilde{\xi}(\theta) v_\theta(q(\theta), \theta) - \tilde{\xi}'(\theta) v(0, \theta) \right] d\theta \\ \mathcal{B} &= \int_{\hat{\theta}_z}^{\tilde{\theta}_z} \left[v(q(\theta), \theta) - (1-\theta) v_\theta(q(\theta), \theta) - \tilde{\xi}(\theta) v_\theta(q(\theta), \theta) - \tilde{\xi}'(\theta) v(0, \theta) \right] d\theta \\ \mathcal{C} &= \int_{\tilde{\theta}_z}^{1} \left[v(q(\theta), \theta) - (1-\theta) v_\theta(q(\theta), \theta) \right] d\theta. \end{aligned}$$

To prove that our guess is correct, we must show that (i) $q^E(\theta)$ maximizes $\mathcal{A}$; (ii) $q^*(\tilde{\theta}_z)$ maximizes $\mathcal{B}$, (iii) $q^*(\theta)$ maximizes $\mathcal{C}$; (iv) $\tilde{\xi}(\theta)$ is everywhere continuous.

(i) When $\xi(\theta) = -(1-\theta)$, $\mathcal{A}$ becomes

$$\int_0^{\hat{\theta}_z} \left[v(q(\theta), \theta) - \tilde{\xi}'(\theta) v(0, \theta) \right] d\theta.$$

Since $v\left[q^E(\theta), \theta\right] = u(q^E(\theta), 0, \theta)$, it is immediate to verify that the derivative of the term inside square brackets with respect to q vanishes at $q = q^E(\theta)$, so $q^E(\theta)$ pointwise maximizes $\mathcal{A}$.

(ii) When $\xi(\theta) = \frac{v_q(q^*(\tilde{\theta}_z),\theta)}{v_{q\theta}(q^*(\tilde{\theta}_z),\theta)} - (1-\theta)$, $\mathcal{B}$ becomes

$$\int_{\hat{\theta}_z}^{\tilde{\theta}_z} \left[v(q(\theta),\theta) - \frac{v_q\left[q^*(\tilde{\theta}_z),\theta\right]}{v_{\theta q}\left[q^*(\tilde{\theta}_z),\theta\right]} v_\theta(q(\theta),\theta) - \tilde{\xi}'(\theta) v(0,\theta) \right] d\theta.$$

It is immediate to verify that the derivative of the term inside square brackets with respect to q vanishes at $q = q^*(\tilde{\theta}_z)$, so $q^*(\tilde{\theta}_z)$ pointwise maximizes $\mathcal{B}$.

(iii) By definition, the non-linear pricing equilibrium quantities $q^*(\theta)$ and the associated equilibrium schedule $P^*(q)$ maximize a firm's profit given the pricing strategy of its rival in the non-linear pricing game without shopping costs. That is, they solve the problem

$$\begin{aligned} \max_{P(q)} \pi_i &= \int_0^1 P[q(\theta)] d\theta \\ \text{s. t. } q(\theta) &= \arg\max_q \left[v^*(q,\theta) - P(q)\right] \\ \max_q \left[v^*(q,\theta) - P(q)\right] &\geq v^*(0,\theta). \end{aligned}$$

where

$$v^*(q,\theta) = \max_x \{u(q,x,\theta) - P^*(x)\}$$

is the indirect utility function in the non-linear pricing game without shopping costs. Once again, we can restate this optimal control problem using $U(\theta)$ as our state variable and $q(\theta)$ as the corresponding control variable. Using these variables, the firm's objective becomes to maximize

$$\int_0^1 \left[v^*(q(\theta),\theta) - U(\theta)\right] f(\theta) d\theta$$

subject to $U(\theta) \geq v^*(0,\theta)$. As in Martimort and Stole (2009), this latter constraint is never binding and so can be neglected. Integrating by parts the objective becomes

$$\int_0^1 \left[v^*(q(\theta),\theta) - (1-\theta) v^*_\theta(q(\theta),\theta)\right] d\theta.$$

By construction, $q^*(\theta)$ maximizes this function. In fact, since $\frac{dq^*(\theta)}{d\theta} > 0$, $q^*(\theta)$ must pointwise maximizes any function $\int_{\tilde{\theta}}^1 \left[v^*(q(\theta),\theta) - (1-\theta) v^*_\theta(q(\theta),\theta)\right] d\theta$. This implies that $q^*(\theta)$ pointwise maximizes $\mathcal{C}$ with respect to $q(\theta)$.

(iii) Since by construction $q^E(\hat{\theta}_z) = q^*(\tilde{\theta}_z)$, we have $v_q(q^*(\tilde{\theta}_z), \theta) = 0$, which implies that $\tilde{\xi}(\theta)$ is continuous at $\theta = \hat{\theta}_z$. Again by contruction, $\frac{v_q(q^*(\tilde{\theta}_z),\tilde{\theta}_z)}{v_{q\theta}(q^*(\tilde{\theta}_z),\tilde{\theta}_z)} = (1-\tilde{\theta}_z)$, which implies that $\tilde{\xi}(\theta)$ is continuous at $\theta = \tilde{\theta}_z$. It follows that $\xi(\theta)$ is everywhere continuous. This completes the proof of the Proposition. ■

Proof of Proposition 5

The proof is similar to that of Proposition 2, except that the preference for variety is now effectively $\ell(q, \theta) - z$. This implies that for sufficiently low types there is no preference for variety to be extracted, and hence both exclusive and non-exclusive prices must vanish.

■