Competitive quantity discounts*

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Abstract

We analyse the effects of competition with quantity discounts in a duopoly model with asymmetric firms. Consumers are privately informed about demand, so firms use quantity discounts as a price discrimination device. However, a dominant firm may also use quantity discounts to weaken or eliminate its competitor. We find that when firms are asymmetric quantity discounts can have anticompetitive effects. In particular, quantity discounts often harm the smaller firm and reduce consumer surplus. They can even decrease social welfare (i.e. the sum of producers' and consumers' surplus) for a set of parameter values. However, the circumstances in which a ban of quantity discounts may increase social welfare are difficult to identify in practice.

Keywords: Quantity discounts, Non-linear pricing, Exclusion, Dominant firm

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1 Introduction

Many firms offer quantity discounts, i.e. lower prices for large purchases. It is generally recognised that these discounts can simply be a way to pass economies of scale on to buyers, or to allow firms to better extract consumer surplus. However, antitrust authorities are sometimes concerned that dominant firms may use quantity discounts to eliminate or weaken competition. These concerns have fueled unresolved policy debates.[1]

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[1] See Kolay, Shaffer and Ordover (2004) and Beard, Ford and Kaserman (2007). The debate has also centred around other forms of loyalty discounts, including bundled discounts (where

In the U.S., the courts have typically been reluctant to prohibit single-product quantity discounts under the antitrust laws. Viewing quantity discounts as a way to deprive a rival of economies of scale so as to drive it out of business, U.S. courts tend to apply the demanding standards required in predatory pricing cases. That is to say, plaintiffs must be able to demonstrate that the incumbent had suffered a loss by pricing below cost, and that it had a reasonable prospect of recouping this loss in the future.[2]

The European case law is different, and much harsher. In landmark decision *Michelin II*, the European Court of Justice did not require proof that price was less than unit cost, and ruled that any quantity discount that does not reflect cost efficiencies is presumed to be abusive if practiced by a dominant firm.[3] This approach is based on the notion that quantity discounts can have exclusionary effects even if they are not part of a predatory strategy; or, as the European Commission states in the 2008 *Guidance Paper*, "without necessarily entailing a sacrifice for the dominant undertaking" (§ 37). The *Guidance Paper* also stresses, however, that the possible exclusionary effects of quantity discounts can only materialise when firms are sufficiently asymmetric.

The purpose of this paper is to inquire whether the concerns of the European antitrust authorities are well founded. We analyse a model where a dominant firm competes with a smaller rival. The model precludes the possibility of predatory pricing, allowing us to focus on the "European view" of anticompetitive quantity discounts. In the model, both firms can use quantity discounts as a price discrimination device (a relatively innocent rationale), but the dominant firm might also have exclusionary purposes. To detect possible anticompetitive effects, we then contrast the non-linear pricing equilibrium with the one that would be obtained if firms were constrained to linear pricing.

The analysis shows that in principle the European view is correct: when firms are sufficiently asymmetric, quantity discounts can have anticompetitive effects even if price is never below unit cost. More precisely, quantity discounts:

(i) allow the dominant firm to more easily exclude the rival than linear pricing;

(ii) are likely to harm consumers;

(iii) can decrease social welfare for a limited range of parameter values.

However, the *possibility* of anticompetitive effects is a necessary but not sufficient condition for a harsh policy, such as that currently adopted in Europe. In our model, quantity discounts actually increase social welfare for a wide range

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price discounts are conditioned on the customer's total purchases of various products supplied by the firm), market-share discounts (i.e. discounts conditioned on the firm's share of the customer's total purchases), and exclusivity discounts. Still other forms of loyalty rebates are conditioned on the customer's past purchasing history (see Fudenberg and Villas-Boas, 2006). For an overview of the current debate, see Heimler (2005), Kobayashi (2005), Spector (2005), Faella (2006), Ordover and Shaffer (2007), and Ahlborn and Bailey (2008).

[2] Two recent decisions where this view has been articulated by the courts are *Brooke Group* and *Concord Boats*: see Klein and Lerner (2008).

[3] However, *Michelin II* and all the subsequent cases where quantity discounts were found abusive involved all-units discounts. Incremental discounts are generally treated more leniently: see Waelbroeck (2005).

of parameter values. And the circumstances in which a ban of quantity discounts would improve social welfare are extremely difficult to identify, even in a highly structured model like ours. This casts doubts on the desirability of a prohibition of quantity discounts.

There is an extensive literature on monopolistic non-linear pricing: see Mussa and Rosen (1978) for a pioneering contribution, and Wilson (1993) for a comprehensive treatment of the matter. The analysis of the oligopoly case, however, is much less developed. Early contributions restricted the firms' strategy space in various ways. For example, Yin (2004) analysed a model in which firms compete in two-part tariffs. Kolay, Shaffer and Ordover (2004) modeled all-units discounts assuming that firms can offer only piecewise linear price schedules.

The first paper that allows firms to offer any non-linear price schedule is Martimort and Stole (2009).[4] However, their analysis is limited to the symmetric case, where exclusionary effects cannot possibly occur, and does not compare the equilibrium with non-linear pricing to that arising when firms can only use linear prices. To address the policy issues discussed above, we extend the Martimort and Stole analysis to the case of asymmetric firms and perform a thorough comparison between the equilibria with linear and non-linear pricing.

Another strand of the literature (Armstrong and Vickers, 2001; Rochet and Stole, 2002; Yang and Ye, 2008) focuses on models of one-stop shopping, where consumers patronise only one firm. In this case, firms effectively compete in utility space, and in equilibrium they typically offer two-part tariffs with a marginal price equal to marginal cost. Armstrong and Vickers (2010) extend the analysis to the case in which consumers can purchase from both firms at an extra cost. However, they focus on the case of multiproduct firms, so their contribution properly belongs to the bundled discounts literature (see also Greenlee, Reitman and Sibley, 2008). There is also a small literature on market-share discounts, which includes Majumdar and Shaffer (2009), Ordover and Shaffer (2007), and Calzolari and Denicolò (2009).

Going back to quantity discounts, Beard, Ford and Kaserman (2007) have argued that quantity discounts can be used as an imperfect substitute for exclusive contracts. In their model, the timing of moves is asymmetric: the incumbent can contract with the buyers before an entrant enters the market. Their analysis highlights an additional channel whereby quantity discounts can have anti-competitive effects, which is not considered in this paper.

The rest of the paper is organised as follows: section 2 sets up the model. In section 3 we derive the equilibrium with linear pricing, and in section 4 that with non-linear pricing. Section 5 compares these two modes of competition in terms of profits, consumer surplus and social welfare. Section 6 analyses the case of a selective ban imposed on the dominant firm only. Section 7 discusses the policy implications of our results and concludes the paper. All proofs are in the Appendix.[5]

[4] Another recent contribution is Hoerning and Valletti (2011), who focus on the case where total demand is fixed.

[5] A Web Appendix available at http://www2.dse.unibo.it/calzolari/ provides the detailed derivation of the closed form solutions.

2 The model

Two firms, denoted by $i = A, B$, supply differentiated products to a final consumer. The consumer's utility function in monetary terms, $u(q_A, q_B, \theta)$,[6] depends on consumption of the two goods q_A and q_B and a parameter, θ, which is the consumer's private information. The qualitative properties of the solution do not depend on the exact specification of demand and the distribution of types. However, we need explicit solutions to perform a detailed comparison between the linear and non-linear pricing equilibria. Following Martimort and Stole (2009), we then posit a quadratic utility function[7]

$$u(q_A, q_B, \theta) = \theta(q_A + q_B) - \frac{1-\gamma}{2}(q_A^2 + q_B^2) - \gamma q_A q_B \tag{1}$$

and assume that θ is uniformly distributed over the interval $[0, 1]$. This is not really restrictive, as the main result of the paper is that anticompetitive effects are possible but difficult to identify. If this is so in the uniform-quadratic model, it seems unlikely that anticompetitive effects may become easier to identify with more general utility functions and/or distribution of types.

In equation (1), the parameter $\gamma \in [0, \frac{1}{2})$ captures the degree of substitutability between the goods. The goods are independent when $\gamma = 0$ and perfect substitutes in the limiting case $\gamma = \frac{1}{2}$. The factor $\frac{1-\gamma}{2}$ that multiplies the middle term in (1) serves to prevent changes in γ to affect the size of the market. As suggested by Shubik and Levitan (1980), this helps clarify the interpretation of comparative statics results.

Firms have constant marginal costs c_A and c_B, with $c_A \leq c_B$. There are no fixed costs.[8] With no loss of generality, we normalise c_A to zero and denote $c_B = c \geq 0$. Thus, the parameter c captures the degree of asymmetry among the firms.[9] As firm B is never active when $c > 1$, we can focus on the case $c \leq 1$, again with no loss of generality.

Firms simultaneously and independently offer a price schedule. With linear pricing, price schedules must take the simple form $P_i(q_i) = p_i q_i$, so that firm i's strategy is simply its price $p_i \in \Re_+$. When firms can use quantity discounts, a strategy for firm i is a lower semi-continuous[10] function $P_i(q_i) : [0, q_{\max}] \to \Re_+$ where q_i is the quantity that firm i is willing to supply, $P_i \geq 0$ is the

[6] In an alternative interpretation of the model, A and B are manufacturers that sell their products through a common retailer, and the function u is the retailer's gross profit.

[7] This is equivalent to assuming that each consumer's demand for the goods is a linear function of their prices.

[8] The static nature of the model and the absence of fixed costs rule out the possibility of predatory pricing. Thus, the model is silent on the possibility that quantity discounts may be used as a cost-effective way to engage in a predatory strategy. This allows us to focus on the specific concerns of the European antitrust authorities discussed in the introduction.

[9] The parameter c may also capture demand asymmetries. For example, one could add a linear term $(\alpha_A q_A + \alpha_B q_B)$ in the utility function, where $\alpha_A - \alpha_B$ can be interpreted as an index of vertical product differentiation. Setting $c_A - \alpha_A = 0$ and $c = c_B - \alpha_B \geq 0$ one re-obtains the formulation used in the paper.

[10] Lower semi-continuity allows for upward jumps in the price schedule and yet guarantees that the net utility function U (see equation (3) below) has a maximum.

corresponding total payment requested, and $q_{\max}$ is a finite upper bound large enough so that no consumer may ever want to buy more than $q_{\max}$.

