On the Optimality of Privacy in Sequential Contracting

May 14, 2004

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MOTIVATION

- Agents often enter a sequence of bilateral relationships

  - consumers procure products from multiple sellers

  - workers hired by several employers

  - entrepreneurs contract with venture capitalists, suppliers, retailers, regulators

  - defense suppliers contract with different administrations

- Two ways an upstream relationship can influence downstream contracting:

  → decisions (**contractual externalities**)

  → information disclosed (**informational externalities**)

- This paper

  (a) environment with contractual and informational externalities

  (b) conditions for the optimality of **privacy** and **information disclosure**

- Two types of disclosure:

→ *endogenous private information* (upstream decisions)

→ *exogenous private information* (learned prior to upstream contracting)

Example: E-commerce

- Trade is not anonymous → personal information is recorded:

  - path in the website
  - choice of product / request of service
  - price
  - advertisement

- Cookies

- Privacy-policy statement → major concern for most e-vendors

- Debate about privacy-protecting laws: US vs Europe

Other examples

- Auctions with resale

- Labor contracts

- Insurance contracts

- Financial contracts

- Private and public procurement

- Multinational regulation

QUESTIONS

- Why and when do parties optimally commit to privacy?
- When do they release information to outsiders?
- What is the influence of disclosure on upstream and downstream contract design?
- Who benefits from information disclosure?

Technically: mechanism design to write contracts that control for

- upstream information flow (screening)

  from the agent → principal

- downstream information flow (signaling)

  from principal → third party (in this paper: another principal)

RELATED LITERATURE

- **Dynamic Common Agency**

  - Baron ('85), Martimort ('99), Bergman & Välimäki ('98), Prat & Rustichini ('98)

- **Vertical Relationships / Contracting with Externalities**

  - Caillaud-Jullien-Picard ('95), Segal ('99), Segal and Whinston ('03)

- **Information Sharing and Certification Intermediaries**

  - Raith ('98), Padilla & Pagano ('98), Lizzeri ('99), Peyrache & Quesada ('03)

- **Privacy Policies**

  - Acquisti & Varian ('02), Calzolari & Pavan ('01), Dodds ('02), Taylor ('02, '03)

- **Auctions followed by downstream strategic interactions**

  - Katzman & Rhodes-Kropf ('02), Zhong ('02), Calzolari & Pavan ('03)

PLAN

1. The model

2. Main theorem: optimal privacy

3. Profitable disclosure

   (1) Direct externalities

   (2) Horizontal differentiation

   (3) Complements

   (4) Substitutes

4. Conclusions

THE MODEL

- **Players**

  Single agent, A, and two principals, P_1 (leader) and P_2 (follower)

  Think of P_1 and P_2 as two differentiated sellers

- **Allocations**

  - *decision* $x_i \in X_i = \{0, 1\}$ $\quad$ $x_i = 0$: status quo $\quad$ $x_i = 1$: decision to trade

  - *monetary transfer* $t_i \in T_i \equiv \mathbb{R}$ paid by A to P_i

- **Preferences**

  $U_A = v_A(x_1, x_2, \theta) - t_1 - t_2$

  $U_i = v_i(x_1, x_2, \theta) + t_i$ for $i = 1, 2$

  $v_A = v_1 = v_2 = 0$ for any $\theta \in \Theta$ if $(x_1, x_2) = (0, 0)$

  Only A knows $\theta \in \Theta = \{\underline{\theta}, \overline{\theta}\}$, with $\Delta\theta \equiv \overline{\theta} - \underline{\theta} > 0$, and $\Pr(\overline{\theta}) = p$

- **Contracts**

  $\phi_2 : \mathcal{M}_2 \mapsto X_2 \times T_2$ $\qquad\qquad$ $\phi_1 : \mathcal{M}_1 \mapsto \Delta(X_1 \times S) \times T_1$

  S: set of signals P_1 discloses to P_2

  $d(s|m_1) = \sum_{x_1 \in X_1} \delta_1(x_1, s|m_1)$: probability P_1 discloses s

  $\delta_1(1|m_1) = \sum_{s \in S} \delta_1(1, s|m_1)$: probability of trade

- **Timing**

  $t = 0 :$ A learns θ

  $t = 1 :$ P_1 announces a public mechanism ϕ_1. If A rejects $\phi_1 \rightarrow$ game over!

