

REGULATION OF MULTINATIONAL BANKS:
A THEORETICAL INQUIRY

Supplementary Material

Available at:

<http://www2.dse.unibo.it/calzolari/web/papers/MNBRegulationSM.pdf>

GIACOMO CALZOLARI

GYONGYI LORANTH

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This supplementary material contains complete proof for all the propositions in the paper in the parametric case $R + L \geq 2$.

Proof of Proposition 1- ($R + L \geq 2$). The foreign regulator's payoff are as in case $R + L < 2$, as well as those of regulator h when $d^f = I$. On the contrary, when $d^f = O$ regulator h obtains $-(1 - p_f)(1 - L)$ with decision $d^h = I$ and $-(1 - p_h)[p_f(1 - (R - 1)) + (1 - p_f)]$ with $d^h = O$.

Note that with decisions (O, I) , if unit h succeeds, the MNB still retains unit h profit $R - 1$. Nothing then changes in the analysis except for the boundary δ'_h which now is $\delta'_h \equiv \frac{p_h - L}{p_h(R-1) - (L+R-2)}$ with $\delta'_h = 1$ for $p_h = 1$; $\delta'_h \geq 0$ if either $p_h \geq L$ or $p_h \leq \frac{L+R-2}{R-1}$; $\delta'_h \leq 1$ if $p_h \geq \frac{L+R-2}{R-1}$. Since $\frac{L+R-2}{R-1} < L$ we finally have that the four sets for the regulators' equilibrium decisions are defined as in case $R + L < 2$, by simply replacing δ'_h with the previous definition. Furthermore, since δ'_h share the same qualitative properties in the two parametric cases, then results (i) and (ii) immediately follow. ■

Proof of Proposition 2- ($R + L \geq 2$). The regulator's payoff are $-2(1 - L)$ with decisions (I, I) , $-(1 - p_i)(2 - L)$ with $d^i = O$ and $d^j = I$, and $-[(1 - p_h)p_f + (1 - p_f)p_h](2 - R) - (1 - p_h)(1 - p_f)2$ with decisions (O, O) . Hence, with $d^j = I$ the optimal decision for unit i is $d^i = O$ iff $p_i \leq L/(2 - L) (< L)$, whilst with $d^j = O$ the optimal decision is $d^i = O$ iff $p_i \geq \varphi_i \equiv \frac{L - p_j(R + L - 2)}{R - 2(R - 1)p_j}$ with $\varphi'_i \equiv \frac{L - p_i R}{R + L - 2 - 2p_i(R - 1)}$. Note that φ'_h and φ_f intersect at $p_h = p_f = 1$ and at $p_h = p_f = \frac{L}{2(R - 1)}$. Furthermore, since in this case for low values of p_j we have $\varphi_i \leq L/(2 - L)$, we will have to directly compare also the payoff of decisions (I, I) with that of (O, O) (this is clearly not necessary if $R + L < 2$). As for this comparison, decision (I, I) are better than (O, O) if $p_h < \varphi \equiv \frac{L(R + 2L - 4)}{L(3R - 2) - 2R}$.

We can have two sub-cases.

Sub-case 1: $L \geq 2(R - 1)$. For any $p_i \in [0, 1]$, $i = h, f$, φ'_h and φ_f intersect only at $p_h = p_f = 1$, and the set of probabilities such that the optimal decisions are (O, O) is empty in this case (see also footnote 14 in the text). The regulator intervenes in both units if $p_i < L/(2 - L)$ $i = h, f$, and intervenes in unit i but not in unit j if $p_j \geq \max\{p_i, L/(2 - L)\}$.