Each firm maximises its expected profits

$$\pi_i = E_\theta[P_i(q_i(\theta)) - c_i q_i(\theta)], \tag{2}$$

and the consumer of type θ chooses $q_A \geq 0$ and $q_B \geq 0$ so as to maximise his net utility

$$U(q_A, q_B, \theta) = u(q_A, q_B, \theta) - P_A(q_A) - P_B(q_B). \tag{3}$$

given his type θ. Given the timing of the game, it is natural to focus on subgame perfect equilibria where the consumer maximises U for any possible pair of price schedules submitted by the firms.

In what follows, we shall say that a firm is *active* if its equilibrium output is strictly positive, and that it is *excluded* if its equilibrium output is zero.

3 Linear pricing

We start from the case in which firms compete in linear pricing, seeking the Bertrand equilibrium of the pricing game.

The calculation of the equilibrium is complicated, because even if the individual demand functions are linear, the total demand functions obtained aggregating over the heterogeneous consumers are non linear. The equilibrium prices are reported in the Appendix. We have:

Proposition 1 *When firms compete in linear prices, there is a unique equilibrium in pure strategies. If $c \geq \hat{c}^\ell$, where*

$$\hat{c}^\ell \equiv \frac{3 - 5\gamma}{3(1-\gamma)} \tag{4}$$

firm B is not active and firm A charges the monopoly linear price $p_M = \frac{1}{3}$. If instead $c < \hat{c}^\ell$, both firms are active. Equilibrium prices satisfy $p_A^* \leq p_B^*$, with a strict inequality when $c > 0$. In the symmetric case $c = 0$, equilibrium prices are

$$p_A^* = p_B^* = \frac{1-2\gamma}{3-4\gamma}. \tag{5}$$

Proof. See the Appendix.

When firm B is active, in equilibrium consumers are partitioned into three groups: low demand consumers, $\theta \in [0, p_A^*]$, who do not buy any product; intermediate demand consumers, $\theta \in [p_A^*, \hat{\theta}^\ell]$, who buy only product A; and high demand consumers, $\theta \in [\hat{\theta}^\ell, 1]$, who buy both products.[11] Notice that the

[11] The possibility that some consumers may buy product A but not product B arises since firm A, having a lower unit cost, can profitably price below firm B. The threshold $\hat{\theta}^\ell$ is defined so that the demand for product B of types $\theta \leq \hat{\theta}^\ell$ vanishes.

inframarginal consumers $\theta < \hat{\theta}^{\ell}$ are "captive," in the sense that a small increase in p_A leads these consumers to reduce q_A, but not to purchase product B.

This latter observation is crucial to understand a *prima facie* surprising property of the equilibrium, namely the absence of a limit pricing region. Relatedly, notice that firm B is excluded only if its cost c is greater than firm A's monopoly price $p_M = \frac{1}{3}$. One would expect that if the products are sufficiently close substitutes, firm B should be excluded even if the cost gap c is smaller than A's monopoly price. For, if c is still relatively large, firm A should then find it profitable to engage in limit pricing, as in models of product differentiation with homogeneous consumers (see, for instance, Zanchettin 2006).

The intuitive reason why firm A does not engage in limit pricing is that in our model consumers are heterogeneous, and low-type consumers are effectively captive to firm A. By reducing its price, firm A may increase its profit on consumers of type $\theta \in [\hat{\theta}^{\ell}, 1]$, but it will certainly decrease its profit on its captive consumers, as long as p_A is below the monopoly price. This effect makes firm A less aggressive, and it is stronger, the closer is $\hat{\theta}^{\ell}$ to 1.

Consider, then, whether a limit pricing strategy might ever be optimal for firm A. Such a strategy involves setting p_A so that $\hat{\theta}^{\ell} = 1$. But when $\hat{\theta}^{\ell}$ is close to 1, almost all consumers are captive to firm A. Therefore, firm A will have an incentive to charge the monopoly price p_M. This means that firm A will drive firm B out of the market, only if it can do so by charging the monopoly price.[12]

Another property of the equilibrium is that an increase in the degree of product substitutability may increase p_A^* when γ is already sufficiently large. The intuition here is as follows. When γ increases, the products become better substitutes. As a result, competition is more intense, decreasing both prices. However, in our model a countervailing effect is at work: when $c > 0$, an increase in γ enlarges the set of consumers that purchase only product A. This reinforces firm A's incentive to exploit these captive consumers, reducing the intensity of competition. This latter effect can prevail on the standard effect of greater substitutability when γ is large enough, leading to an increase in p_A^*.

4 Non-linear pricing

Now we turn to the case where firms can offer non-linear price schedules $P_i(q_i)$. The difficulty in finding the equilibrium with non-linear prices is that the strategy space is very large. We overcome this difficulty by using a guess-and-check strategy. We start by guessing a specific functional form of the equilibrium price schedules, so that they are fully identified by a few parameters. If the initial guess is correct, the equilibrium of the original game will coincide with that of a restricted game, where firms can only choose those parameters. The equilibrium

[12] Only in the limiting case $\gamma = \frac{1}{2}$ does one obtain the familiar limit pricing equilibrium, where firm B prices at c and firm A prices at $c - \epsilon$ and serves the entire market. When γ approaches $\frac{1}{2}$, p_B^* converges to c from above and p_A^* converges to c from below. The equilibrium quantity Q_B^* converges to zero, but it is positive for any $\gamma < \frac{1}{2}$.

of the restricted game then becomes the candidate equilibrium of the original game. Finally, we verify that the candidate equilibrium strategies satisfy the best response property *over the unrestricted strategy space*. The drawback of the guess-and-check strategy is that it fails to locate equilibria that do not conform to the initial guess, if there are any.[13]

To simplify the exposition, we shall present first the equilibrium when firm B is active and then when it is excluded. We start by heuristically determining the conditions under which firm B is active (a rigorous proof is provided in the Appendix). Since there are no fixed costs and the marginal utility is decreasing, firm B will be excluded if and only if for all θ

$$c \geq u'_{q_B}\left(\tilde{q}_A(\theta), 0, \theta\right) \tag{6}$$

where $\tilde{q}_A(\theta)$ is the optimal quantity when the consumer does not purchase from firm B. Next we can exploit two well known properties of the monopoly solution, which carry over to the duopoly case: monotonicity and no distortion at the top. Monotonicity means that equilibrium quantities increase with θ, implying that if firm B is active it must find it profitable to serve consumer $\theta = 1$. The no-distortion-at-the-top property implies that if consumer $\theta = 1$ does not consume product B, he must be consuming the efficient stand-alone quantity of product A, which is $\frac{1}{1-\gamma}$. It follows that firm B stays active if

$$c < u'_{q_B}\left(\frac{1}{1-\gamma}, 0, 1\right). \tag{7}$$

Condition (7) is equivalent to $c < \hat{c}^{n\ell}$ where

$$\hat{c}^{n\ell} \equiv \frac{1-2\gamma}{1-\gamma}. \tag{8}$$

The threshold $\hat{c}^{n\ell}$ is equal to 1 when $\gamma = 0$, decreases with the degree of product substitutability γ, and converges to 0 when γ approaches 1/2.

Now, let us consider the case where firm B is active. As the utility function is quadratic, we guess that firm B will offer a quadratic price schedule

$$P_B(q_B) = \alpha_{1,B} q_B + \alpha_{2,B} q_B^2. \tag{9}$$

We could add a positive fixed fee, which would imply an upward jump of the price schedule at $q_B = 0$. However, as argued by Wilson (1993) for the case of monopoly and by Martimort and Stole (2009) for that of duopoly, in equilibrium the fixed fee must necessarily vanish, so there is no loss of generality in ruling it out from the outset.

[13] However, adapting the techniques of Hoerning and Valletti (2011) it is easy to proof that the equilibrium we find is unique if firms are restricted to *differentiable* price schedules. It is also possible to rule out non-differentiable equilibria in which firm A uses a fixed fee to impose a minimal purchase requirement, similar to an exclusivity requirement. This pattern cannot emerge in equilibrium because it is a general property of any best response price schedule that the marginal type must should purchase a quantity $q \to 0$.

Firm A, being more efficient, may serve more consumers than firm B. Thus, we guess that its price schedule may consist of various quadratic branches. The upper branch, which is intended for high demand consumers who purchase both products, is similar to the price schedule of firm B, i.e.:

$$P_A^d(q_A) = \alpha_{1,A} q_A + \alpha_{2,A} q_A^2. \tag{10}$$

In addition, firm A may also engage in limit pricing (to induce some intermediate demand consumers to purchase only product A) and in monopoly pricing (to exploit those low demand consumers who are effectively captive). The monopoly part of the price schedule can be calculated easily using standard monopolistic screening techniques and is

$$P_A^m(q_A) = \frac{1}{2} q_A - \frac{1-\gamma}{4} q_A^2. \tag{11}$$

The limit-pricing schedule must lead intermediate demand consumers to purchase the limit quantity $q_A^{\lim}(\theta)$. This is defined as the quantity q_A so large that it induces consumers not to purchase product B. To be more precise, the limit quantity $q_A^{\lim}(\theta)$ is such that type θ's marginal willingness to pay for product B is just equal to the marginal price of product B at $q_B = 0$, i.e. $u'_{q_B}(q_A^{\lim}(\theta), 0, \theta) = P_B'^*(0)$. It is easy to show that in order to induce consumer θ to purchase this limit quantity, firm A's price schedule must be

$$P_A^{\lim}(q_A, \alpha_{1,B}) = \alpha_{1,B} q_A - \left(\frac{1}{2} - \gamma\right) q_A^2. \tag{12}$$

Summarising, our guess is that firm A will submit a price schedule that combines the duopoly, limit pricing and the monopoly branches described above. For those branches of the price schedule that do not apply when q_A is close to zero, we must also allow for the possibility of constant terms.

Proposition 2 describes the equilibrium that conforms to this guess. Even if our guess allows for the possibility that the price schedules exhibit upward jumps, the equilibrium schedules are everywhere continuous and differentiable.