  If A accepts, he chooses m_1, pays $t_1(m_1)$ and receives a lottery $\delta_1(x_1, s|m_1)$

  $t = 2 :$ P_2 pays $\tau(\phi_1)$, receives information s w/ prob. $d(s|m_1)$ and offers A

  a mechanism $\phi_2(\phi_1, s)$

  If A rejects $\phi_2 \rightarrow$ game over!

  If A accepts, he chooses m_2 which induces $t_2(m_2)$ and $x_2(m_2)$

- Price for information: $\tau(\phi_1) = \gamma[\mathbb{E}U_2(\phi_1) - \mathbb{E}U_2^{ND}(\phi_1)]$ $\qquad \gamma \in [0, 1]$

CONTRACT DESIGN

→ Revelation principle is not valid

[Epstein & Peters ('99), Martimort & Stole ('02), Peters ('01)]

→ Pavan and Calzolari ('03)

Definition (MDM): *A Markovian direct mechanism is a mechanism in which the message space coincides with the* **extended type space**. *Formally,* $\mathcal{M}_1 = \Theta_1^E \equiv \Theta$, *and* $\mathcal{M}_2 = \Theta_2^E \equiv \Theta \times X_1$

Theorem: (Markovian Revelation Principle)

Let Γ *be any sequential common agency game in which A follows Markov strategies and* Γ_{Θ^E} *a sequential common agency game in which principals are restricted to use MDM.*

An outcome function $\pi : \Theta \to \Delta(X_1 \times T_1 \times X_2 \times T_2)$ *can be sustained as an equilibrium of* Γ **if and only if** *it can be sustained as an equilibrium of* Γ_{Θ^E} *in which A truthfully reports his extended type to each principal with probability one.*

Mechanism design problem for P_2

Let $U_A^2(\theta_2^E, \widehat{\theta}_2^E; s) \equiv v_A\left(x_2(\widehat{\theta}_2^E; s), \theta_2^E\right) - v_A(0, \theta_2^E) - t_2(\widehat{\theta}_2^E; s)$ be the marginal surplus of $\theta_2^E = (\theta, x_1)$ in the downstream relationship when he reports $\widehat{\theta}_2^E = (\widehat{\theta}, \widehat{x}_1)$

Optimal contract for P_2 solves

$$\mathcal{P}_2(s) : \begin{cases} \max_{\phi_2 \in \Phi_2} \sum_{\theta_2^E} [v_2(x_2(\theta_2^E; s), \theta_2^E) + t_2(\theta_2^E; s)] \Pr(\theta_2^E; s) \\ \\ \text{s.t. for } \textit{any } \theta_2^E \text{ and } \widehat{\theta}_2^E \in \Theta_2^E \\ \\ U_A^2(\theta_2^E, s) \equiv v_A(x_2(\theta_2^E; s), \theta_2^E) - v_A(0, \theta_2^E) - t_2(\theta_2^E; s) \geq 0, \\ \\ U_A^2(\theta_2^E; s) \geq U_A^2(\theta_2^E, \widehat{\theta}_2^E; s) \end{cases}$$

Mechanism design problem for P_1

P_1 chooses ϕ_1 and $\phi_2(s)$ so as to solve $\mathcal{P}_1$:

$$\text{Max } \mathbb{E}U_1 = \sum_{\theta}\left\{\sum_{x_1,s}[v_1(x_1,x_2(\theta_2^E;s),\theta)]\delta_1(x_1,s|\theta) + t_1(\theta)\right\}\Pr(\theta)$$

s.t.

$$U_A(\theta) \equiv \sum_{x_1,s}[v_A(x_1,0,\theta) + U_A^2(\theta_2^E;s)]\delta_1(x_1,s|\theta) - t_1(\theta) \geq 0, \text{ for any } \theta \in \Theta,$$

$$U_A(\theta) \geq \sum_{x_1,s}[v_A(x_1,0,\theta) + U_A^2(\theta_2^E;s)]\delta_1(x_1,s|\hat{\theta}) - t_1(\hat{\theta}), \text{ for any } \theta \text{ and } \widehat{\theta} \in \Theta,$$

$$\phi_2(s) \text{ solves } \mathcal{P}_2(s) \text{ for any } s \in S$$

Definition 1:

Mechanism ϕ_1 **discloses information** if and only if it assigns measure to s that lead to different posterior beliefs over Θ_2^E

Disclosure is considered **optimal** for P_1 if and only if $\exists$ a ϕ_1 that discloses information and solves $\mathcal{P}_1$, and there are no other solutions to $\mathcal{P}_1$ that induce no disclosure.

