Delegated and intrinsic common agency with full market participation*

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Abstract

We study equilibria in common agency in which a privately informed agent (e.g. a retailer) may or may not be free to choose the principals (the manufacturers) to be active with, situations respectively labelled as delegated or intrinsic agency. We show that the equilibria with full participation can be conveniently characterized in two steps. First, independently of the type of agency, one derives equilibrium activities and the associated total surplus. Then, one determines how the surplus is shared between the agent and the principals, depending on the competition induced by intrinsic or delegated agency. In this respect, the type of agency only matters with substitutable activities, as in this case the agent obtains a larger rent with delegated than with intrinsic common agency.
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1 Introduction

The framework of common agency has many important applications. A retailer may choose to be the agent of several producers. A multinational firm may be active in several countries thus being regulated or taxed by different national authorities. A policy maker may accept influence from several lobbies at the same time. In all these examples the agent is free to choose the principals to be active with. In other cases, instead, the agent only has the possibility to interact either with all principals, or with none of them. This naturally arises, for instance, when a regulated firm is subject to the control of both a sectoral regulator and an environmental agency. Similarly, the manager in a firm necessarily acts as an agent of both stockholders and creditors, while workers are often responsible *vis à vis* several superiors (e.g. the heads of the different divisions).

The first type of environment as been indicated as *delegated common agency* since the participation decision with any of the principals is delegated to the agent, while the second environment refers to cases in which common agency is *intrinsic* (Bernheim and Whinston, 1986).

The intrinsic framework has been employed in early theoretical and applied papers on common agency with adverse selection both as a natural requirement of specific environments, but also since it is meant to significantly simplify equilibrium characterization. See for example Bond and Gresik

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(1996), Calzolari (2001), Ivaldi and Martimort (1994), Laffont and Pouyet (2004), Martimort (1992, 1996), Mezzetti (1997), Olsen and Osmundsen (2001), Stole (1991) among others. However, as the previous examples illustrate, the assumption that the agent cannot choose a sub-set of principals to be active with is not always easy to justify from an economic viewpoint.

When not comfortable with the intrinsic agency assumption, one is then left with the more complex environment of delegated agency, in which the decision to participate with a principal also depends on other principals' contracts and on the agent's private information. In this environment, the agent can decide not to participate if a principal's contract is unfavorable, while still earning a surplus with other principals. This surplus may also depend on his private information (i.e. the outside option is type-dependent).

In this short paper we provide a few results which may be useful for applications. We investigate to what extent equilibria in intrinsic and delegated common agency differ when, *in equilibrium*, the agent decides to participate with all principals (full participation). We show that when preferences are such that this is the case, one can decompose equilibrium characterization in two steps. First, independently of whether agency is intrinsic or delegated, one can derive the equilibrium activities that are object of contracting and the surplus generated by such activity levels. Then, as a second and separate step, one determines how equilibrium surplus is shared between the agent and the principals depending on the particular type of agency framework, intrinsic or delegated.

The first result is obtained by showing that the programs to be solved to characterize activity levels are the same with intrinsic and with delegated agency, so that the sets of equilibrium activities are also the same in the two cases. As for the second result, we prove constructively how the agent's rent differ in the two cases. We show that the rent is the same with intrinsic and with delegated agency (and so is the sum of the principals' surpluses) if activities are complements. On the contrary, when substitutability is not too strong (as assumed by almost all applied papers), we show that the agent is able to get a strictly larger rent with delegated than with intrinsic agency.[1] In addition to the rent accruing in the intrinsic environment, any type of agent is able to secure for himself the payoff increase that the least-efficient type could obtain by *separately* contracting with the principals as opposed to common agency.

Although our analysis is limited in scope since we consider equilibria with full participation, we think these results are useful for applied works. In addition to the illustrated property of separability for equilibrium characterization, our results imply that the many papers that have assumed intrinsic common agency to simplify the analysis (even in environments that would have been best illustrated with a different assumption) have characterized equilibria with full participation that make perfect sense also when one considers the more adequate environment of delegated agency, with a possible qualifier concerning surplus sharing.