Sub-case 2: $L < 2(R - 1)$. For any $p_i \in [0, 1]$, $i = h, f$, φ'_h and φ_f intersect at $p_h = p_f = 1$ but also at $p_h = p_f = \frac{L}{2(R - 1)} < 1$ so that the set of probabilities such that the optimal decisions are (O, O) is not empty. Furthermore, note that $\frac{L}{2 - L} \leq \frac{L}{2(R - 1)}$ iff $L \leq 2 - 2(R - 1)$. Hence, if $R < 3/2$ which implies $2(R - 1) < 2 - 2(R - 1)$, the optimal decisions are as follows: (I, I) if $p_i < \frac{L}{2 - L}$, (O, O) if $p_h \geq \varphi_h$ and $p_f \geq \varphi_f$, (I, O) if $p_f \geq \frac{L}{2 - L}$ and $p_h < \max\{p_f, \varphi_h\}$ and, finally, (O, I) if $p_h \geq \frac{L}{2 - L}$ and $p_f < \max\{p_h, \varphi_f\}$. If instead $R \geq 3/2$, when $2 - 2(R - 1) < L < 2(R - 1)$ optimal decisions are (I, I) for $p_i \leq \min\{\frac{L}{2 - L}, \varphi\}$ $i = h, f$, (I, O) for $p_f \geq \frac{L}{2 - L}$ and $p_h \leq \varphi_h$, (O, I) for $p_h \geq \frac{L}{2 - L}$ and $p_f \leq \varphi_f$, and (O, O) for $p_i \geq \max\{\varphi, \varphi_i\}$ $i = h, f$. Finally, if instead $L \leq 2 - 2(R - 1)$, then decisions are exactly as in the previous case with $R < 3/2$.

This description of the optimal decisions then confirms points (i) and (ii) in the Proposition. Concerning the comparative statics of decisions on L , let us first assume $d^f = I$. An increase in L

has two effects on d^h now: (a) it increases the value of the assets the regulator can receive from $d^h = I$; (b) it makes the residual assets of a successful unit h less valuable to support the unit f (which are worth $p_h(1 - L)$ in expected terms since no more than $1 - L$ can be obtained). Clearly, both these effects work in the direction of a tougher regulation on the unit h . Consider now the case when $d^f = O$. The equity stake effect is also present now and a larger L makes it less pronounced by reducing the value of foreign residual assets which can be obtained with $d^h = I$ (i.e. $p_f(1 - L)$ in expected terms). However, it is clear that a larger L will have a higher impact on the regulator's direct cost from $d^h = I$ (point (a) above) than on the regulator's equity share in unit f . Hence, a larger L again results in tougher regulatory behavior towards unit h (symmetrically for unit f). ■

Proof of Proposition 3- $(R + L \geq 2)$. We need to compare the decisions for the two representations considering the two sub-cases with a branch MNB illustrated in the proof of Proposition 2- $(R + L \geq 2)$. It is immediate to see that sub-case 2 (i.e. $L < 2(R - 1)$) is qualitatively equivalent to the benchmark case with $R + L < 2$, discussed in the proof of Proposition 3. Considering instead sub-case 1, we have that unit f is subject to softer regulation with branch than subsidiary representation if and only if $p_h \leq L$. Now we show that unit h is always subject to softer regulation with branch representation. To this end, note that with subsidiary MNB the equilibrium decision is $d^h = O$ if either $p_h \geq L$ when $p_f < L$, or $p_h \geq \delta'_h$ when $p_f \geq L$. With branch, if $p_f < L$ the decision is $d^h = O$ when also $p_h \geq \max\{L/(2 - L), p_f\} (< L)$, which proves the result in this case. When instead $p_f \geq L$, the decision is $d^h = O$ if $p_h \geq p_f$ and to prove the result it suffices to note that $\delta'_h \geq p_h$. In fact, we have $\delta'_h < p_h$ for any $p_h \geq \acute{p}_h \equiv \frac{L}{R-1}$, where \acute{p}_h is the value of p_h such that $\delta'_h = p_h$. However, in the current sub-case 1 we have $\acute{p}_h \geq 1$ since $L \geq 2(R - 1)$ and therefore there is no p_h such that $\delta'_h < p_h$ which concludes the proof. ■