Definition 2:

Independence. *Player i's preferences are independent of* x_j *if* $v_i(x_i,x_j,\theta) = v_i(x_i,\theta)$

(Additive) separability. *Player i's preferences are separable if*

$$v_i(x_1,x_2,\theta) = v_i^1(x_1,\theta) + v_i^2(x_2,\theta)$$

Sign of Single Crossing Condition. *The single crossing condition in the agent's preferences has the same sign for* x_1 and x_2 *if* a *higher (lower)* θ *indicates a higher (lower) valuation for both* x_1 *and* x_2*, i.e.*

$$sign\{\Delta_\theta[\Delta_{x_1} v_A(\mathbf{x},\theta)]\} = sign\{\Delta_\theta[\Delta_{x_2} v_A(\mathbf{x},\theta)]\}$$

MAIN RESULT

Theorem: *Assume the following hold:*

(a) P_1*'s preferences are independent of* x_2;

(b) *the sign of the single crossing condition is the same for* x_1 and x_2;

(c) P_2*'s and A's preferences are separable.*

Then **no disclosure** *is optimal for* P_1 *for any rational price* $\tau_2(\phi_1)$ *that* P_2 *is willing to pay.*

If any of these conditions is violated, there exist preferences for which disclosure is strictly optimal even if P_1 *can not make* P_2 *pay for the information she receives*

Effects of disclosure on $\mathbb{E}[U_1]$

(1) *rent-shifting effect*

(2) *information trade effect*

(3) *upstream revelation effect*

The rent shifting effect

$$v_1(x_1,\theta) = m_1(\theta)x_1$$

$$v_2(x_1,x_2,\theta) = e_2(\theta)x_1 + m_2(\theta)x_2$$

$$v_A(x_1,x_2,\theta) = a(\theta)x_1 + b(\theta)x_2$$

with $\Delta b \equiv b(\overline{\theta}) - b(\underline{\theta}) > 0$ and $\Delta a \equiv a(\overline{\theta}) - a(\underline{\theta}) > 0$

WLOG $W_2(\theta) \equiv m_2(\theta) + b(\theta) > 0$ for any θ

P_2's optimal mechanism consists in setting a price for $x_2 = 1$ equal to

$$t_2(s) = \begin{cases} b(\underline{\theta}) & \text{if } W_2(\underline{\theta}) \geq W_2(\overline{\theta})\Pr(\bar{\theta};s), \\ b(\overline{\theta}) & \text{otherwise} \end{cases}$$

If $W_2(\overline{\theta})p > W_2(\underline{\theta})$, no surplus for A with P_2

P_1 could disclose information according to

$$\underline{\theta}$$

$$\overline{\theta}$$

For $\overline{\delta}$ sufficiently low $\rightarrow t_2(s_2) = b(\underline{\theta})$

$\overline{\theta}$ obtains an expected rent with P_2 equal to $\overline{\delta}\Delta b > 0$

Problem: by announcing $\widehat{\theta} = \underline{\theta}$, $\overline{\theta}$ can get Δb with certainty

To make ϕ_1 incentive compatible P_1 must reduce $\overline{t}_1$ by Δb

Net effect of disclosure on $\overline{t}_1$: $\overline{\delta}\Delta b - \Delta b < 0$

P_1 is better off by keeping all information secret!