To state the proposition, it is convenient to define the price schedule which is obtained in the symmetric equilibrium where $c = 0$ (the case analysed by Martimort and Stole, 2009), which is:

$$P^*(q) = \alpha_1^* q - \frac{\alpha_1^*}{2} q^2, \tag{13}$$

where

$$\alpha_1^* = \frac{1}{4}\left[3(1-\gamma) - \sqrt{1 - 2\gamma + 9\gamma^2}\right]. \tag{14}$$

We are now ready to state:

Proposition 2 *When firm B is active, i.e. if $c < \hat{c}^{n\ell}$, the following is an equilibrium in non-linear prices. Firm B offers the price schedule*

$$P_B^*(q_B) = P^*(q_B) + c\left(1 - \alpha_1^* \frac{1-\gamma}{1-2\gamma}\right) q_B \quad \text{for } 0 \le q_B \le 1 - c\frac{1-\gamma}{1-2\gamma}, \tag{15}$$

and firm A offers the price schedule

$$P_A^*(q_A) = \begin{cases} P_A^m(q_A) & \textit{for } 0 \le q_A \le \tilde{q}_A^* \\ \alpha_{0,A}^{\lim *} + P_A^{\lim}(q_A, \alpha_{1,B}^*) & \textit{for } \tilde{q}_A^* \le q_A \le \hat{q}_A^* \\ \alpha_{0,A}^{d*} + P^*(q_A) + c\frac{\gamma\alpha_1^*}{1-2\gamma} q_A & \textit{for } \hat{q}_A^* \le q_A \le 1 + c\dfrac{\gamma}{1-2\gamma}. \end{cases} \tag{16}$$

The constants $\alpha_{0,A}^{\lim *}$, $\alpha_{0,A}^{d*}$, $\tilde{q}_A^*$ and $\hat{q}_A^*$ are such that firm A's price schedule is continuous and differentiable at $\tilde{q}_A^*$ and $\hat{q}_A^*$; the precise expressions are given in the Appendix.

Proof. See the Appendix.

In accordance with our guess, the equilibrium price schedule of firm A involves monopoly pricing for small purchases, limit pricing for intermediate purchases, and duopoly pricing for large purchases.

In equilibrium, consumers are divided into four groups: low demand consumers, who do not buy any product; captive consumers, who purchase only product A and are not contested by firm B; intermediate demand consumers, who in equilibrium purchase only product A but are contested by firm B; and high demand consumers, who buy both products. The equilibrium quantities are

$$q_A^*(\theta) = \begin{cases} \max\left\{0, \dfrac{2\theta - 1}{1-\gamma}, \dfrac{\theta - \left[\alpha_1^* + c\left(1 - \alpha_1^* \frac{1-\gamma}{1-2\gamma}\right)\right]}{\gamma}\right\} & \text{for } \theta \le \hat{\theta}^* \\ q^*(\theta) + c\dfrac{\gamma}{1-2\gamma} & \text{for } \theta > \hat{\theta}^*. \end{cases} \tag{17}$$

$$q_B^*(\theta) = q^*(\theta) - c\frac{1-\gamma}{1-2\gamma}, \tag{18}$$

where

$$q^*(\theta) = \frac{\theta - \alpha_1^*}{1 - \alpha_1^*} \tag{19}$$

is the symmetric equilibrium quantity, and

$$\hat{\theta}^* = \alpha_1^* + c\,(1 - \alpha_1^*)\,\frac{1-\gamma}{1-2\gamma}. \tag{20}$$

Equilibrium quantities are increasing in θ, as depicted in Figure 1.

Various comments are in order. First, the equilibrium price schedules are concave everywhere. This means that firms always offer quantity discounts and never charge quantity premia. Second, the equilibrium satisfies the no-distortion-at-the-top property, which implies that type $\theta = 1$ must purchase the efficient quantities $q_i^{fb}(1)$.[14] For consumer $\theta = 1$ to be willing to purchase the efficient quantities, the slopes of the equilibrium price schedules at the efficient

[14] The efficient quantities are implicitly defined by the conditions

$$u'_{q_i}(q_i^{fb}, q_j^{fb}, \theta) = c_i$$

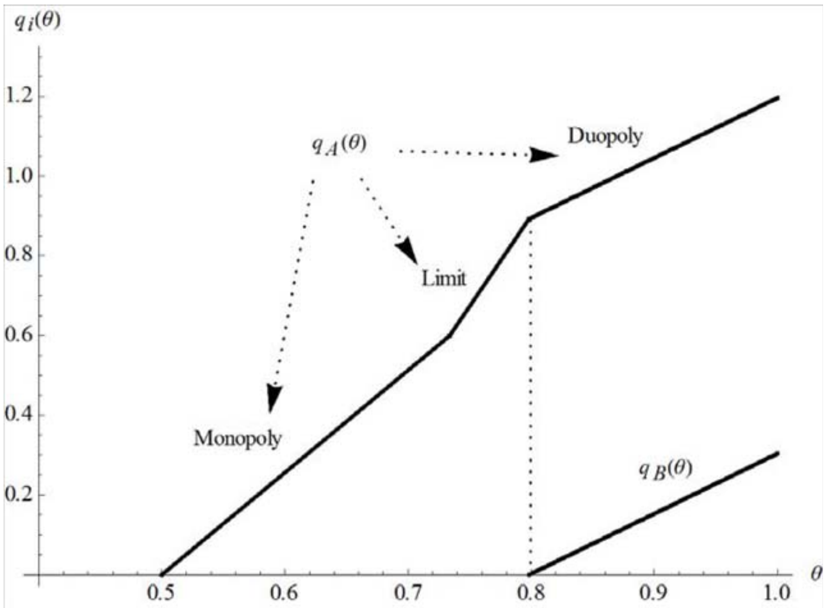

Figure 1: Equilibrium quantities when the less efficient firm is active; specifically, the case $\gamma = 0.22$ and $c = 0.5$ is depicted.

quantities must equal the marginal costs, i.e. $P_i'(q_i^{fb}) = c_i$. Third, marginal prices are never smaller than marginal costs. This confirms that there is no room for predatory pricing in our model.[15] Fourth, the intensity of quantity discounts (as measured by the degree of concavity of the price schedules, $\frac{P''(q)q}{P'(q)}$) is the same for both firms when consumers purchase both products, and is smaller than under monopoly. Finally, an increase in c shifts up the price schedules of both firms (as $\alpha_1^* < \frac{1-2\gamma}{1-\gamma}$), demonstrating that prices are strategic complements even in the presence of quantity discounts.

Next, let us turn to the case in which firm B is not active. Without any substantial loss of generality, we can posit that when there is no demand for its product, firm B stands ready to supply consumers at marginal cost. That is to say, firm B offers a price schedule $P_B(q_B) = cq_B$.

We have:

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and hence are

$$q_A^{fb}(\theta) = \theta + \frac{\gamma c}{1-2\gamma}$$
$$q_B^{fb}(\theta) = \theta - \frac{c(1-\gamma)}{1-2\gamma}.$$

[15] However, the no-distortion-at-the-top property means that marginal prices are equal to marginal costs for type $\theta = 1$. Thus, if a firm approximates the true equilibrium price schedule with a piecewise linear schedule, it might inadvertently price below marginal cost over a range of quantity levels.

Proposition 3 *If $c \geq \hat{c}^{n\ell}$, firm B is not active and in equilibrium it offers the competitive price schedule $P_B^*(q_B) = cq_B$. Firm A offers the price schedule*

$$P_A^*(q_A) = \begin{cases} P_A^{\lim}(q_A, c) & \text{for } 0 \leq q_A \leq \tilde{q}_A^* \\ \alpha_{0,A}^{m*} + P_A^m(q_A) & \text{for } \tilde{q}_A^* \leq q_A \leq \frac{1}{1-\gamma} \end{cases} \tag{21}$$

where the constants $\tilde{q}_A^*$ and $\alpha_{0,A}^{m*}$ are such that firm A's price schedule is continuous and differentiable at $\tilde{q}_A^*$; the precise expressions are given in the Appendix.

Proof. See the Appendix.

Obviously, firm A's price schedule now comprises only two parts, the monopoly branch and limit pricing branch. The twist is that now the limit pricing branch applies to small purchases (and hence to low demand consumers) and the monopoly branch to large purchases (and hence to high demand consumers).

To understand this new pattern, it is useful to consider whether B effectively threatens to enter the market when A is charging the monopoly price schedule and, if so, whether its entry threat is stronger for low- or high-type consumers. Since B is willing to serve any consumer at the constant price c, its threat of entry depends on consumers' willingness to pay for product B. Now, the willingness to pay for the first unit of product B, conditional on the monopoly quantity of product A being consumed, is:

$$u'_{q_B}(q_A^m(\theta), 0, \theta) = \begin{cases} \theta & \text{for } 0 \leq \theta \leq \frac{1}{2} \\ \frac{\gamma + (1-3\gamma)\theta}{1-\gamma} & \text{for } \frac{1}{2} \leq \theta \leq 1. \end{cases} \tag{22}$$

This expression is increasing in θ when $\gamma < \frac{1}{3}$, but it is decreasing when $\gamma > \frac{1}{3}$ (and $\theta > \frac{1}{2}$). In the former case, one can easily check that the willingness to pay is always lower than $\hat{c}^{n\ell}$. Thus, if firm B is not active (i.e. $c \geq \hat{c}^{n\ell}$), it does not threaten to enter either, and firm A can safely charge the monopoly price schedule. The same is true when $\gamma > \frac{1}{3}$ and $c \geq \frac{1}{2}$. If instead $c < \frac{1}{2}$ and $\gamma > \frac{1}{3}$,[16] firm B's threat of entry is effective and is stronger for lower types. In this case, firm A must engage in limit pricing for low types, and it can charge the monopoly price schedule only for high types.[17]

The equilibrium quantities now are $q_B^*(\theta) = 0$ and

$$q_A^*(\theta) = \max\left\{0, \frac{\theta - c}{\gamma}, \frac{2\theta - 1}{1 - \gamma}\right\}. \tag{23}$$

We conclude the presentation of the non-linear pricing equilibrium by noting that certain branches of firm A's price schedule may vanish for a range of

[16] If $c < \frac{1}{2}$ and $\gamma \leq \frac{1}{3}$, we are in the region where firm B is active and hence Proposition 2 applies.

[17] In this case, the consumer with the highest marginal willingness to pay for product B is the one indiffierent between the monopoly and the limit pricing quantities, i.e. $\theta = \frac{\gamma - c(1-\gamma)}{3\gamma - 1}$. This is lower than one if $c \geq \hat{c}^{n\ell}$.