A comment on the notion of full participation is in order. Full participation is here an equilibrium property which follows from the players' preferences and, clearly, it is not simply assumed. In a similar vein, the majority of standard principal-agent models employs assumptions on preferences, such that the agent always participates with the principal, independently of his type and of distortions that may arise. Many of those papers have provided very valuable insights although assuming full participation. The assumption that leads here to full participation has also an equivalent in the literature on competition with differentiated sellers that deals with the common case of "full market coverage" (see for instance Wauthy, 1996).

Clearly, one may be interested also to know how competition with delegated agency affects participation when the market is "uncovered" (i.e. partial participation), such as in the single-principal

[1] We also argue that when instead substitutability is strong enough, then both equilibrium activities and rents may be affected.

agent model of Champsaur and Rochet (1989). Martimort and Stole (2007) show that delegated common agency matters to equilibrium participation only when the activities are substitutes: assuming partial participation in equilibrium, then delegated agency induces more participation than intrinsic agency. Furthermore, since they consider preferences such that in equilibrium some types do not participate, the least-efficient type of agent that is active is just indifferent between participating with both, one or none of the principals and thus earns no rent.

As in standard single principal-agent models, full and partial participation are both potentially interesting for applied works and we are convinced that this is also true in the case of common agency. Hence, our paper and that of Martimort and Stole complement each other providing a broad and complete picture on the effects of delegated agency for equilibrium characterization.

The next Section illustrates the agency model as a relationship between a common retailer of two manufacturers. Section 3 contains our results.

2 The model

Two manufacturers (the principals) P_1, P_2 simultaneously contract with a single retailer (the agent) A (the arguments can be extended to $n \geq 2$ principals). For any quantity q_i of the good she produces, P_i asks A a total payment $X_i(q_i)$ (the contract) defined over a compact set $\mathcal{Q} \subset \Re_+$. After observing the offers $X_1(\cdot)$ and $X_2(\cdot)$, A then chooses q_1,q_2 and contracts are executed. The agent's payoff is $\Pi(q_1, q_2, \theta) \equiv R(q_1, q_2, \theta) - X_1(q_1) - X_2(q_2)$ where the retailer's gross profits $R(\cdot)$ is strictly concave in q_1, q_2 with (subscripts denoting partial derivatives) $R_\theta > 0$, $R_{i\theta} > 0$ for $i = 1, 2$ and $\theta \in [\underline{\theta}, \overline{\theta}]$ is A's private information (e.g. the market size) distributed according to cumulative $G(\theta)$ and density $g(\theta) > 0$. When $R_{ij} > 0$, goods are (demand) *Complements*, otherwise if $R_{ij} < 0$ they are (demand) *Substitutes* and we assume that R_{ij} has a constant sign. The principals' profits $V_i = X_i(q_i) - c(q_i)$ are quasi-linear with identical (concave) cost $c(q_i)$.

To be consistent with the idea that A should not pay anything in case he does not purchase, we let $X_i(q_i) = 0$ for $q_i = 0$ and $X_i(q_i) = \bar{x}_i + x_i(q_i)$ for $q_i > 0$, where $\bar{x}_i$ is a (possibly nil) constant that will be useful in the sequel and $x_i(\cdot)$ is a continuous function. Clearly, A always (weakly) prefers to take both contracts although he may decide not *to be active* with P_i which we define as the decision of consuming $q_i = 0$ (A is active with P_i if $q_i > 0$).[2] As in most of the existing common agency papers with asymmetric information (for example all those quoted above), the contract of a principal does not depend on other principals' activities since they may not be observable. Furthermore, those "conditional offers" can be seen as a way to increase other principals' costs by affecting the agent's willingness to participate with them and seem at odd with some observed situations.[3]

As explained in the Introduction, we will focus on the comparison between intrinsic and delegated agency. Hence, we will not consider equilibria in which some types θ are active with one principal but not with the other, since, clearly, any such equilibrium with asymmetric participation in delegated common agency (DCA henceforth) is by definition impossible in intrinsic common agency (ICA henceforth). Furthermore, we will consider preferences so that each principal prefers to have all types active *in equilibrium*. As standard in single principal-agent models, P_i prefers any type θ to be active if the virtual surplus (to be defined later) associated to the lowest type $\underline{\theta}$

[2] Alternatively, one could discard the condition $X_i(0) = 0$ so that participation would correspond to A's decision to be active. The two approaches are equivalent.

