This is an online Appendix to the paper "On regulation and competition: pros and cons of a diversified monopolist" by G. Calzolari and C. Scarpa

Model Setup: Betrand competition with differentiated products

```
(*NOTATION: variables with index 1 are of the conglomerate,
variables without index are of the rivals*)
```

```
(*Consumer
 preferences: using symmetry and we explicitate y1 and all other (n-1) firms' output*)
```

$$u = \mu\,(y1 + (n-1)\,y) - 1/2\,\left(y1^2 + (n-1)\,y^2\right) - \gamma\,((n-1)\,y1\,y + y\,y\,(n-2)\,(n-1));$$

```
(*The restrictions on the parameters are: γ belongs to [-1,1],
negative are complements,
positive are substitutes, =0 independent and =1 perfect substitutes*)
```

```
(*with these preferences demand functions are as
  follows (they are obtained solving consumer's optimization,
   before imposing symmetry, as in Hackner 1999):*)
```

$$\text{y1param} = \text{FullSimplify}\left[\mu\,\frac{1}{(\gamma\,(n-1)+1)} - \frac{\gamma\,(n-2)+1}{(1-\gamma)\,(\gamma\,(n-1)+1)}\,p1 + \frac{\gamma\,(n-1)}{(1-\gamma)\,(\gamma\,(n-1)+1)}\,p\right]$$

$$\text{yparam} = \text{FullSimplify}\left[\mu\,\frac{1}{(\gamma\,(n-1)+1)} - \frac{\gamma\,(n-2)+1}{(1-\gamma)\,(\gamma\,(n-1)+1)}\,p + \frac{\gamma}{(1-\gamma)\,(\gamma\,(n-1)+1)}\,p1 + \frac{\gamma\,(n-2)}{(1-\gamma)\,(\gamma\,(n-1)+1)}\,pj\right]$$

```
(*Note: writing the demand of the generic competitor we must differentiate the price of
   firm 1 and the prices of the other n-2 non-conglomerate rivals using the price pj*)
```

$$\frac{p1 + (-2+n)\,p1\,\gamma - \mu + \gamma\,(p - n\,p + \mu)}{(-1+\gamma)\,(1 + (-1+n)\,\gamma)}$$

$$\frac{p + (-2+n)\,p\,\gamma - (p1 + (-2+n)\,pj)\,\gamma + (-1+\gamma)\,\mu}{(-1+\gamma)\,(1 + (-1+n)\,\gamma)}$$

```
(*Identification of the parameters of the
 standard linear demand with differentiated products*)
y1 = m - b p1 + s (n - 1) p;
y = m - b p + s p1 + s (n - 2) pj;
```

$$\text{mparam} = \mu \frac{1}{(\gamma (n-1)+1)};$$

$$\text{bparam} = \frac{\gamma (n-2)+1}{(1-\gamma)(\gamma (n-1)+1)};$$

$$\text{sparam} = \frac{\gamma}{(1-\gamma)(\gamma (n-1)+1)};$$

```
(*Defining profits*)
Profit1 = y1 p1 - c y1 + θ q y1;
Profit = y p - c y;
```

Analysis of competition

```
(*COMPETITION WITH INCOMPLETE INFORMATION: regulation is uniform and
   does not transmit information to the rivals which remain uninformed*)
Profit1uAI = Profit1 /. {θ -> θu , p1 → p1u};
Profit1dAI = Profit1 /. {θ -> θd , p1 → p1d};

ProfituAI = Profit /. p1 → p1u;
ProfitdAI = Profit /. p1 → p1d;
EProfitAI = v ProfituAI + (1 - v) ProfitdAI;


DEProfitAI = Simplify[D[EProfitAI, p] /. pj → p];
FullSimplify[
 Solve[{DEProfitAI == 0, D[Profit1uAI, p1u] == 0, D[Profit1dAI, p1d] == 0}, {p, p1u, p1d}]]
```

$$\Big\{\Big\{p1u \to \frac{s\,(2\,m + (-1+n)\,q\,s\,(-1+v)\,(\theta d - \theta u)) + 4\,b^2\,(c - q\,\theta u) + 2\,b\,(2\,m + s\,(c + (-2+n)\,q\,\theta u))}{2\,(2\,b+s)\,(2\,b+s-n\,s)},$$

$$p \to \frac{2\,b^2\,c + m\,s + b\,(2\,m + s\,(c + q\,((-1+v)\,\theta d - v\,\theta u)))}{(2\,b+s)\,(2\,b+s-n\,s)},$$