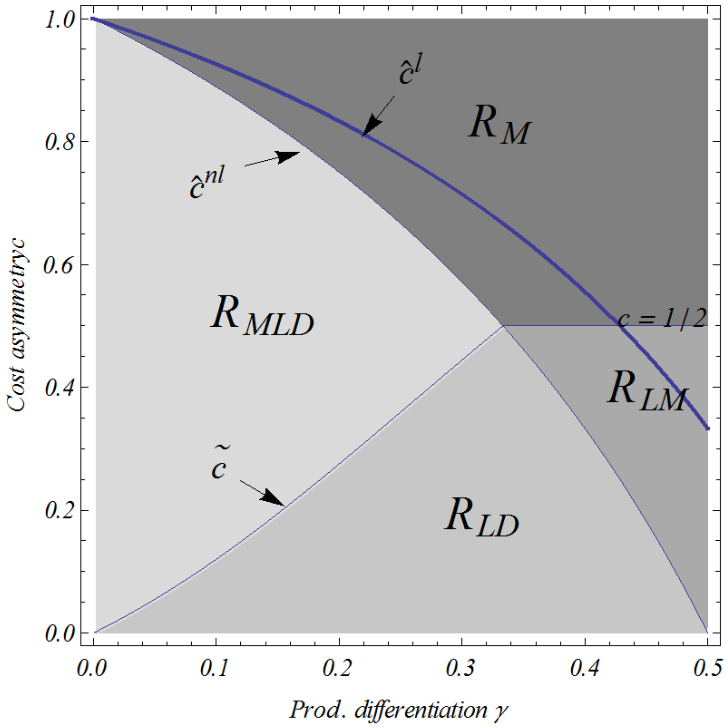

Figure 2: The four possible equilibrium patterns with non linear pricing.

parameter values. In particular, when B is active, the monopoly part of A's price schedule vanishes (that is, $\tilde{q}_A^* = 0$) when $c > \tilde{c}$, where

$$\tilde{c} \equiv \frac{1}{2} \frac{(1-2\alpha_1^*)(1-2\gamma)}{1-\alpha_1^*(1-\gamma)-2\gamma}.$$

Likewise, when B is not active, the limit pricing part of A's price schedule vanishes (that is, $\tilde{q}_A^* = 0$) when $c > \frac{1}{2}$.

Figure 2 illustrates how the parameter space (c, γ) splits into four regions that correspond to the possible equilibrium patterns: the *monopoly-limit-duopoly region,* $R_{MLD} \equiv \left\{(c,\gamma) : \hat{c}^{n\ell} > c > \tilde{c}\right\}$, where firm B is active but some consumers are captive to firm A; the *limit-duopoly region,* $R_{LD} \equiv \left\{(c,\gamma) : c < \min\left\{\hat{c}^{n\ell}, \tilde{c}\right\}\right\}$, where firm B is active and there are no captive consumers; the *limit-monopoly region,* $R_{LM} \equiv \{(c,\gamma) : \hat{c}^{n\ell} \leq c < \frac{1}{2}\}$, where firm B is not active but still competes with firm A for some low-type consumers; and the *monopoly region,* $R_M \equiv \{(c,\gamma) : c \geq \max\{\hat{c}^{n\ell}, \frac{1}{2}\}\}$, where firm A is an unconstrained monopolist.

5 Comparison

This section compares the equilibria with linear and non-linear pricing in terms of profits, consumer surplus and welfare. The effects of quantity discounts are generally ambiguous, even in a highly structured model like ours. Luckily, however, there are only two parameters in our model, the degree of product substitutability γ and the degree of asymmetry c. Thus, we can calculate and plot the regions of the parameter space (c, γ) where the relevant variables are smaller or larger with quantity discounts. These regions are presented below in a series of figures.[18]

5.1 The symmetric case

It is useful to start from the symmetric case $c = 0$, where we can easily determine the effects of quantity discounts in terms of the only remaining parameter, the degree of product substitutability γ.

Proposition 4 *In the symmetric case, there exist two thresholds, $\gamma_f > \gamma_c$, such that firms are better off with quantity discounts if the products are weak substitutes ($0 \leq \gamma \leq \gamma_f \simeq 0.25$), and consumers are better off if the products are close substitutes ($\gamma_c \leq \gamma < \frac{1}{2}$, where $\gamma_c \simeq 0.14$). Social welfare is always larger with quantity discounts.*

Proof. See the Web Appendix.

These conclusions follow from two opposing effects of quantity discounts, which are also at work in the asymmetric case. On the one hand, quantity discounts intensify competition. This reduces profits and benefits consumers. On the other hand, quantity discounts also allow firms to better extract consumer surplus. This increases profits and harms consumers. The former effect prevails when the products are close substitutes. In this case, firms are caught in a prisoners' dilemma: both would gain from a ban of quantity discounts, but if quantity discounts are permitted, each firm has a unilateral incentive to use them. The latter effect prevails when the products are weak substitutes.[19] However, quantity discounts increase social welfare for any degree of product differentiation.[20] Thus, in the symmetric case quantity discounts cannot be anti-competitive in our model.

[18] These figures have been drawn using a Mathematica file available in the Web Appendix.

[19] This result depends on the specific functional forms we have used in this paper. When the products are weak substitutes, quantity discounts tend to have the same effects as under monopoly. As is well known, however, these effects may well be ambiguous for different specifications of demand.

[20] This of course presumes that both firms continue to be active even when quantity discounts are permitted. In the presence of fixed costs, an increase in the intensity of competition may leave room for only one firm to be active. The effect on social welfare may then be negative. Indeed, since there is no definite relation between a firm's marginal contribution to social welfare and its profits, both over- and under-participation seem possible.

5.2 The asymmetric case

The asymmetric case is more interesting, as it opens the possibility that the dominant firm may use quantity discounts to eliminate (or weaken) its competitor.

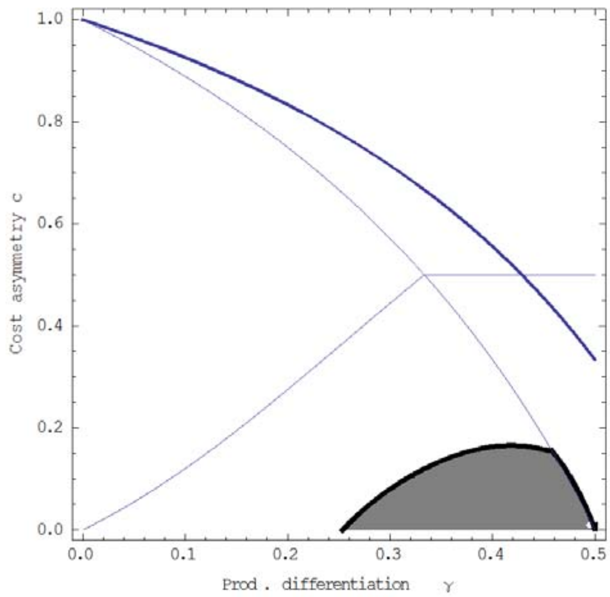

Figure 3: In the grey region, quantity discounts decrease the profit of the dominant firm.

5.2.1 Exclusion

Quantity discounts increase the likelihood that the less efficient firm is excluded from the market. However, this outcome results from inefficient participation of the less efficient firm under linear pricing, not from inefficient exclusion with quantity discounts.

Proposition 5 *With non-linear pricing, firm B is excluded for a larger set of parameter values than with linear pricing. However, with non-linear pricing firm B is excluded precisely for those parameter values for which it should be excluded in the first best solution.*

The proof is very simple. Consider the circumstances under which exclusion is socially efficient. Because the products are differentiated, even the product supplied by the less efficient firm should be consumed unless c is large enough. To be precise, the condition is that c should be lower than the maximum marginal willingness to pay for product B when the efficient quantity of product

A is already being consumed. The maximum is achieved at $\theta = 1$ and is equal to $\hat{c}^{n\ell}$, so exclusion is socially desirable only if $c \geq \hat{c}^{n\ell}$. This means that with quantity discounts, firm B is excluded precisely when exclusion is socially efficient.[21] With linear pricing, by contrast, firm A prices above marginal cost, so consumer $\theta = 1$ buys less than the efficient quantity of product A. Because the products are substitutes, the demand for product B is inefficiently high. Hence, with linear pricing firm B may be active even if it should actually be excluded. This is, indeed, what happens when $\hat{c}^{n\ell} \leq c < \hat{c}^{\ell}$. This is a genral argument that holds not only in our uniform-quadratic model, but also with more general utility functions and distribution of types.

5.2.2 Profits

As we have seen above, in the symmetric case both firms benefit from quantity discounts if the products are weak substitutes. In the asymmetric case, the picture is quite different. Figures 3 and 4 depict the region of parameter values where the profit of the more and the less efficient firm, respectively, are lower with quantity discounts. It appears that the more efficient firm is much more likely to benefit from quantity discounts than the less efficient one.

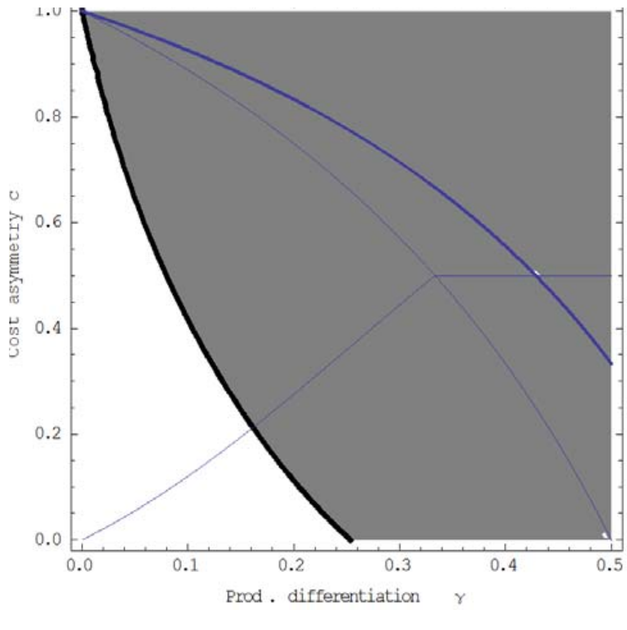

Figure 4: In the grey region, quantity discounts decrease, or do not affect, the less efficient firm's profit.

[21] This property of the equilibrium follows from the no-distortion-at-the-top property, which implies that type $\theta = 1$ consumes the efficient quantity of product A.

As in the symmetric case, an increase in the degree of substitutability γ reduces the likelihood that firms may gain from quantity discounts. An increase in the degree of asymmetry c, by contrast, increases the likelihood that quantity discounts benefit the more efficient firm and harm the less efficient one. Unless firms are almost symmetric, or products almost independent, quantity discounts harm the less efficient firm.

5.2.3 Consumer surplus

The effect of quantity discounts on consumer surplus is illustrated in Figure 5. An increase in the degree of asymmetry increases the likelihood of a negative effect. Consumers may benefit from quantity discounts only if the degree of asymmetry is low and the products are close substitutes.