[3] For example, those type of contracts are usually sanctioned by antitrust laws in the manufacturer-retailer relationship. In the case of international regulation they would be considered an undue extension of domestic jurisdiction outside national boundaries.

is positive for some q_i. This is implied by a no "shut-down" condition (see Fudenberg and Tirole, 1991) which, in the case of more than one good, is

$$R_i\left(0, q_j, \underline{\theta}\right) - c_i\left(0\right) - R_{i\theta}\left(0, q_j, \underline{\theta}\right) \frac{1 - G\left(\underline{\theta}\right)}{g\left(\underline{\theta}\right)} > 0, \tag{1}$$

for any q_j, $i, j = 1, 2$ and $i \neq j$. It is worth stressing that although in the following we will assume that (1) holds, we do not impose that A *must* be active with both principals, as in ICA, and this will be an equilibrium feature of the game.

3 Results

Taking as given $X_j(\cdot)$, P_i chooses $X_i(\cdot)$ that maximizes $E[V_i]$ (expectation is taken over θ), subject to A's incentive compatibility (IC) and individual rationality (IR).[4] Constraint (IC) controls A's choice over q_i for any θ and can be conveniently written as

$$q_i\left(\theta\right) \in ArgMax\{\hat{R}\left(q_i, \theta\right) - X_i\left(q_i\right)\} \; \forall \theta, \tag{2}$$

where $\hat{R}\left(q_i, \theta\right) \equiv R\left(q_i, q_j\left(q_i, \theta\right), \theta\right) - X_j\left(q_j\left(q_i, \theta\right)\right)$ and $q_j\left(q_i, \theta\right)$ maximizes Π given any q_i.[5] Constraint (IR) controls A's activity (or participation) decision and depends on whether we are considering ICA or DCA. In particular, let $\Pi\left(\theta\right) \equiv \Pi\left(q_i\left(\theta\right), q_j(q_i\left(\theta\right), \theta), \theta\right)$, the rationality constraint then writes as

$$\Pi\left(\theta\right) \geq \mathcal{I}_{CA} \Pi\left(0, q_j(0, \theta), \theta\right) \; \forall \theta \tag{3}$$

where $\mathcal{I}_{CA}$ is an indicator function $\mathcal{I}_{CA} = 0$ when considering ICA and $\mathcal{I}_{CA} = 1$ with DCA. With ICA, the principals must make sure that A obtains a non-negative payoff, whilst with DCA they must assure a non-negative *gain*, defined as the difference between the left and the right hand sides in (3).

The solution to this program faces two potential issues. First, independently of the type of participation (ICA or DCA), the "single crossing" condition is here endogenous since the sign of $\hat{R}_{i\theta}\left(q_i, \theta\right) = R_{i\theta}\left(\cdot\right) + R_{j\theta}\left(\cdot\right) \partial q_j / \partial q_i$ depends (also) on X_j. Second, as shown in (3) with DCA the A's outside option is type dependent.[6] The first issue (which emerges both in ICA and in DCA) is addressed in the literature by assuming that the following condition holds and then verifying that the (candidate) equilibrium actually satisfies it. This is the approach we will use in the sequel.[7]

Condition 1 *$\hat{R}_{i\theta}$ has the same constant sign as $R_{i\theta}$.*

With complements, Condition 1 is always satisfied since $\partial q_j / \partial q_i$ is positive so that $\hat{R}_{i\theta} \geq 0$. With substitutes, instead, it requires that substitutability is not too strong.

Let us now turn to the second aforementioned issue, which is instead specific to DCA, namely the type-dependency of the agent's outside-option. It is important to notice that the same Condition

[4] The standard Revelation Principle is invalid in common agency. Peters (2001, 2003) and Martimort and Stole (2002) (for simultaneous contracting) have shown that one can safely consider nonlinear prices $X_i(q_i)$ (see also Calzolari and Pavan, 2007, for related results in sequential common agency games).