$$p1d \to \frac{4\,b^2\,(c - q\,\theta d) + 2\,b\,(2\,m + s\,(c + (-2+n)\,q\,\theta d)) + s\,(2\,m + (-1+n)\,q\,s\,v\,(\theta d - \theta u))}{2\,(2\,b+s)\,(2\,b+s-n\,s)}\Big\}\Big\}$$

```
(*COMPETITION WITH FULL
 INFORMATION: information has been transmitted by discriminatory regulation*)
DProfit = D[Profit, p] /. pj → p;
stmp = Simplify[Solve[{D[Profit1, p1] == 0, DProfit == 0}, {p1, p}]]
p1FI = p1 /. stmp[[1, 1]];
pFI = p /. stmp[[1, 2]];

y1FI = FullSimplify[y1 /. {p1 -> p1FI, p -> pFI}];
yFI = FullSimplify[y /. pj → p /. {p1 -> p1FI, p -> pFI}];

Profit1FI = FullSimplify[Profit1 /. {p1 -> p1FI, p -> pFI}]
ProfitFI = FullSimplify[Profit /. pj → p /. {p1 -> p1FI, p -> pFI}]
```

$$\left\{\left\{p1 \to \frac{m\,s + 2\,b^2\,(c - q\,\theta) + b\,(2\,m + s\,(c + (-2 + n)\,q\,\theta))}{(2\,b + s)\,(2\,b + s - n\,s)},\ p \to \frac{2\,b^2\,c + m\,s + b\,(2\,m + s\,(c - q\,\theta))}{(2\,b + s)\,(2\,b + s - n\,s)}\right\}\right\}$$

$$\frac{b\left((2\,b + s)\,(-m + c\,(b + s - n\,s)) + q\left(-2\,b^2 + b\,(-2 + n)\,s + (-1 + n)\,s^2\right)\theta\right)^2}{(2\,b + s)^2\,(2\,b + s - n\,s)^2}$$

$$\frac{b\,((2\,b + s)\,(-m + c\,(b + s - n\,s)) + b\,q\,s\,\theta)^2}{(2\,b + s)^2\,(2\,b + s - n\,s)^2}$$

```
(*Note: p1 is decreasing in qθ as well as p and
  profit1FI is increasing and profitFI is decreasing in it*)

(*With screening regulation, off the equilibrium the conglomerate reports untruthfully
 to the regulator and hence the information passed to the rivals is manipulated:
  we use the notation that the first letter refers
   to the true type and the second to the announced one,
i.e. p1ud is the  price of the conglomerate when its true type is θu but it announces θd
*)

(*Low type when announcing truthfully*)
p1dd = p1FI /. {θ -> θd , q → qd};
pdd = pFI /. {θ -> θd , q → qd};
Profit1dd = FullSimplify[Profit1 /. {p1 → p1dd, p → pdd} /. {θ -> θd , q → qd}]
Profitdd = FullSimplify[Profit /. pj → p /. {p1 → p1dd, p → pdd} /. {θ -> θd , q → qd}]
```

$$\frac{b\left((2\,b + s)\,(-m + c\,(b + s - n\,s)) + qd\left(-2\,b^2 + b\,(-2 + n)\,s + (-1 + n)\,s^2\right)\theta d\right)^2}{(2\,b + s)^2\,(2\,b + s - n\,s)^2}$$

$$\frac{b\,((2\,b + s)\,(-m + c\,(b + s - n\,s)) + b\,qd\,s\,\theta d)^2}{(2\,b + s)^2\,(2\,b + s - n\,s)^2}$$

```
(*High type when announcing truthfully*)
p1uu = p1FI /. {θ -> θu , q → qu};
puu = pFI /. {θ -> θu , q → qu};
Profit1uu = FullSimplify[Profit1 /. {p1 → p1uu, p → puu} /. {θ -> θu , q → qu}]
Profituu = FullSimplify[Profit /. pj → p /. {p1 → p1uu, p → puu} /. {θ -> θu , q → qu}]
```

$$\frac{b\left((2\,b + s)\,(-m + c\,(b + s - n\,s)) + qu\left(-2\,b^2 + b\,(-2 + n)\,s + (-1 + n)\,s^2\right)\theta u\right)^2}{(2\,b + s)^2\,(2\,b + s - n\,s)^2}$$