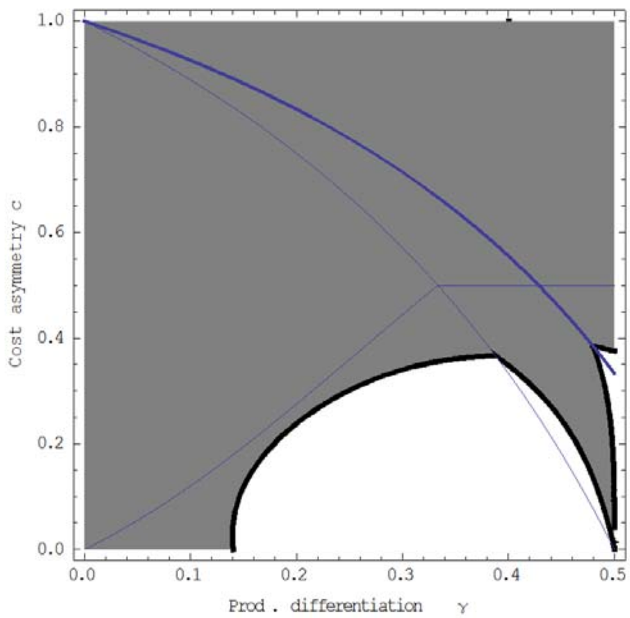

Figure 5: In the grey region, quantity discounts decrease consumer surplus.

5.2.4 The Carlton and Waldman test

Though the maximisation of consumer surplus is sometimes advocated as the proper goal of competition policy, this criterion may be misleading in price discrimination cases. Carlton and Waldman (2008) argue that

> [...] an antitrust claim involving exclusion requires that there be (i) harm to a rival, (ii) harm to consumers and (iii) a linkage between the harm to the rival and the harm to consumers. For example, a

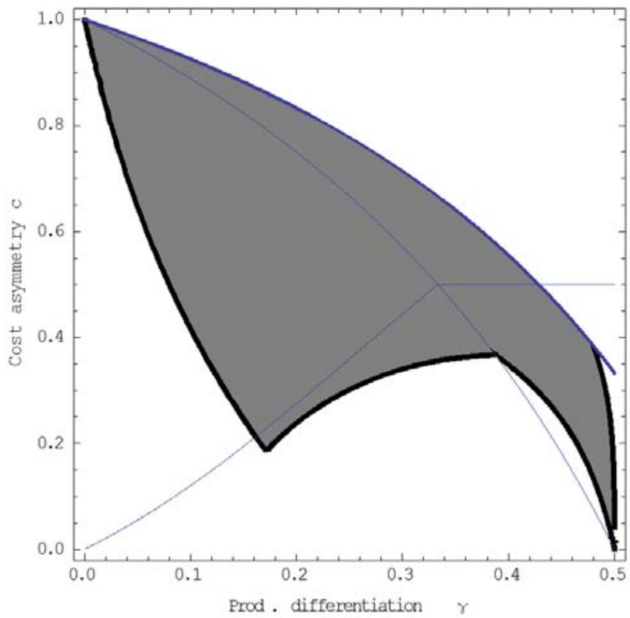

Figure 6: In the grey region, conditions (i) and (ii) of the Carlton and Waldman test are satisfied.

> monopolist who switches from simple monopoly pricing to discriminatory pricing may harm consumers but because no rival is affected should not (and is not) regarded as violating the antitrust laws. (Carlton and Waldman, 2008, p. 1 [Roman numbering added])

Figure 6 represents the region where conditions (i) and (ii) of the Carlton and Waldman test are both met. If condition (iii) is also satisfied (which, however, should not be taken for granted), the Carlton and Waldman test seems to imply that quantity discounts may be anti-competitive in a sizeable region of the parameter space.

5.2.5 Social welfare

Finally, consider the classic criterion of social welfare maximisation, where social welfare is defined as the sum of producers' and consumers' surplus. Unlike the symmetric case, there does exist a region where quantity discounts decrease social welfare – the grey area in Figure 7. However, social welfare can decrease only for intermediate values of c and γ. Thus, identifying the circumstances in which a ban is desirable may be a formidable task in practice.

The reason why prohibiting quantity discounts cannot increase social welfare for "extreme" values of the parameters is easy to grasp. When c is low, firms are almost symmetric and hence the results for the symmetric case apply. When c is high, the less efficient firm exerts little competitive pressure and so the market resembles a monopoly, where quantity discounts are welfare increasing. When γ is close to zero, the products are almost independent. There are effectively two separate monopolies, so quantity discounts are again welfare increasing. When γ is large, the products are close substitutes, and the positive "competition-enhancing" effect of quantity discounts prevails.

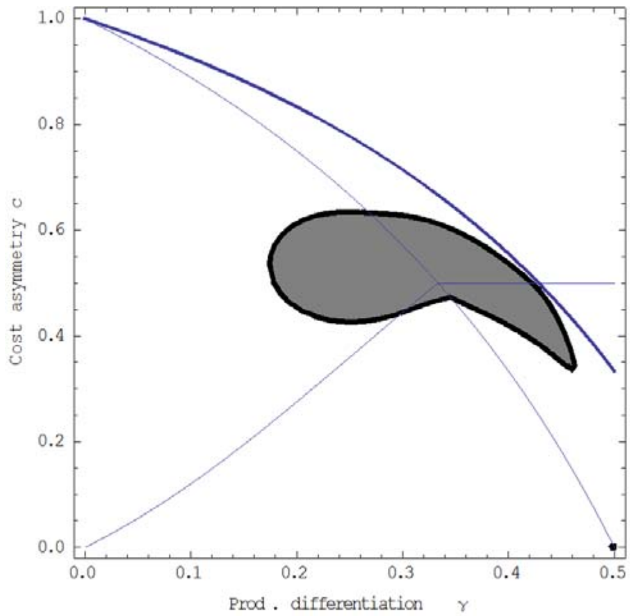

Figure 7: Quantity discounts decrease social welfare in the grey region.

For intermediate values of c and γ, the picture is less clear. To illustrate how quantity discounts can decrease welfare, consider for instance the point $c = \frac{1}{2}$ and $\gamma = \frac{2}{5}$, where social welfare is higher with linear pricing. With non-linear pricing, firm A behaves as an unconstrained monopolist. With linear pricing, by contrast, firm B is active. As we have seen above, the intuitive reason is that firm A prices above marginal cost and so high type consumers purchase an inefficiently low amount of product A. Although firm B is not very efficient, it exerts some competitive pressure on firm A. As a result, firm A lowers its price, and a much larger set of consumer types than under non-linear pricing is served. This increases consumer surplus significantly and leads to a welfare improvement.

6 A selective ban

So far we have compared the case that both firms can engage in quantity discounts to the case where both are constrained to use linear prices. However, competition policy can regulate the unilateral behaviour of dominant firms only. In the legal jargon, the less efficient firm, which cannot possibly have a dominant position, has no "special responsibility." Hence, it should always be free to use quantity discounts. For these reasons, it is also interesting to compare the case that both firms can engage in quantity discounts to the case where only firm B can.

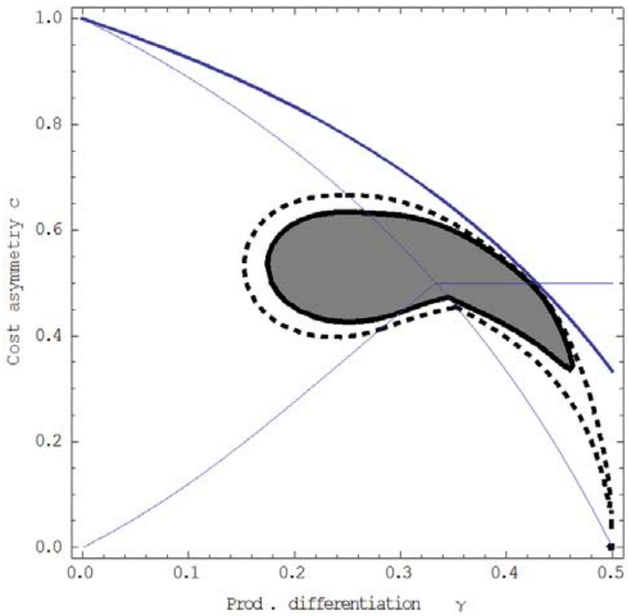

Figure 8: A selective ban on quantity discounts decreases social welfare in the region within dashed contours.

In the latter case, firm B is excluded when $c \geq \hat{c}^{\ell}$, as with linear pricing. The reason is exactly the same as before. It is also obvious that if firm B is excluded, the equilibrium (where firm A is a *de facto* monopolist) is the same as under linear pricing. We therefore focus on the case in which firm B is active.

Proposition 6 *With a selective ban, when firm B is active, i.e. if $c < \hat{c}^{\ell}$, the following is an equilibrium. Firm B offers the price schedule*

$$P_B^{**}(q_B) = \frac{1}{2}\left[1 + \frac{p_A^{**}\gamma - c(1-\gamma)}{1-\gamma}\right] q_B - \frac{1-2\gamma}{4(1-\gamma)} q_B^2 \quad \textit{for } 0 \leq q_B \leq \breve{q}_B,$$

where p_A^{**} is firm A's equilibrium price, which can be calculated explicitly and is reported in the Appendix.

Proof. See the Appendix.

Although firm B alone can now use quantity discounts, low demand consumers still purchase product A only, irrespective of the level of c. The intuition is that quantity discounts entail not only greater marginal prices, but also bigger quantity distortions for lower types. This means that the set of consumer types who purchase exclusively from firm A will be even larger than in the linear pricing equilibrium.

With a selective ban, firm B still charges no fixed fee, but the no-distortion-at-the-top property no longer holds. The intuitive reason is that firm A now prices above marginal cost, so consumer $\theta = 1$ purchases an inefficiently low amount of product A. Firm B does not further distort the consumption of consumer $\theta = 1$; that is to say, the marginal price faced by consumer $\theta = 1$ is equal to c. However, because this consumer buys an inefficiently low amount of product A and the products are substitutes, he will purchase an inefficiently large amount of product B.

The effects of a selective ban on producers' and consumers' surplus and social welfare are as follows. Firm A is almost always harmed by a selective ban: it can only benefit in a small subset of the (already small) region where non-linear pricing would generate a prisoners' dilemma for the firms. Firm B, by contrast, always benefits from a selective ban. There is also an increase in the region of the parameter space in which banning quantity discounts increases social welfare, as shown in Figure 8. However, the region is still relatively small and difficult to identify.