[5] This way of writing A's payoff w.r.t. q_i is known as the "residual-utility" approach for common agency models and has been introduced by Martimort and Stole.

[6] See Jullien (2001) for a single principal-agent model with this property.

[7] To our knowledge, all common agency papers with asymmetric information follow this approach thus assuming Condition 1.

1 also solves this second issue. In fact, differentiating both sides of (3) w.r.t. θ and rearranging, we obtain

$$\frac{d}{d\theta}[\Pi(\theta) - \mathcal{I}_{CA}\Pi(0, q_j(0,\theta),\theta)] = \hat{R}_\theta(q_i(\theta),\theta) - \mathcal{I}_{CA}\hat{R}_\theta(0,\theta) \geq 0,$$

where the sign is implied by Condition 1. Hence, one only needs to take care of the rationality constraint of the least-efficient type $\underline{\theta}$: when the latter is satisfied, the constraints (IR) for higher types are also necessarily met, exactly as with ICA.

Hence, with Conditions 1 the program of P_i can be solved as a standard (single) principal-agent model independently of whether agency is intrinsic or delegated. In particular, for any θ the quantity $q_i(\theta)$ maximizes P_i's virtual surplus

$$\hat{V}_i(q_i,\theta) \equiv \hat{R}(q_i,\theta) - c(q_i) - \hat{R}_\theta(q_i,\theta)\frac{1-G(\theta)}{g(\theta)} \tag{4}$$

with (IR) binding for type $\underline{\theta}$. This discussion immediately leads to the following.

Proposition 1 *In equilibria with full-participation, quantities are the same whether the agency relationship is intrinsic or delegated.*

Since the set of equilibrium quantities is the same, the only difference between ICA and DCA possibly rests on A's rent.

Proposition 2 *Consider equilibria with full-participation. If goods are complements the agent obtains the same rent in both types of agency. With substitutes and delegated agency, any type of agent obtains the rent accruing in the intrinsic agency* plus *the payoff increase that type $\underline{\theta}$ would get by separately contracting with principals.*

Proof. See the Appendix. ■

To understand why with substitutes competition between the principals guarantees the agent a profit level strictly greater with DCA than with ICA, notice that with substitutes increasing the output of one principal makes the other principal's good less valuable. As a consequence, the threat to leave one principal and being active only with the other may be credible. Any type of agent is then left with a strictly larger payoff than with intrinsic agency. On the contrary, with complements, the threat to participate only with the other principal is not credible since the agent himself prefers to get both goods. The rent is then the same he can get when he is obliged to deal with both principals (or none).

Martimort and Stole (2007) consider a DCA model with preferences such that in equilibrium some types are excluded (also with full information). Denoting with $\tilde{\theta} > \underline{\theta}$ the least-efficient type that participates in equilibrium, with a continuity argument they show type $\tilde{\theta}$ obtains no rent also with substitutes because this type is just indifferent between participating with both, one, or none of the principals (in which case he would obtain no rent). When instead the least-efficient type $\underline{\theta}$ participates in equilibrium, as in our analysis, he is able to profit from competition between principals. In particular, we show he secures for himself the payoff increase he would obtain by *separately* contracting with the principals as opposed to common agency (see (11) in the Appendix). Furthermore, since rents are non decreasing in θ for (IC), also all the more efficient types benefit of this positive shift in their rents.

With complements, since making sure that the agent obtains a *non-negative gain* is less demanding for principals than granting him a *non-negative payoff*, any equilibrium in ICA that gives A a non-negative gain is an equilibrium in DCA as well. One may then wonder if other equilibria exist in DCA with respect to ICA and what can be said for substitutes. In this respect, an immediate implication of Propositions 1 and 2 is the following Corollary that illustrates useful results for applied common agency papers.