Idea of the proof

Given any ϕ_1 that solves $\mathcal{P}_1$ there exists another mechanism ϕ_1^{ND} that induces the same distribution over X_1, it does not disclose information, and s. t.

$$\mathbb{E}U_1(\phi_1) - \mathbb{E}U_1(\phi_1^{ND}) = (1-\gamma)\sum_{\theta\in\Theta}\left[\sum_{s\in S} U_A^2(\theta;s)d(s|\theta) - U_A^{2ND}(\theta)\right]\Pr(\theta) +$$

$$+\gamma\sum_{\theta\in\Theta}\left[\sum_{s\in S} W_2(x_2(\theta;s),\theta)d(s|\theta) - W_2(x_2^{ND}(\theta),\theta)\right]\Pr(\theta) +$$

$$-\sum_{\theta\in\Theta}[U_A(\theta;\phi_1) - U_A(\theta;\phi_1^{ND})]\Pr(\theta) \leq 0.$$

After substituting for the upstream rents

$$\mathbb{E}U_1(\phi_1) - \mathbb{E}U_1(\phi_1^{ND}) = (1-\gamma)p\left[\sum_{s\in S} U_A^2(\overline{\theta};s)d(s|\overline{\theta}) - \sum_{s\in S} U_A^2(\overline{\theta};s)d(s|\underline{\theta})\right] +$$
$$+\gamma\sum_{s\in S}[(1-p)W_2(x_2(\underline{\theta};s),\underline{\theta}) - p\Delta_\theta v_A^2(x_2(\underline{\theta};s),\theta)]d(s|\underline{\theta}) +$$
$$-\gamma[(1-p)W_2(x_2^{ND}(\underline{\theta}),\underline{\theta}) - p\Delta_\theta v_A^2(x_2^{ND}(\underline{\theta}),\theta)]$$

$\sum_{s\in S} U_A^2(\overline{\theta},s)d(s|\overline{\theta}) - \sum_{s\in S} U_A^2(\overline{\theta},s)d(s|\underline{\theta}) \le 0 \rightarrow$ from standard representation theorems

$$x_2(\underline{\theta};s) = \arg\max_{x_2\in X_2}\{\Pr(\underline{\theta};s)W_2(x_2,\underline{\theta}) - \Pr(\overline{\theta};s)\Delta_\theta v_A^2(x_2,\theta)\}$$

$$x_2^{ND}(\underline{\theta}) = \arg\max_{x_2\in X_2}\{(1-p)W_2(x_2,\underline{\theta}) - p\Delta_\theta v_A^2(x_2,\theta)\}$$

SUMMARIZING

- Incentives for truthful revelation are *negatively* affected by disclosure!
- Under (a)-(c) these negative effect offsets both rent shifting and information trade effects
  → optimal policy: full privacy!

REMARKS

- Result extends to $\Theta \equiv [\underline{\theta}, \overline{\theta}]$ and $X_1 = X_2 = \mathbb{R}_+$.
- When $\gamma = 1$, result embeds Baron & Besanko ('84) as a special case.
  Under (a)-(c), the optimal ϕ_2 from P_1's viewpoint is the static optimal contract.
  When preferences are separable, this can be sustained by committing not to disclose information!

PROFITABLE DISCLOSURE

- Disclosure of *exogenous information*: θ

  (1) Direct externalities

  (2) Horizontal differentiation

- Disclosure of *endogenous information*: x_1

  (3) Complements

  (4) Substitutes

- We want to show that disclosure is optimal even if $\tau(\phi_1) = 0$ for any ϕ_1

Disclosure of exogenous private information

1- DIRECT EXTERNALITIES

Assume

$v_1(x_1, x_2, \theta) = m_1 x_1 + e x_2$

$v_2(x_1, x_2, \theta) = m_2 x_2$

$v_A(x_1, x_2, \theta) = a(\theta) x_1 + b(\theta) x_2$ with $\Delta b > 0$ and $\Delta a > 0$

Idea: P_1 may accept to give A extra rents if this leads to more favorable outcomes in downstream contracting

- **Definition**: P_2's prior beliefs are **unfavorable** to A if and only if P_2 asks a high price if she receives no information from P_1, i.e.

$$p(\bar{b} + m_2) > \underline{b} + m_2$$

Favorable beliefs

$$p(\bar{b} + m_2) \leq \underline{b} + m_2$$

- P_2's beliefs can be manipulated through a (noisy) disclosure policy that WLOG has the following structure:

$$\begin{array}{ccccc} \overline{\theta} & \longrightarrow & s_1 & \Rightarrow & t_2(s_1) = \overline{b} \\ & \searrow & & & \\ \underline{\theta} & \longrightarrow & s_2 & \Rightarrow & t_2(s_2) = \underline{b} \end{array}$$