Proof of Proposition 4- $(R + L \geq 2)$. The proof of Proposition 2- $(R + L \geq 2)$ shows that regulatory decisions at $t = 1$ are qualitatively different from those with $R + L < 2$ only in sub-case 1, since then decisions (O, O) are always dominated for the regulator of a branch MNB. Hence, letting $L \geq 2(R - 1)$ and considering the different values of p_h we have the following. If p_h such that $0 \leq p_h < L/(2 - L)$, then since $L/(2 - L) \leq L$, we immediately have $\Pi_B > \Pi_S > \Pi_D = 0$. If $L/(2 - L) \leq p_h < L$, then $\Pi_B = (R + L - 2)\frac{1}{2}(1 + p_h^2)$, $\Pi_S = (R + L - 2)\frac{1}{2}(1 - L^2)$ and $\Pi_D = 0$, so that $\Pi_B > \Pi_S > \Pi_D$. We can then state that B is optimal for any $p_h \leq \underline{p}_h \equiv L$. If $L \leq p_h < \frac{L(-3+L+R)}{-1+L(R-1)}$, where the right hand side is the value of p_h such that $\delta'_h = L$, then $\Pi_D = p_h(R - 1)$, $\Pi_S = p_h(R - 1) \int_0^L dF(p_f) + \int_L^1 p_f(R + L - 2)dF(p_f)$ and $\Pi_B = (R + L - 2)[p_h \int_0^{p_h} dF(p_f) + \int_{p_h}^1 p_f dF(p_f)]$, with $\Pi_D > \Pi_S > \Pi_B$ since $(R - 1) \geq (R + L - 2)$ and $p_h \geq L$. Finally, if $\frac{L(-3+L+R)}{-1+L(R-1)} \leq p_h \leq 1$ the comparison between B and D is as in the previous case, whilst we have $\Pi_S - \Pi_D = p_h(R - 1)(\delta'_h + \frac{\delta_h'^2}{2} - 1 - \frac{L^2}{2}) + (R + L - 2)(\frac{1}{2} - \frac{\delta_h'^2}{2})$. The last term is positive independently of p_h , and the first is increasing in p_h and positive at $p_h = 1$. Furthermore, $\Pi_S - \Pi_D$

is negative at $p_h = \frac{L(-3+L+R)}{-1+L(R-1)}$ and positive at $p_h = 1$. It then follows that, by the intermediate value theorem, there exists a \bar{p}_h with $\frac{L(-3+L+R)}{-1+L(R-1)} \leq \bar{p}_h \leq 1$ so that $\Pi_S \leq \Pi_D$ iff $p_h \leq \bar{p}_h$. ■

Proof of Proposition 5-($R+L \geq 2$). Clearly, regulator f prefers B to S or D for the same reasons discussed in case $R+L < 2$. Consider now regulator h and let $L \geq 2(R-1)$ so that, as shown in the proof of Proposition 2-($R+L \geq 2$), with branch-MNB decisions can never be (O, O) . We then have the following.

Case 1: If $0 \leq p_h < \frac{L}{2-L}$, we have $W_D = L - 1$,

$$\begin{aligned} W_S &= \int_0^L (L-1) dF(p_f) + \int_L^1 (1-p_f)(L-1) dF(p_f), \\ W_B &= \int_0^{\frac{L}{2-L}} 2(L-1) dF(p_f) + \int_{\frac{L}{2-L}}^1 (1-p_f)(L-2) dF(p_f). \end{aligned}$$

so that $W_S \geq W_D \geq W_B$.

Case 2: If $\frac{L}{2-L} \leq p_h < L$, we have W_D and W_S are as in *Case 1*, and

$$W_B = \int_0^{p_h} (1-p_h)(L-2) dF(p_f) + \int_{p_h}^1 (1-p_f)(L-2) dF(p_f) = (L-2)\left(\frac{1}{2} - \frac{p_h^2}{2}\right),$$

which reaches its maximum at $p_h = L$. Now, note that for $p_h = L$, $W_D - W_B = L\frac{(1-L)^2}{2}$ so that for any $p_h \in [\frac{L}{2-L}, L)$, $W_D > W_B$.

Case 3: If $L \leq p_h < \frac{L(3-L-R)}{1-L(R-1)}$, where for $p_h = \frac{L(3-L-R)}{1-L(R-1)}$ we have $\delta_h = p_h$, then W_B is as in *Case 2*, $W_D = (1-p_h)(-1)$, and

$$W_S = (1-p_h) \int_0^L (-1) dF(p_f) + \int_L^1 (1-p_f)(L-1) dF(p_f).$$

We now prove that $(1-p_f)(L-1) > (1-p_h)(-1)$ from which it follows $W_S > W_D$. Recall that, conditional on the foreign unit being open, the home regulator's optimal decision at $t = 1$ is $d^h = I$. This means that the home regulator's cost associated to $d^h = I$, i.e., $(1-p_f)(L-1)$, should be lower than that of $d^h = O$, i.e., $(1-p_h)[p_f(R-2) + (1-p_f)(-1)]$ which is equal to $(1-p_h)p_f(R-1) + (1-p_h)(-1)$. As we have $(1-p_h)p_f(R-1) > 0$, it then follows that $(1-p_f)(L-1) > (1-p_h)(-1)$. Finally, note that $W_D - W_B = \frac{p_h(2-L)-L}{2} > 0$ since $L \leq p_h$.