Corollary 1 *Consider equilibria with full participation.* (i) *With complements the set of implementable outcomes is the same with intrinsic and delegated agency. The sets differ with substitutes uniquely for the larger rent the agent obtains with delegated agency;* (ii) *Characterizing implementable outcomes, one can first derive equilibrium quantities independently of ICA and DCA and then determine how the total surplus is shared between the agent and the principals.*

Result (ii) illustrates that, independently of the type of agency, one can conveniently split the derivation of equilibrium outcomes into two steps. First one can derive equilibrium activities (which is independent of ICA and DCA) and the total surplus generated by such activity levels. Then, one determines the way the equilibrium surplus is shared between the agent and the principals on the basis of the particular type of competition that takes place: ICA or DCA and complements or substitutes.[8] We view this as a useful result for applications. It highlights that when principals may effectively compete for the agent (i.e. in DCA) and the market is covered, the unique effect of competition, if any, is to reduce their rents and increase that of the agent. In this respect Corollary 1 represents a generalization of similar results in Chiesa and Denicolò (2008) to our environment of asymmetric information with and without intrinsic agency.

In addition to providing a simple approach for the potentially complex delegated agency environment, our previous results also show that the many papers that in the past have assumed intrinsic agency to simplify the analysis have in fact characterized sensible equilibria also for the delegated environment.

Finally, abandoning Condition 1 a new difference may emerge between ICA and DCA. With DCA the agent's outside option is type dependent and, if Condition 1 is not verified, the rationality constraint may be binding for types other than the lowest. In fact, since $R_\theta > 0$ any P_i would like to contract with a "high" type. However, if R_{ij} is negative and "large", P_i may also prefer an agent who chooses low q_j and thus possibly a low type agent. "Efficient" agents (i.e. with high θ) would be *per se* desirable, but with $R_{ij} < 0$ they become less so since they tend to choose a large quantity of the other principal's output. If the latter effect prevails, their equilibrium rent might be lower than the one of less efficient types and constraint (IR) may bind for types $\theta > \underline{\theta}$. Furthermore, as shown by Jullien (2001) for single principal-agent with type-dependent outside option, when this is the case equilibrium allocations may also be affected. Clearly, this possible inversion of roles of the agent's types (and associated effects on the equilibrium) cannot take place with ICA since the outside option is not type-dependent.[9] We leave this interesting issue for future research.

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4 Appendix

Proof of Proposition 2. As usual, the rent $\Pi(\theta)$ for any θ is obtained integrating the following necessary condition $d\Pi(\theta)/d\theta = \hat{R}_\theta(q_i(\theta), \theta)$ so that

$$\Pi(\theta) = \Pi(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} \hat{R}_\theta(q_i(u), u)\, du. \tag{5}$$

Since from Proposition 1 quantities are the same with ICA and DCA, the only difference in the rents of these two cases may possibly originate from $\Pi(\underline{\theta})$ in (5). Furthermore, since with ICA $\Pi(\underline{\theta}) = 0$, we simply have to study whether $\Pi(0, q_j(0, \underline{\theta}), \underline{\theta})$ for j = 1,2 is strictly positive or not (it cannot be negative given the definition of X_i).

Let define

$$\pi_j(\theta) \equiv \max_{q_j>0}\{R(0, q_j, \theta) - \bar{x}_j - x_j(q_j)\} \tag{6}$$

as the maximal (but possibly negative) payoff that type θ can obtain by being active with P_j and inactive with P_i so that $\Pi(0, q_j(0, \theta), \theta) = \max\{0, \pi_j(\theta)\}$. We now show that $\pi_j(\underline{\theta}) > 0$ with substitutes and $\pi_j(\underline{\theta}) = 0$ with complements.

(i) Substitutes.[10] We proceed by showing first that $\max\{\pi_1(\underline{\theta}), \pi_2(\underline{\theta})\} < 0$ is impossible. Assume to the contrary that $\max\{\pi_1(\underline{\theta}), \pi_2(\underline{\theta})\} < 0$. This implies $\Pi(\underline{\theta}) = 0$ and also $\pi_1(\underline{\theta}) + \pi_2(\underline{\theta}) < 0$. By definition we have

$$\Pi(\underline{\theta}) = \max_{q_i>0, q_j>0}[R(q_i, q_j, \underline{\theta}) - \bar{x}_i - x_i(q_i) - \bar{x}_j - x_j(q_j)] = 0 \tag{7}$$