$$\frac{b\,((2\,b + s)\,(-m + c\,(b + s - n\,s)) + b\,qu\,s\,\theta u)^2}{(2\,b + s)^2\,(2\,b + s - n\,s)^2}$$

```
(*High type when announcing untruthfully*)
p1ud = p1 /.
   FullSimplify[Solve[{0 == D[Profit1, p1] /. p -> pdd}, {p1}] /. {q → qd, θ → θu}][[1, 1]];
pud = pdd;
Profit1ud = FullSimplify[Profit1 /. {p1 → p1ud, p → pud} /. {θ -> θu , q → qd} ];
Profitud = FullSimplify[Profit /. pj → p /. {p1 → p1ud, p → pud} /. {θ -> θu , q → qd}] ;
```

```
(*These semplifications allow to rewrite the prices as in the paper*)
FullSimplify[p1uu - p1ud]
```

$$\text{FullSimplify}\left[\text{p1ud} - \left(\text{p1uu} - \left(\frac{2\,b\,(qu - qd)\,(-2\,b + (-2 + n)\,s)\,\theta u + qd\,(-1 + n)\,s^2\,(\theta d - \theta u)}{2\,(2\,b + s)\,(2\,b + s - n\,s)}\right)\right)\right]$$

$$\frac{2\,b\,qu\,(-2\,b + (-2 + n)\,s)\,\theta u + qd\,\left((-1 + n)\,s^2\,(\theta d - \theta u) + 4\,b^2\,\theta u - 2\,b\,(-2 + n)\,s\,\theta u\right)}{2\,(2\,b + s)\,(2\,b + s - n\,s)}$$

0

```
FullSimplify[puu - pud]
```

$$\text{FullSimplify}\left[\text{pud} - \left(\text{puu} - \left(\frac{b\,s\,(qd\,(\theta d - \theta u) - (qu - qd)\,\theta u)}{(2\,b + s)\,(2\,b + s - n\,s)}\right)\right)\right]$$

$$\frac{b\,s\,(qd\,\theta d - qu\,\theta u)}{(2\,b + s)\,(2\,b + s - n\,s)}$$

0

```
(*Low type when announcing untruthfully*)
p1du = p1 /.
   FullSimplify[Solve[{0 == D[Profit1, p1] /. p -> puu}, {p1}] /. {q → qu, θ → θd}][[1, 1]];
pdu = puu;
Profit1du = FullSimplify[Profit1 /. {p1 → p1du, p → pdu} /. {θ -> θd , q → qu}] ;
Profitdu = FullSimplify[Profit /. pj → p /. {p1 → p1du, p → pdu} /. {θ -> θd , q → qu} ];
```

```
(*These semplifications allow to rewrite the prices as in the paper*)
FullSimplify[p1dd - p1du]
```

$$\text{FullSimplify}\left[\text{p1du} - \left(\text{p1dd} - \left(\frac{(-qd + qu)\,\theta d\,\left(4\,b^2 - 2\,b\,(-2 + n)\,s\right) - (-1 + n)\,qu\,s^2\,(\theta d - \theta u)}{2\,(2\,b + s)\,(2\,b + s - n\,s)}\right)\right)\right]$$

$$\frac{4\,b^2\,(-qd + qu)\,\theta d + 2\,b\,(-2 + n)\,(qd - qu)\,s\,\theta d - (-1 + n)\,qu\,s^2\,(\theta d - \theta u)}{2\,(2\,b + s)\,(2\,b + s - n\,s)}$$

0

```
FullSimplify[puu - pud]
```

$$\text{FullSimplify}\left[\text{pud} - \left(\text{puu} - \left(\frac{b\,s\,(qd\,(\theta d - \theta u) - (qu - qd)\,\theta u)}{(2\,b + s)\,(2\,b + s - n\,s)}\right)\right)\right]$$

$$\frac{b\,s\,(qd\,\theta d - qu\,\theta u)}{(2\,b + s)\,(2\,b + s - n\,s)}$$

0

```
(*incidentally notice that when goods become perfect substitutes,
prices converge to marginal cost, as expected*)
Limit[puu /. θu → 0 /. {m → mparam, b → bparam, s → sparam}, γ → 1]
Limit[p1uu /. θu → 0 /. {m → mparam, b → bparam, s → sparam}, γ → 1]
Limit[pdd /. θd → 0 /. {m → mparam, b → bparam, s → sparam}, γ → 1]
Limit[p1dd /. θd → 0 /. {m → mparam, b → bparam, s → sparam}, γ → 1]
```

c

c

c

c

```
(*The informational Rent is defined as follows...*)
Rent = FullSimplify[Profit1ud - Profit1dd]

(*and values zero when the theta are identical*)
FullSimplify[Rent /. θu → θd]
```

$$-\frac{1}{4 (2 b + s) (2 b + s - n s)} b\, qd (\theta d - \theta u) \left(4 b^2 (-2 c + qd (\theta d + \theta u)) + 2 b (4 m + s (c (-6 + 4 n) - (-2 + n) qd (\theta d + \theta u))) + s (4 m + (-1 + n) s (4 c - qd (3 \theta d + \theta u)))\right)$$