Agent's payoff:

$$U_A(\overline{\theta}) = \delta_1(1|\overline{\theta})\bar{a} + d(s_2|\overline{\theta})\Delta b - t_1(\overline{\theta}),$$
$$U_A(\underline{\theta}) = \delta_1(1|\underline{\theta})\underline{a} - t_1(\underline{\theta}),$$

$\delta_1(1|\theta)$: probability of trade

$d(s|\theta)$: probability of s when A reports θ

As standard, at the optimum constraints ($\underline{IR}_1$) and ($\overline{IC}_1$) bind:

$$\mathbb{E}[U_1] = p\delta_1(1|\overline{\theta})(m_1 + \bar{a}) + (1-p)\delta_1(1|\underline{\theta})\left(m_1 + \underline{a} - \tfrac{p}{1-p}\Delta a\right) +$$
$$+ pe + (1-p)d(s_2|\underline{\theta})e - p[d(s_2|\underline{\theta}) - d(s_2|\overline{\theta})]\Delta b$$

subject to the constraints

$$[\delta_1(1|\overline{\theta}) - \delta_1(1|\underline{\theta})]\Delta a \geq [d(s_2|\underline{\theta}) - d(s_2|\overline{\theta})]\Delta b \qquad (\underline{IC}_1)$$
$$d(s_1|\overline{\theta}) \geq Hd(s_1|\underline{\theta}), \qquad (SR_1)$$
$$d(s_2|\overline{\theta}) \leq Hd(s_2|\underline{\theta}). \qquad (SR_2)$$

where $H \equiv \left(\frac{1-p}{p}\right)\left(\frac{m_2+\underline{b}}{\Delta b}\right)$

Extra cost of disclosure: $\delta_1(1|\underline{\theta}) < 1$

Proposition 1:

If P_2's prior beliefs are **unfavorable** *to A*,

- *disclosure is optimal for* $e > E^* > 0$

- *disclosure is welfare increasing: P_1 and A are strictly better off, P_2 is indifferent*

If P_2's prior beliefs are **favorable**,

- *disclosure is optimal for* $e < E^{**} < 0$

- *disclosure is welfare increasing only for large negative externalities:*
  A is worse off, P_1 better off, P_2 indifferent

Intuition

Assume unfavorable beliefs

Costs of disclosure: (1) larger rent for $\overline{\theta}$: $p[d(s_2|\underline{\theta}) - d(s_2|\overline{\theta})]\Delta b$

(2) inefficient trade: $\delta_1(1|\overline{\theta}) > \delta_1(1|\underline{\theta})$

P_1 is ready to pay these costs if the externality gain $(1-p)d(s_2|\underline{\theta})e$ is large enough!

Effect of disclosure on $U_A(\overline{\theta}) = \delta_1(1|\underline{\theta})\Delta a + d(s_2|\underline{\theta})\Delta b$ can be positive!

Disclosure may result in a Pareto improvement!

2- **HORIZONTAL DIFFERENTIATION**

Suppose θ measures relative preferences for two horizontally differentiated sellers

Formally,

$v_1(x_1,x_2,\theta) = m_1x_1$

$v_2(x_1,x_2,\theta) = m_2x_2$

$v_A(x_1,x_2,\theta) = a(\theta)x_1 + b(\theta)x_2$ with $\Delta b > 0$, BUT $\Delta a < 0$!

By announcing $\bar{\theta}$, A reveals a higher valuation for x_2, but a lower valuation for x_1.

$$\text{Max } \mathbb{E}U_1 = p\{\delta_1(1|\overline{\theta})(m_1 + \overline{a}) + d(s_2|\overline{\theta})\Delta b - U_A(\overline{\theta})\} + \\ + (1-p)\{\delta_1(1|\underline{\theta})(m_1 + \underline{a}) - U_A(\underline{\theta})\}$$

subject to $U_A(\overline{\theta}) \geq 0$, $U_A(\underline{\theta}) \geq 0$, and

$$U_A(\overline{\theta}) \geq U_A(\underline{\theta}) + d(s_2|\underline{\theta})\Delta b - \delta_1(1|\underline{\theta})|\Delta a| \qquad (\overline{IC}_1)$$

$$U_A(\underline{\theta}) \geq U_A(\overline{\theta}) - d(s_2|\overline{\theta})\Delta b + \delta_1(1|\overline{\theta})|\Delta a| \qquad (\underline{IC}_1)$$

Disclosure → Countervailing incentives !