7 Conclusion

Our analysis has revealed that quantity discounts can have anti-competitive effects even if they are not part of a predatory strategy. This means that competition policy should not necessarily apply the standards required in predatory pricing cases to quantity discount cases.

Having said this, we must stress that our results do not lend support to the harsh policy currently adopted in Europe, which comes close to a *per se* prohibition of quantity discounts. In particular, we have found that while quantity discounts do facilitate the exclusion of the less efficient firm, in the absence of fixed costs exclusion occurs if and only if it is, indeed, socially efficient. This argument puts to rest the legal motivation for prohibiting non-linear pricing by a dominant firm as a means to prevent socially inefficient exclusion, unless the dominant firm is depriving the rival of economies of scale.

We have also found that quantity discounts decrease social welfare (i.e. the sum of producers' and consumers' surplus) only for a limited range of intermediate values of the degree of asymmetry and of product substitutability. Thus, even in a highly structured model as ours, identifying the circumstances in which banning quantity discounts increases social welfare can be an exceedingly difficult task for antitrust authorities.

The case for prohibiting quantity discounts might be stronger if competition policy aimed at the maximisation of consumer surplus (in which case quantity discounts should be prohibited when firms are highly asymmetric *or* the products are weak substitutes), or if they adopted the Carlton and Waldman test, according to which exclusionary abuses require (i) harm to a rival, (ii) harm to consumers and (iii) a linkage between the two. (In this latter case, quantity discounts should be prohibited when firms are highly asymmetric *and* the products are fairly close substitutes.) However, these alternative welfare criteria are still controversial.

Our analysis indicates that quantity discounts often have positive welfare effects even if practiced by a dominant firm that faces a smaller rival, and hence suggests that the policy adopted by European antitrust authorities is excessively harsh.

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Appendix

Proof of Proposition 1.

Consumer θ's demand for product i is

$$q_i = \max\left[\frac{\gamma p_j - (1-\gamma)p_i}{1-2\gamma} + \theta, 0\right]$$

if $q_j > 0$, and

$$q_i = \max\left[\frac{\theta - p_i}{1-\gamma}, 0\right]$$

if $q_j = 0$.

Next we aggregate the individual demands into the total demand functions. To proceed, we assume that $p_B \geq p_A$ as firm B has a greater unit cost than firm A. Later, we verify that this inequality holds in equilibrium.

When $p_B \geq p_A > 0$, there are three groups of consumers: low-θ types, $\theta \in [0, p_A]$, who do not buy any product; intermediate-θ types, $\theta \in [p_A, \hat{\theta}^{\ell}]$, who buy only product q_A; and high-θ types, $\theta \in [\hat{\theta}^{\ell}, 1]$, who buy both products. The critical threshold $\hat{\theta}^{\ell}$ is implicitly defined by the condition that $q_B = 0$ when $q_A > 0$, which gives:

$$\hat{\theta}^{\ell} = \frac{(1-\gamma)p_B - \gamma p_A}{1-2\gamma}$$

Summing across consumers one obtains the total demand for the two products:

$$\begin{aligned} Q_A &= \int_{p_A}^{\hat{\theta}^{\ell}} \frac{\theta - p_A}{1-\gamma} d\theta + \int_{\hat{\theta}^{\ell}}^{1} \left[\theta + \frac{\gamma p_B - (1-\gamma)p_A}{1-2\gamma}\right] d\theta \\ &= \frac{(1-\gamma)(p_B - p_A)^2 + [1-2\gamma-(1-\gamma)p_B + gp_A]\,[1-2\gamma+(1+\gamma)p_B+\gamma p_A]}{2(1-2\gamma)^2} \end{aligned}$$

$$\begin{aligned} Q_B &= \int_{\hat{\theta}^{\ell}}^{1} \left[\theta + \frac{\gamma p_A - (1-\gamma)p_B}{1-2\gamma}\right] d\theta \\ &= \frac{[1-2\gamma-(1-\gamma)p_B+\gamma p_A]^2}{2(1-2\gamma)^2} \end{aligned}$$

Firms' profits are $\pi_A = p_A Q_A$ and $\pi_B = (p_B - c)\, Q_B$, respectively.

The best response function of firm B is

$$p_B = \frac{1-2\gamma+2c(1-\gamma)+\gamma p_A}{3(1-\gamma)}.$$

This has a positive intercept and is increasing, with a slope equal to $\frac{\gamma}{3(1-\gamma)} < 1$. The best response function of firm A is

$$p_A = \frac{2[1-(3-\gamma)\gamma] + 2\gamma^2(1-p_B) - \sqrt{\Xi}}{3[1-(3-\gamma)\gamma]}$$

where

$$\Xi = 1+\gamma\left\{-3+\gamma+4\gamma^3+2(1-2\gamma)\left[-3+\gamma(5+\gamma)\right]p_B+\left[3-12\gamma+\gamma^2(12+\gamma)\right]p_B^2\right\}.$$

A's best response function has a positive intercept too. It is always increasing, with a slope lower than one.

Equilibrium prices are

$$p_A^* = \frac{2(3-\gamma)(1-2\gamma)(3-5\gamma)+10\gamma^2(1-\gamma)c-3\sqrt{\Omega}}{(3-4\gamma)[9-4(6-\gamma)\gamma]},$$

where

$$\begin{aligned}\Omega \equiv\ & \{3-2\gamma\left[5+\gamma(2\gamma-5)\right]\}^2 - 8(1-\gamma)\gamma(1-2\gamma)\{3-\gamma\left[7-\gamma(2+\gamma)\right]\}c + \\ & +4(1-\gamma)^2\gamma\{3-\gamma\left[12-\gamma(12+\gamma)\right]\}c^2,\end{aligned}$$

and

$$p_B^* = \frac{1-2\gamma+2c(1-\gamma)+\gamma p_A^*}{3(1-\gamma)}.$$

The calculations leading to these results are reported in the Web Appendix, which also shows that $p_B^* \geq p_A^*$.

The above analysis assumes that firm B is active. It is immediate to check that the optimal linear price for firm A when it is a pure monopolist is $p_M = \frac{1}{3}$. Now consider firm B's best response to $p_A = \frac{1}{3}$. This is

$$p_B = \frac{5+6c-\frac{2}{(1-\gamma)}}{9}.$$

It is easy to verify that $p_B - c \geq 0$ if and only if $c \leq \hat{c}^\ell$. This means that when $c \geq \hat{c}^\ell$ firm A can engage in monopoly pricing. The We Appendix verifies that firm A's linear limit price coincides with the linear monopoly price $p_M = \frac{1}{3}$, confirming that in equilibrium there is no limit pricing region. ■

Proof of Proposition 2.

We start by reporting the equilibrium value of the coefficients $\alpha_{0,A}^d$, $\alpha_{0,A}^{\lim}$, $\hat{q}_A$ and $\tilde{q}_A$. As stated in the proposition, these guarantee that the smooth pasting conditions hold, that is, that A's price schedule be continuous and continuously differentiable at $\hat{q}_A$ and $\tilde{q}_A$. Continuity of firm A's price schedule at $\hat{q}_A$ and $\tilde{q}_A$ requires

$$\alpha_{0,A}^d + \alpha_{1,A}^*\hat{q}_A - \frac{\alpha_{1,A}^*}{2}\hat{q}_A^2 = \alpha_{0,A}^{\lim} + \alpha_{1,B}^*\hat{q}_A - \left(\frac{1}{2}-\gamma\right)\hat{q}_A^2$$

and

$$\alpha_{0,A}^{\lim} + \alpha_{1,B}^{*}\tilde{q}_A - \left(\frac{1}{2} - \gamma\right)\tilde{q}_A^2 = \frac{1}{2}\tilde{q}_A - \frac{1-\gamma}{4}\tilde{q}_A^2,$$

respectively. Differentiability of A's price schedule is equivalent to continuity of the equilibrium quantity, which requires

$$\hat{q}_A = q_A^{d*}(\hat{\theta}) = q_A^{\lim}(\hat{\theta})$$

and

$$\tilde{q}_A = q_A^{\lim}(\tilde{\theta}) = q_A^{m}(\tilde{\theta}),$$

where

$$q_A^{\lim}(\theta) = \frac{\theta - \left[\alpha_1^* + c\left(1 - \alpha_1^*\frac{1-\gamma}{1-2\gamma}\right)\right]}{\gamma}$$

is the limit quantity, and

$$q_A^{m}(\theta) = \frac{2\theta - 1}{1-\gamma}$$

is the monopoly quantity.

When $c > \tilde{c}$, where

$$\tilde{c} \equiv \frac{1}{2}\frac{(1-2\alpha_1^*)(1-2\gamma)}{1-\alpha_1^*(1-\gamma)-2\gamma},$$

the solution to this system of equations is

$$\alpha_{0,A}^{\lim *} = -\frac{[(1-2\gamma)(1-2c-2\alpha_1^*) + 2\alpha_1^* c(1-\gamma)]^2}{4(1-2\gamma)^2(1-3\gamma)} < 0,$$

$$\alpha_{0,A}^{d*} = \alpha_{0,A}^{\lim} - \frac{c^2(1-2\gamma-\alpha_1^*)}{2(1-2\gamma)^2} < 0,$$

$$\hat{q}_A^* = \frac{c}{1-2\gamma} > 0$$

and

$$\tilde{q}_A^* = \frac{2\alpha_1^* c}{1-2\gamma} - \frac{(1-2\alpha_1^*)(1-2c)}{1-3\gamma} > 0$$

When instead $c \leq \tilde{c}$, firm B's cost is so small that the monopoly region vanishes, $\tilde{q}_A^* = 0$, and firm A is always constrained by its competitor. The fixed fee $\alpha_{0,A}^{\lim}$ also vanishes, so that the no-fixed-fee property $P_A^*(0) = 0$ continues to hold, and we have

$$\alpha_{0,A}^{d*} = -\frac{c^2(1-2\gamma-\alpha_1^*)}{2(1-2\gamma)^2}.$$

The cutoff quantity is $\hat{q}_A^*$ is the same as above.

Next we show that the price schedules specified in the proposition satisfy the best response property in the set of all feasible price schedules (that is, not necessarily piecewise quadratic). The verification procedure is as follows. Given

P_j^*, firm i faces a monopolistic screening problem where type θ has an indirect utility function

$$v_i^*(q_i,\theta) = \max_{q_j \geq 0} \left[u(q_i, q_j, \theta) - P_j^*(q_j) \right],$$

which accounts for any benefit he can obtain by optimally trading with firm j. As u is quadratic and P_j^* is piecewise quadratic, v_i^* is also piecewise quadratic. It may have kinks, but we shall show that any such kink preserves concavity, so the indirect utility function is globally concave.