0

```
(*The Rent can be conveninently rewritten as follows, as in the paper...*)
FullSimplify[
 b qd (θu - θd) / (4 (2 b + s) (2 b + s - n s))
   (4 (2 b + s) (m - c (b + s - n s))
     + qd (  (2 b + s) (2 b + s - n s) (θd + θu) - s^2 (-1 + n) 2 θd  )
   )
  - Rent]
```

0

```
(*The Rent is positive since the term qd
  (   (2 b+s) (2 b+s-n s) (θd+θu)-s^2 (-1+n) 2 θd    ) is positive. To see this
  take the most unfavorable cases of θu=θd and b at its lowest admissible level,
b=(n-1)s, then the expression becomes 4 (-1+n)^2 qd s^2 θd>0 *)

FullSimplify[qd (  (2 b + s) (2 b + s - n s) (θd + θu) - s^2 (-1 + n) 2 θd  ) /. b → (n - 1) s /. θu -> θd]
```

$4 (-1 + n)^2\, qd\, s^2\, \theta d$

$4 (-1 + n)^2\, qd\, s^2\, \theta d$

```
(*Now we derive incentive compatibility constraint*)
IC = FullSimplify[Profit1uu - Profit1du - (Profit1ud - Profit1dd)]
```

$$\frac{1}{4 (2 b + s) (2 b + s - n s)} b (\theta d - \theta u) \left(4 b^2 (qd - qu) (-2 c + (qd + qu) (\theta d + \theta u)) - 2 b (qd - qu) (-4 m + s (c (6 - 4 n) + (-2 + n) (qd + qu) (\theta d + \theta u))) + s \left(4 m (qd - qu) + (-1 + n) s \left(4 c (qd - qu) - qd^2 (3 \theta d + \theta u) + qu^2 (\theta d + 3 \theta u)\right)\right)\right)$$

```
(*We now rewrite IC in a compact way...*)
```

```
K = (qd + qu) (θd + θu) (4 b^2 - 2 b s (-2 + n)) + 4 (2 b + s) (m - c (b + s - n s));
H = -s^2 (n - 1) ((qu^2 - qd^2) (3 θd + θu) + 2 qu^2 (θu - θd));
FullSimplify[b (θu - θd) / (4 (2 b + s) (2 b + s - n s)) ((qu - qd) K + H) - IC]
```

0

```
(*since K>0 and, with monotonicity i.e. qu>qd, H<0,
it follows that monotonicity is here necessary but no sufficient for IC>0*)
```

Now we move to regulation

```
(*First we verify monotonicity of regulated outputs by
 directly studying the derivative of the info rent w.r.t. qd*)
FullSimplify[D[Rent, qd]]
```

$$-\frac{1}{2 (2 b + s) (2 b + s - n s)} b (\theta d - \theta u) \left(4 b^2 (-c + qd (\theta d + \theta u)) + 2 b (2 m + s (c (-3 + 2 n) - (-2 + n) qd (\theta d + \theta u))) + s (2 m + (-1 + n) s (2 c - qd (3 \theta d + \theta u)))\right)$$

```
(*Using the same reasoning we used to show that Rent>0,
we can show that the Rent is increasing in qd*)

(*hence since the Rent enters with a minus in the maximand in the social welfare
 for qd, and since qu instead is undistorted, it follows that indeed qu>qd*)

(*WELFARE*)
(*regulated demand and regulated consumer surplus*)
pr = mr - q;
CSRu = Integrate[pr, q] /. q → qu;
CSRd = Integrate[pr, q] /. q → qd;

(*unregulated consumers' surplus*)
CSUu = FullSimplify[u /. pj → p /. {p1 → p1uu, p → puu} /. {m → mparam, b → bparam, s → sparam}];
CSUd = FullSimplify[u /. pj → p /. {p1 → p1dd, p → pdd} /. {m → mparam, b → bparam, s → sparam}];

(*the costs of all firms*)
Costconglomu = FullSimplify[c (q + y1) - θ q y1 /. q → qu /. θ → θu /. {p1 → p1uu, p → puu} /.
    {m → mparam, b → bparam, s → sparam}];
Costconglomd = FullSimplify[c (q + y1) - θ q y1 /. q → qd /. θ → θd /. {p1 → p1dd, p → pdd} /.
    {m → mparam, b → bparam, s → sparam}];

Costrivalsu =
  FullSimplify[c y (n - 1) /. {p1 → p1uu, p → puu} /. {m → mparam, b → bparam, s → sparam}];
Costrivalsd = FullSimplify[c y (n - 1) /. {p1 → p1dd, p → pdd} /.
    {m → mparam, b → bparam, s → sparam}];
```

```
(*the profits when separation is imposed*)
ProfitSep = FullSimplify[Profit1 /. q → 0 /. p1 → p1uu /. p → puu /. qu → 0]

(*and a simple check*)
FullSimplify[Profit /. pj → p /. p1 → p1uu /. p → puu /. qu → 0]
```