from which we obtain an expression for $\bar{x}_i + \bar{x}_j$. Substituting into $\pi_1(\underline{\theta}) + \pi_2(\underline{\theta}) < 0$, gives

$$\begin{aligned} \pi_1(\underline{\theta}) + \pi_2(\underline{\theta}) = \max_{q_j>0}[R(0, q_j, \underline{\theta}) - x_j(q_j)] + \max_{q_i>0}[R(q_i, 0, \underline{\theta}) - x_i(q_i)] + \\ - \max_{q_i>0, q_j>0}[R(q_i, q_j, \underline{\theta}) - x_i(q_i) - x_j(q_j)] < 0. \end{aligned} \tag{8}$$

But this is impossible since, from substitutability, for any q_i and q_j

$$R(q_i, q_j, \underline{\theta}) - x_i(q_i) - x_j(q_j) < R(0, q_j, \underline{\theta}) - x_j(q_j) + R(q_i, 0, \underline{\theta}) - x_i(q_i) \tag{9}$$

which holds *a fortiori* when we take the maximum of both sides as in (8).

Since it cannot be that $\max\{\pi_1(\underline{\theta}), \pi_2(\underline{\theta})\} < 0$, at least one of the two expressions in curly bracket must be weakly positive. Let this be $\pi_i(\underline{\theta}) \geq 0$. We now show that this implies $\pi_j(\underline{\theta}) > 0$ so that finally $\Pi(0, q_j(0, \underline{\theta}), \underline{\theta}) > 0$. From $\Pi(\underline{\theta}) = \Pi(q_i(\underline{\theta}), 0, \underline{\theta})$ we obtain

$$\bar{x}_j = \max_{q_i, q_j}[R(q_i, q_j, \underline{\theta}) - x_i(q_i) - x_j(q_j)] - \max_{q_i}[R(q_i, 0, \underline{\theta}) - x_i(q_i)] \tag{10}$$

Substituting $\bar{x}_j$ into (6) we have

$$\begin{aligned} \pi_j(\underline{\theta}) = \max_{q_i>0}[R(q_i, 0, \underline{\theta}) - x_i(q_i)] + \max_{q_j>0}[R(0, q_j, \underline{\theta}) - x_j(q_j)] - \\ - \max_{q_i>0, q_j>0}[R(q_i, q_j, \underline{\theta}) - x_i(q_i) - x_j(q_j)] \end{aligned} \tag{11}$$

>From (9) we then know that $\pi_j(\underline{\theta}) > 0$ so that, finally, $\Pi(\underline{\theta}) = \pi_j(\underline{\theta}) > 0$.

[10] The strategy we follow is related to Ivaldi and Martimort (1994).

(ii) Complements. We prove again our claim by contradiction. Assume $\min\{\pi_1(\underline{\theta}), \pi_2(\underline{\theta})\} > 0$ so that $\Pi(\underline{\theta}) > 0$. Using (10), we can obtain $\bar{x}_j$ and similarly $\bar{x}_i$. Summing $\bar{x}_j$ and $\bar{x}_i$ and rearranging we obtain

$$\begin{aligned} &-(\bar{x}_j + \bar{x}_i) + \max_{q_i>0, q_j>0} [R(q_i, q_j, \underline{\theta}) - x_i(q_i) - x_j(q_j)] = \\ &= \max_{q_i>0} [R(q_i, 0, \underline{\theta}) - x_i(q_i)] + \max_{q_j>0} [R(0, q_j, \underline{\theta}) - x_j(q_j)] - \\ &- \max_{q_i>0, q_j>0} [R(q_i, q_j, \underline{\theta}) - x_i(q_i) - x_j(q_j)] \end{aligned} \tag{12}$$

Following similar steps as in case (ii), the right hand side is negative so that the left hand side is also negative. But this implies $\Pi(\underline{\theta}) < 0$, a contradiction. Hence, it must be $\pi_1(\underline{\theta}) \leq 0$, $\pi_2(\underline{\theta})\} \leq 0$ so that $\Pi(\underline{\theta}) = 0$. ■