However, the consumer's reservation utility $v_i^*(0,\theta)$ is now type dependent. Thus, in order to apply the standard approach of pointwise maximisation of the virtual surplus function, we will have to check not only that the sorting condition is satisfied, but also that the equilibrium rent increases with θ more quickly than $v_i^*(0,\theta)$ so that the consumer's participation constraint $v_i^*(q_i,\theta) \geq v_i^*(0,\theta)$ binds only at the lowest participating type $\hat{\theta}_i$ (see Jullien, 2000).

This latter property, however, is implied by the sorting condition. To show this, notice that when the indirect utility function is differentiable, the sorting condition is satisfied if the following condition holds

$$\int_0^{q_i} \frac{\partial^2 v_i^*(u,\theta)}{\partial\theta \partial q_i} du \geq 0.$$

This is equivalent to

$$\frac{\partial v_i^*(q_i,\theta)}{\partial\theta} \geq \frac{\partial v_i^*(0,\theta)}{\partial\theta}.$$

But this inequality implies that if $v_i^*(q_i,\theta_i) \geq v_i^*(0,\theta_i)$ then the participation constraint is, indeed, satisfied for any $\theta \geq \hat{\theta}_i$.

Thus, provided that the sorting condition holds, firm i's problem reduces to finding a function $q_i^+(\theta)$ that pointwise maximises the "indirect virtual surplus"

$$s_i(q_i,\theta) = v_i^*(q_i,\theta) - c_i q_i - (1-\theta)\frac{dv_i^*}{d\theta}.$$

We check ex-post, as usual, that the maximiser $q_i(\theta)$ satisfies the standard monotonicity condition. The verification of the best response property is completed by checking that $q_i^+(\theta) = q_i^*(\theta)$.

Consider, then, firm A's best response to the equilibrium price schedule of firm B, $P_B^*(q_B)$. The indirect utility function is piecewise quadratic, with two branches corresponding to the case in which the $\arg\max_{q_B \geq 0} [u(q_A, q_B, \theta) - P_B^*(q_B)]$ is 0 or is strictly positive, and a kink between the two branches:

$$v_A^*(q_A,\theta) = \begin{cases} \theta q_A - \dfrac{1-\gamma}{2} q_A^2 & \text{if } q_B = 0 \text{ or, equivalently, } q_A \geq q_A^{\lim}(\theta) \\ A_0 + A_1 q_A + A_2 q_A^2 & \text{if } q_B > 0 \text{ or, equivalently, } q_A < q_A^{\lim}(\theta). \end{cases}$$

The coefficients A_0, A_1 and A_2 can be calculated as

$$\begin{aligned}
A_0 &= \frac{\left[(\theta-c)(1-2\gamma)-\alpha_1^*(1-c(1-\gamma)-2\gamma)\right]^2}{2(1-\gamma-\alpha_1^*)(1-2\gamma)^2},\\
A_1 &= \gamma\frac{c(1-2\gamma)+\alpha_1^*(1-c(1-\gamma)-2\gamma)}{(1-\gamma-\alpha_1^*)(1-2\gamma)}+\theta\frac{1-2\gamma-\alpha_1^*}{1-\gamma-\alpha_1^*}\\
A_2 &= -\frac{1-2\gamma+\alpha_1^*(1-\gamma)}{2(1-\gamma-\alpha_1^*)}<0.
\end{aligned}$$

On both branches of the indirect utility function, the coefficients of the quadratic terms are negative. In addition, it can be checked that

$$\left.\frac{\partial^2 v_A^*(q_A,\theta)}{\partial q_A^2}\right|_{q_A\le q_A^{\lim}(\theta)}=A_2\ge\left.\frac{\partial^2 v_A^*(q_A,\theta)}{\partial q_A^2}\right|_{q_A>q_A^{\lim}(\theta)}=-\left(1-\gamma\right),$$

so the function v_A^* is globally concave. It can also be checked that the sorting condition is satisfied as

$$\frac{\partial^2 v_A^*}{\partial\theta\partial q_A}=\begin{cases}1 & \text{if } q_A\ge q_A^{\lim}(\theta)\\ \dfrac{1-2\gamma-\alpha_1^*}{1-\gamma-\alpha_1^*}>0 & \text{if } q_A<q_A^{\lim}(\theta).\end{cases}$$

We can therefore obtain A's best response by maximising the virtual surplus function $s_A(q_A,\theta)$. Like the indirect utility function, the virtual surplus function is piecewise quadratic with a kink. The maximum can occur in either one of the two quadratic branches, or at the kink. To be precise:

$$q_A^+(\theta)=\begin{cases}\dfrac{2\theta-1}{1-\gamma} & \text{if } \gamma<\frac{1}{3} \text{ and } \frac{1}{2}\le\theta\le\theta_1\\ q_A^{\lim}(\theta) & \text{if } \gamma<\frac{1}{3} \text{ and } \theta_1\le\theta\le\theta_2 \text{ or if } \gamma\ge\frac{1}{3} \text{ and } \alpha_{1,B}^*\le\theta\le\theta_2\\ \dfrac{\theta-\alpha_1^*}{1-\alpha_1^*}+\dfrac{c\gamma}{1-2\gamma} & \text{if } \theta\ge\theta_2,\end{cases}$$

where

$$q_A^{\lim}(\theta)\equiv\frac{\theta-\alpha_1^*}{\gamma}+c\frac{\alpha_1^*\gamma-(1-\alpha_1^*)(1-2\gamma)}{(1-2\gamma)\gamma},$$

$$\theta_1=\frac{\alpha_1^*(1-\gamma)-\gamma}{1-3\gamma}+c(1-\gamma)\frac{1-\alpha_1^*(1-\gamma)-2\gamma}{1+\gamma(6\gamma-5)}(=\tilde{\theta}^*)$$

and

$$\theta_2=\frac{c(1-\gamma)+\alpha_1^*(1-2\gamma-c(1-\gamma))}{1-2\gamma}(=\hat{\theta}^*).$$

When $\gamma\ge\frac{1}{3}$ or $\gamma<\frac{1}{3}$ and $\theta_1<\frac{1}{2}$, which happens for $c<\tilde{c}$, the optimum is never achieved on the upper branch of the indirect utility function. In other words, firm A's best response never involves setting the quantity at the monopoly level.

Finally, one can easily check that $q_A^+(\theta)=q_A^*(\theta)$, which implies that in order to implement the quantities $q_A^+(\theta)$ firm A must, indeed, offer the equilibrium price schedule $P_A^*(q_A)$.

Consider now firm B. The procedure is the same as for firm A, but now we must distinguish between two cases, depending on whether A's price schedule comprises also the lower (monopoly) branch or not. Consider first the case in which $\tilde{q}_A^* = 0$ and hence there is no monopoly branch of A's price schedule.

The indirect utility function of a consumer who trades with firm B then is

$$v_B^*(q_B, \theta) = \begin{cases} \theta q_B - \frac{1-\gamma}{2} q_B^2 & \text{if } q_A = 0 \text{ or, equivalently, if } q_B \geq q_B^{\lim}(\theta) \\ \hat{B}_0 + \hat{B}_1 q_B + \hat{B}_2 q_B^2 & \text{if } 0 < q_A \leq \hat{q}_A \text{ or, equivalently, if } \check{q}_B(\theta) \leq q_B < q_B^{\lim}(\theta) \\ B_0 + B_1 q_B + B_2 q_B^2 & \text{if } q_A > \hat{q}_A \text{ or, equivalently, if } 0 < q_B \leq \check{q}_B(\theta) \end{cases}$$

where

$$\begin{aligned} q_B^{\lim}(\theta) &= \frac{\theta - \alpha_1^*}{\gamma} - \frac{\alpha_1^* c}{1-2\gamma} \\ \check{q}_B(\theta) &= \frac{\theta - \alpha_1^* - c(1-\alpha_1^*)}{\gamma} + \frac{\alpha_1^* c}{1-2\gamma}. \end{aligned}$$

The coefficients of the lower branches of the indirect utility functions are

$$\begin{aligned} \hat{B}_0 &= \frac{(\theta - c)^2}{2\gamma} \\ \hat{B}_1 &= c \\ \hat{B}_2 &= -\frac{1-2\gamma}{2} \end{aligned}$$

and

$$\begin{aligned} B_0 &= \frac{2\theta - 1}{2(1-\gamma)} \\ B_1 &= \theta - \frac{\gamma}{1-\gamma} \\ B_2 &= -\frac{1-\gamma}{2}. \end{aligned}$$

All branches are concave, and global concavity can be checked by comparing the left and right derivatives of $v_B^*(q_B, \theta)$ at the kinks. The sorting condition can also be checked as for firm A. We can therefore find B's best response by pointwise maximisation of the virtual surplus function.

One can easily check that there is never an interior maximum on the upper or intermediate branch of the virtual surplus function. This is equivalent to saying that firm B is active only when firm A supplies the duopoly quantity $q_A^{d*}(\theta)$. Pointwise maximisation of the virtual surplus function leads to

$$q_B^+(\theta) = \frac{\theta - \alpha_1^*}{1 - \alpha_1^*} - c\frac{1-\gamma}{1-2\gamma}.$$

This coincides with $q_B^*(\theta)$, thereby confirming that $P_B^*(q_B)$ is the best response.

The case where firm A's price schedule comprises also the lower (monopoly) branch is similar. The indirect utility function $v_B^*(q_B, \theta)$, and hence the virtual surplus $s_B(q_B,\theta)$, now comprise four branches (all quadratic). The equation of the fourth branch, which corresponds to $0 < q_A < \tilde{q}_A^*$, is

$$v_B^*(q_B,\theta) = \tilde{B}_0 + \tilde{B}_1 q_B + \tilde{B}_2 q_B^2$$

where

$$\begin{aligned}
\tilde{B}_0 &= \frac{(2\theta-1)^2}{4(1-\gamma)} \\
\tilde{B}_1 &= \frac{\theta+\gamma(1-3\gamma)}{1-\gamma} \\
\tilde{B}_2 &= -\frac{1-\gamma(2+\gamma)}{2(1-\gamma)}.
\end{aligned}$$

However, it turns out that the optimum still lies on the lower branch where $q_A > \hat{q}_A$ and that it entails $q_B^+(\theta) = q_B^*(\theta)$. This observation completes the proof of Proposition 2. ■

Proof of Proposition 3.