- *Rent shifting* effect: P_1 discloses information to increase A's downstream informational rent.

This never pays when A's valuations for x_1 and x_2 are positively correlated

(increase in downstream surplus more than compensated by the increase in $U_A(\overline{\theta})$).

When x_1 and x_2 are horizontally differentiated,

(+) disclosure helps reducing $\underline{\theta}$'s rent $U_A(\underline{\theta}) = |\Delta a| - d(s_2|\underline{\theta})\Delta b$!

(-) cost of disclosure: reduction in the level of trade with $\overline{\theta}$ required by incentive compatibility.

Proposition 2:

Disclosure is optimal for P_1 if and only if

(i) *P_2's prior beliefs are* **unfavorable** *to A*

(ii) $p\left\{H\Delta b - (1-H)\frac{\Delta b}{|\Delta a|}(m_1 + \overline{a})\right\} + (1-p)\Delta b \geq 0$

Disclosure is welfare increasing if and only if $m_1 + \overline{a} - \frac{1-p}{p}|\Delta a| < 0$

- P_1 strictly benefits from disclosure

- P_2 is indifferent

- A is indifferent if $m_1 + \overline{a} - \frac{1-p}{p}|\Delta a| < 0$ *and worse off otherwise*

Disclosure of endogenous private information

$$v_1(x_1, x_2, \theta) = m_1 x_1$$

$$v_2(x_1, x_2, \theta) = m_2 x_2$$

$$v_A(x_1, x_2, \theta) = a(\theta)x_1 + bx_2 + gx_1x_2, \text{ with } \Delta a \geq 0.$$

3- **Complements**: $g > 0$

$$t_2(s) = \begin{cases} b + g & \text{if } (m_2 + b + g)\Pr(x_1 = 1; s) > m_2 + b, \\ b & \text{if } (m_2 + b + g)\Pr(x_1 = 1; s) \leq m_2 + b. \end{cases}$$

4- **Substitutes**: $g < 0$

$$t_2(s) = \begin{cases} b + g & \text{if } m_2 + b + g \geq (m_2 + b)\Pr(x_1 = 0; s), \\ b & \text{if } m_2 + b + g < (m_2 + b)\Pr(x_1 = 0; s). \end{cases}$$

IDEA

- When P_2 is interested in upstream decisions, P_1 can create a downstream rent for A by introducing uncertainty on x_1.

- To control P_2's offer, P_1 can use *stochastic allocations* and/or a (noisy) disclosure policy

  - Stochastic allocations are costly!

- Information disclosure allows P_1 to control downstream decisions without imposing excessive distortions on x_1

The **Complements** case

Proposition 3:

Disclosure is optimal only for strong complementarity, i.e. for $g > g^*$

Disclosure either makes P_2 *and A worse off and reduces welfare,*

or it leads to a Pareto improvement

- The **Substitutes** case

Proposition 4:

Disclosure is profitable only for intermediate level of substitutability, i.e. for

$$g_1 < |g| < g_2$$

Disclosure has ambiguous effect on welfare

CONCLUSION

- Interaction between two principals who sequentially contract with the same agent
- Focus: determinants of information disclosure
- Optimal disclosure policy consists in keeping all information secret when

  (a) preferences are separable

  (b) the sign of the single crossing is the same for x_1 and x_2

  (c) the upstream principal is not personally interested in downstream decisions
- Result is robust to the possibility of information trade!

- If any of these conditions is violated, there exist preferences for which disclosure is strictly optimal, even if the upstream principal can not sell information to the downstream principal

- Full disclosure is never optimal

- (Partial) disclosure need not necessarily harm the agent, but has in general ambiguous effects on welfare

Applications

- Auctions with resale

- Labor contracts

- Insurance contracts

- Financial contracts

- Private and public procurement

- Multinational regulation