Case 4: If $\frac{L(3-L-R)}{1-L(R-1)} \leq p_h < 1$ then W_D and W_B are as in Case 3, and

$$W_S = (1 - p_h) \int_0^L (-1) dF(p_f) + (1 - p_h) \int_L^{\delta'_h} [p_f(R - 2) + (1 - p_f)(-1)] dF(p_f) + \int_{\delta'_h}^1 (1 - p_f)(L - 1) dF(p_f).$$

We then have $W_S > W_D$ since, as discussed above, $\int_L^1 (1 - p_f)(L - 1) dF(p_f) > \int_L^1 (1 - p_h)(-1) dF(p_f)$, and also $\int_L^{\delta'_h} [p_f(R - 2) + (1 - p_f)(-1)] dF(p_f) > \int_L^{\delta'_h} (-1) dF(p_f)$. This concludes the proof. ■

Proof of Proposition 6-($R + L \geq 2$). With branch representation the analysis is as in case $R + L < 2$. With subsidiary MNB, the payoffs are as in case $R + L < 2$ except for decisions (I, O) , where they are $-(1 - p_f) + p_f(R + L - 2)$ and $-(1 - p_f)(1 - L)$ respectively for the foreign and the home regulator. Hence, the only boundaries that are different in this case are: δ_h becoming $\delta_h \equiv \frac{p_f(L+R-2)-L}{R-p_f(R-1)}$ with $\delta'_h \equiv \frac{p_h R - L}{2-L-R+p_h(R-1)}$ and the one discriminating between decisions $d^f = I$ and $d^f = O$ when $d^h = I$, with $d^f = I$ now optimal iff $p_f \leq \frac{L}{R-(1-L)}$ with $L \geq \frac{L}{R-(1-L)} \geq \frac{L}{R}$. Note that δ_h increases in p_f if $(2 - R)R \geq L$ and decreases in p_f otherwise, so that we have two cases. If $(2 - R)R \geq L$, then δ'_h and δ_f intersect at a value of p_h that is larger than the one at which δ_f and $\frac{L}{R-(1-L)}$ intersects, and this implies that the description of the optimal decisions is (qualitatively) the same as in the proof of Proposition 6 in the text. If instead $(2 - R)R < L$ (so that δ_h decreases in p_f), then δ'_h and δ_f intersect at a value of p_h that is smaller than the one at which δ_f and $\frac{L}{R-(1-L)}$ intersect. In this case, the mixed-strategy (not-empty) region is now defined by $p_h \in [\delta_h, L/R]$ and $p_f \in [\delta_f, L]$. Since with branch representation the home unit is certainly subject to intervention if $p_h \leq L/R$, it follows that in the mixed-strategy region the probability that the home unit is subject to intervention is smaller with subsidiary than with branch representation. ■

Proof of Proposition 7-($R + L \geq 2$). First recall that nothing changes with respect to case $R + L < 2$ regarding B and D . Decisions at $t = 1$ are (qualitatively) unaffected also with S if $(2 - R)R \geq L$, as shown in the proof of Proposition 6-($R + L \geq 2$). Also note that profits associated with decisions (I, I) and (O, O) are clearly as in case $R + L < 2$. On the contrary, profits for decisions (I, O) were $\Pi_S = \Pi_B = 0$ in case $R + L < 2$ and are now $\Pi_S = \Pi_B = p_f(R + L - 2)$. With decisions (O, I) we have $\Pi_S = p_h(R - 1)$, as in case $R + L < 2$, and $\Pi_B = p_f(R + L - 2)$, which is now larger than in case $R + L < 2$, but still smaller than Π_S .

Hence, it follows that when $(2 - R)R \geq L$, the reasoning used for $R + L < 2$ applies, and *a fortiori* we have that the bank prefers to remain domestic if $p_h < \bar{p}_h$ (with $\bar{p}_h \in [c_+^S, L]$), and to

expand abroad with a foreign subsidiary for $p_h \geq \bar{p}_h$, where c_+^S is the value of p_h such that $\delta'_h = \delta_f$ with $c_+^S \leq \frac{L}{L+R(1-L)}$.