Like in the proof of Proposition 2, we start by determining the equilibrium values of the coefficients $\alpha_{0,A}^m$ and $\tilde{q}_A$. They are determined by a smooth pasting condition that imposes the continuity and differentiability of the price schedule at $\tilde{q}_A$. Continuity requires that

$$c\tilde{q}_A - \left(\frac{1}{2}-\gamma\right)\tilde{q}_A^2 = \alpha_{0,A}^m + \frac{1}{2}\tilde{q}_A - \frac{1-\gamma}{4}\tilde{q}_A^2,$$

whereas differentiability, which is equivalent to the continuity of the equilibrium quantity, requires

$$\tilde{q}_A = q_A^{\lim}(\tilde{\theta}) = q_A^m(\tilde{\theta}),$$

where now

$$q_A^{\lim}(\theta) = \frac{\theta-c}{\gamma}.$$

The solution is

$$\tilde{q}_A^* = \frac{2c-1}{1-3\gamma} > 0$$

and

$$\alpha_{0,A}^{m*} = \frac{(2c-1)^2}{4(1-3\gamma)} > 0$$

when $c < \frac{1}{2}$, and $\tilde{q}_A^* = \alpha_{0,A}^{m*} = 0$ when $c \geq \frac{1}{2}$.

The rest of proof is similar to the proof of Proposition 2. Now, however, it suffices to find firm A's best response to $P_B^*(q_B) = cq_B$. The indirect utility function is:

$$v_A^*(q_A,\theta) = \begin{cases} \theta q_A - \frac{1-\gamma}{2}q_A^2 & \text{if } q_B = 0 \text{ or, equivalently, } q_A \geq q_A^{\lim}(\theta) \\ A_0 + A_1 q_A + A_2 q_A^2 & \text{if } q_B > 0 \text{ or, equivalently, } q_A < q_A^{\lim}(\theta), \end{cases}$$

where now

$$A_0 = \frac{(\theta-c)^2}{2(1-\gamma)}$$
$$A_1 = \frac{\theta(1-2\gamma)+c\gamma}{1-\gamma}$$
$$A_2 = -\frac{1-2\gamma}{2(1-\gamma)}.$$

On both branches of the indirect utility function, the coefficients of the quadratic terms are negative. In addition, it can be checked that

$$\left.\frac{\partial^2 v_A^*(q_A,\theta)}{\partial q_A^2}\right|_{q_A \le q_A^{\lim}(\theta)} = A_2 \geq \left.\frac{\partial^2 v_A^*(q_A,\theta)}{\partial q_A^2}\right|_{q_A > q_A^{\lim}(\theta)} = -(1-\gamma),$$

so the function v_A^* is globally concave. It can also be checked that the sorting condition is satisfied as

$$\frac{\partial^2 v_A^*}{\partial\theta\partial q_A} = \begin{cases} 1 & \text{if } q_A \geq q_A^{\lim}(\theta) \\ \frac{1-2\gamma}{1-\gamma} > 0 & \text{if } q_A < q_A^{\lim}(\theta). \end{cases}$$

We can therefore obtain A's best response by maximising the virtual surplus function $s_A(q_A,\theta)$. Like the indirect utility function, the virtual surplus function is piecewise quadratic with a kink. The maximum can occur on the upper branch, or at the kink. It can never occur on the lower branch, where

$$q_A < q_A^{\lim}(\theta) = \frac{\theta-c}{\gamma}.$$

To show this, notice that the optimum on the upper branch is

$$q_A^-(\theta) = 2\left[2\theta - 1 + c\gamma\left(1-2\gamma\right)\right].$$

However, it can easily be checked that $q_A^-(\theta) > \frac{\theta-c}{\gamma}$, a contradiction.

When $\gamma < \frac{1}{3}$ or $\gamma \geq \frac{1}{3}$ and $c > \frac{1}{2}$, the maximisation of the virtual surplus leads to

$$q_A^+(\theta) = \frac{2\theta-1}{1-\gamma}(\equiv q_A^m(\theta)).$$

When instead $\gamma \geq \frac{1}{3}$ and $c \leq \frac{1}{2}$, we have

$$\begin{array}{ll} q_A^+(\theta) = q_A^{\lim}(\theta) & \text{if } c \leq \theta \leq \theta_3 \\ q_A^+(\theta) = q_A^m(\theta) & \text{if } \theta_3 \leq \theta \leq 1 \end{array}$$

where

$$\theta_3 = \frac{\gamma - c(1-\gamma)}{3\gamma-1}(=\tilde{\theta}^*).$$

Finally, one can easily check that $q_A^+(\theta) = q_A^*(\theta)$, which implies that in order to implement the quantities $q_A^+(\theta)$ firm A must, indeed, offer the equilibrium price schedule $P_A^*(q_A)$.

To complete the proof, it suffices to check that $q_A^+(\theta) \geq q_A^{\lim}(\theta)$, which implies that the demand for product B when $p_B = c$ vanishes. ■

Proof of Proposition 6.

To calculate the equilibrium with a selective ban, consider firm B's best response function first. For any given p_A, define the indirect utility function as

$$v(q_B, \theta) = \max_{q_A \geq 0} [u - p_A q_A]$$

This indirect utility function accounts for any benefit the consumer can obtain by purchasing product A at the constant price p_A. The maximum is achieved at

$$q_A = \max\left\{\frac{\theta - p_A - \gamma q_B}{1-\gamma}, 0\right\}.$$

As discussed in the main text, in equilibrium firm B will serve only consumers who purchase positive amounts of product A. Thus, there is no loss of generality in focusing on the solution where $q_A > 0$, where the indirect utility function becomes

$$v(q_B, \theta) = \frac{\theta^2 + (2\theta - p_A - q_B)\left[(1-2\gamma)q_B - p_A\right]}{2(1-\gamma)}.$$

It is easy to verify that the indirect utility function satisfies the sorting condition $\frac{\partial^2 v}{\partial\theta \partial q_B} > 0$, and that the participation constraints binds for the lowest participating type. (Although there is a type-dependent reservation utility $\frac{\theta^2 - p_A(2\theta - p_A)}{2(1-\gamma)}$, this does not affect the solution.) Thus, firm B's best response can be determined as the solution to a well-behaved monopolistic screening problem. Define the virtual surplus function as

$$\begin{aligned} s(q_B, \theta) &= v(q_B, \theta) - (1-\theta)\frac{\partial v(q_B, \theta)}{\partial \theta} - c q_B \\ &= \frac{3\theta^2 - 2\theta + (4\theta - p_A - q_B - 2)\left[(1-2\gamma)q_B - p_A\right]}{2(1-\gamma)} - c q_B. \end{aligned}$$

The optimal quantity $q_B^+(\theta)$ is found by pointwise maximisation of the virtual surplus function, yielding

$$q_B(\theta) = 2\theta - 1 + \frac{p_A\gamma - c(1-\gamma)}{1-2\gamma}.$$

One can finally calculate the price schedule that supports the quantity $q_B^+(\theta)$, given that firm A prices at p_A:

$$P_B(q_B) = \frac{1 - 2\gamma + c(1-\gamma) + \gamma p_A}{2(1-\gamma)} q_B - \frac{1-2\gamma}{4(1-\gamma)} q_B^2.$$

This if firm B's best response function. Notice that the coefficient of the quadratic term is independent of p_A. Thus, the pricing game is effectively equivalent to a standard Bertrand game where firms choose linear prices, with the twist that firm B is actually choosing the coefficient of the linear term of a quadratic pricing function

$$P_B(q_B) = p_B q_B - \frac{1-2\gamma}{4(1-\gamma)} q_B^2.$$

In this pricing game, firm B's best response function is

$$p_B = \frac{1-2\gamma + c(1-\gamma) + \gamma p_A}{2(1-\gamma)},$$

as we have just seen. When firm B offers a non linear price $p_B q_B - \beta_2 q_B^2$ where $\beta_2 = \frac{1-2\gamma}{4(1-\gamma)}$ and firm A offers a linear price p_A, firm A's quantity is

$$Q_A = \int_{p_A}^{\hat{\theta}^{mix}} \frac{\theta - p_A}{1-\gamma} d\theta + \int_{\hat{\theta}^{mix}}^{1} \frac{\theta(1-2\gamma+2\beta_2) - p_A(1-\gamma+2\beta_2) + \gamma p_B}{1-2\gamma+2\beta_2(1-\gamma)} d\theta$$

where the first integral corresponds to types buying only from firm A, the second to types buying from both firms, and

$$\hat{\theta}^{mix} = \frac{(1-\gamma)p_B - \gamma p_A}{1-2\gamma}.$$

maximising $\pi_A = p_A Q_A$ with respect to p_A gives firm A's best response:

$$p_A = \frac{2[1-3\gamma+2\beta_2(1-2\gamma)] + 2\gamma^2(2-p_B) - 1/2\sqrt{\Psi}}{6\beta_2(1-2\gamma) + 3[1-(3-\gamma)\gamma]}$$

where

$$\begin{aligned}\Psi &= 12[(3-\gamma)\gamma - 2\beta_2(1-2\gamma) - 1][1 + 2\beta_2 - (4 - 2p_B + p_B^2 + 4\beta_2)\gamma + (2-p_B)^2\gamma^2] + \\ &\quad +16[1 - 2\beta_2(1-2\gamma) - \gamma(3 + (2-p_B)\gamma)]^2.\end{aligned}$$

The equilibrium price of firm A is (see the Web Appendix for details):

$$p_A^{**} = \frac{4 - \gamma\{16 - \gamma[19 + 3c(1-\gamma) - 6\gamma]\} - \sqrt{2}\sqrt{\Theta}}{6 - \gamma[24(1-\gamma) + 5\gamma^2]}$$

where

$$\begin{aligned}\Theta &= (1-\gamma)\{2 - \gamma[2c(1-2\gamma)(3-6\gamma+2\gamma^3) - c^2(1-\gamma)(3+2\gamma(-6+\gamma(6+\gamma))) + \\ &\quad +(1-\gamma)(-11+2\gamma(7+2\gamma(-5+2\gamma)))]\}.\end{aligned}$$

Substituting this expression into B's best response one gets the coefficient of the linear term in B's price schedule.

Finally, substituting the equilibrium prices into $\hat{\theta}^{mix}$, the Web Appendix verifies that $\hat{\theta}^{mix} < 1$, and hence firm B is active, if and only if $c < \hat{c}^{\ell}$.■