If instead $(2-R)R < L$, recall that B is still dominated by D . Hence, we need to compare being D with S . Consider first $p_h \leq L/R$. Then, $\Pi_D = 0$ and, whatever are the decisions with S (also in the mixed strategy region), $\Pi_S \geq 0 = \Pi_D$. Consider now $p_h \geq L/R$, then $\Pi_S = p_h(R-1)[1 + \int_{\delta_f}^1 p_f dF(p_f)] > \Pi_D$. Hence, the bank always prefers to expand abroad as a subsidiary.

■

Proof of Proposition 8- $(R+L \geq 2)$. First recall that nothing changes with respect to case $R+L < 2$ as for B and D , so that the comparison of payoff for both the regulators is unaffected. Regulatory $t = 1$ decisions are (qualitatively) unaffected also with S if $(2-R)R \geq L$, as shown in the proof of Proposition 6- $(R+L \geq 2)$. Hence, in what follows we illustrate the case $(2-R)R < L$.

Consider first regulator f . Although $t = 1$ decisions are different than in case $(2-R)R \geq L$, the analysis goes through unchanged: when D is preferred to B , then necessarily S is dominated by B ; when B is preferred to D , then B is necessarily also preferred to S .

Consider now regulator h and the following cases.

If $p_h \leq 2-R (< L)$ then $W_S = L-1 + \int_{\frac{L}{R-(1-L)}}^1 p_f(1-L) dF(p_f)$ so that $W_B \leq W_D \leq W_S$.

If $2-R \leq p_h \leq \hat{p}_h^1$ where \hat{p}_h^1 is such that $\delta'_h = \delta_f$ for $p_h = \hat{p}_h^1$ then,

$$W_S = \int_0^{\frac{L}{R-(1-L)}} (L-1) dF(p_f) + \int_{\frac{L}{R-(1-L)}}^{\delta'_h} (1-p_f)(L-1) dF(p_f) + \int_{\delta'_h}^1 [p_h R - 1 + p_f(1-p_h)(R-1)] dF(p_f)$$

Comparing S and D , in the region of $0 \leq p_f \leq \delta'_h$ we have $W_S \geq W_D$. Moreover, in the region $\delta'_h \leq p_f \leq 1$ regulator h could choose decision I obtaining $(1-p_f)(L-1) (\geq W_D)$, but she prefers to choose decision O . This in turn means that $W_S \geq W_D$, so that $W_B \leq W_D \leq W_S$.

If $\hat{p}_h^1 \leq p_h \leq \hat{p}_h^2$, where \hat{p}_h^2 is such that for $p_h = \hat{p}_h^2$, $\delta'_h = \frac{L}{R-(1-L)}$, then

$$W_S = \int_0^{\frac{L}{R-(1-L)}} (L-1) dF(p_f) + \int_{\frac{L}{R-(1-L)}}^{\delta'_h} (1-p_f)(L-1) dF(p_f) + \int_{\delta'_h}^{\delta_f} \tilde{W}_S(p_h, p_f) dF(p_f) + \int_{\delta_f}^1 [p_h R - 1 + p_f(1-p_h)(R-1)] dF(p_f).$$

where $\tilde{W}_S(p_h, p_f)$ is the payoff associated with mixed strategies. The three terms with no mixed strategies are as in the previous case, resulting S to be preferred to D ; in addition, we also have that $\tilde{W}_S(p_h, p_f) = \sigma_f W_S^{IO} + (1-\sigma_f) W_S^{II} \geq L-1 = W_D$, so that $W_B \leq W_D \leq W_S$.

If $\hat{p}_h^2 \leq p_h \leq L/R$ then

$$W_S = \int_0^{\frac{L}{R-(1-L)}} (L-1) dF(p_f) + \int_{\frac{L}{R-(1-L)}}^{\delta_f} \tilde{W}_S(p_h, p_f) dp_f + \int_{\delta_f}^1 [p_h R - 1 + p_f(1-p_h)(R-1)] dF(p_f)$$

and we can follow the same reasoning as in the previous case.

Finally, if $L/R \leq p_h \leq 1$ we have

$$W_S = \int_0^{\delta_f} [p_h R - 1] dF(p_f) + \int_{\delta_f}^1 [p_h R - 1 + p_f(1-p_h)(R-1)] dF(p_f) = p_h R - 1 + \int_{\delta_f}^1 p_f(1-p_h)(R-1) dF(p_f)$$

so that, again, $W_B \leq W_D \leq W_S$. ■