$$\frac{b\,(-b\,c+m+c\,(-1+n)\,s)^2}{(2\,b+s-n\,s)^2}$$

$$\frac{b\,(-b\,c+m+c\,(-1+n)\,s)^2}{(2\,b+s-n\,s)^2}$$

```
(*putting together all the building blocks defining welfare*)
Welfareu = CSRu + CSUu – Costconglomu – Costrivalsu – (1 – a) (n – 1) Profituu –
    (1 – a) (Rent + ProfitSep) /. {m → mparam, b → bparam, s → sparam};

Welfared = CSRd + CSUd – Costconglomd – Costrivalsd –
    (1 – a) (n – 1) Profitdd – (1 – a) (ProfitSep) /. {m → mparam, b → bparam, s → sparam};

(*FINALLY: optimal regulated quantity for type θu*)
qusol = qu /.
   Simplify[Solve[D[Welfareu, qu] == 0, qu] /. {m → mparam, b → bparam, s → sparam}][[1, 1]];

(*and a simple check*)FullSimplify[qusol /. θu → 0]
```

$-c+mr$

```
(*FINALLY: optimal regulated quantity for type θd*)
qdsol = qd /. Simplify[Solve[D[Welfared – (1 – a) v / (1 – v) Rent, qd] == 0, qd] /.
      {m → mparam, b → bparam, s → sparam}][[1, 1]];

(*and a simple check*)
FullSimplify[qdsol /. θd → 0 /. θu → 0]
```

$-c+mr$

```
(*now we define all other the equilibrium main variables*)
yu = y /. pj → p /. {p1 → p1uu, p → puu} /. qu → qusol /. {m → mparam, b → bparam, s → sparam};
y1u = y1 /. pj → p /. {p1 → p1uu, p → puu} /. qu → qusol /. {m → mparam, b → bparam, s → sparam};
yd = y /. pj → p /. {p1 → p1dd, p → pdd} /. qd → qdsol /. {m → mparam, b → bparam, s → sparam};
y1d = y1 /. pj → p /. {p1 → p1dd, p → pdd} /. qd → qdsol /. {m → mparam, b → bparam, s → sparam};

pu = puu /. qu → qusol /. {m → mparam, b → bparam, s → sparam};
p1u = p1uu /. qu → qusol /. {m → mparam, b → bparam, s → sparam};
pd = pdd /. qd → qdsol /. {m → mparam, b → bparam, s → sparam};
p1d = p1dd /. qd → qdsol /. {m → mparam, b → bparam, s → sparam};
```

```
ICsol = IC /. {qd → qdsol, qu → qusol} /. {m → mparam, b → bparam, s → sparam};
Rentsol = Rent /. {qd → qdsol, qu → qusol} /. {m → mparam, b → bparam, s → sparam};


(*DETERMINING UNIFORM REGULATION*)

Welfaredunif = Welfared /. qd → q;
Welfareuunif = Welfareu /. qu → q;
Rentunif = Rent /. qd → q;

quniform = q /.
   Simplify[Solve[D[(1 - v) * Welfaredunif + v * (Welfareuunif - (1 - a) Rentunif), q] == 0, q] /.
      {m → mparam, b → bparam, s → sparam}][[1, 1]];
Rentunifsol = Rentunif /. {m → mparam, b → bparam, s → sparam} /. q -> quniform;

p1uuniform = p1uu /. qu → q /. q -> quniform /. {m → mparam, b → bparam, s → sparam};
p1duniform = p1dd /. qd → q /. q -> quniform /. {m → mparam, b → bparam, s → sparam};
puuniform = puu /. qu → q /. q -> quniform /. {m → mparam, b → bparam, s → sparam};
pduniform = pdd /. qd → q /. q -> quniform /. {m → mparam, b → bparam, s → sparam};


(*Unregulated market with separation*)
ysep = yu /. θu → 0;
psep = pu /. θu → 0;
```

Now we make some comparative statics on main parameters

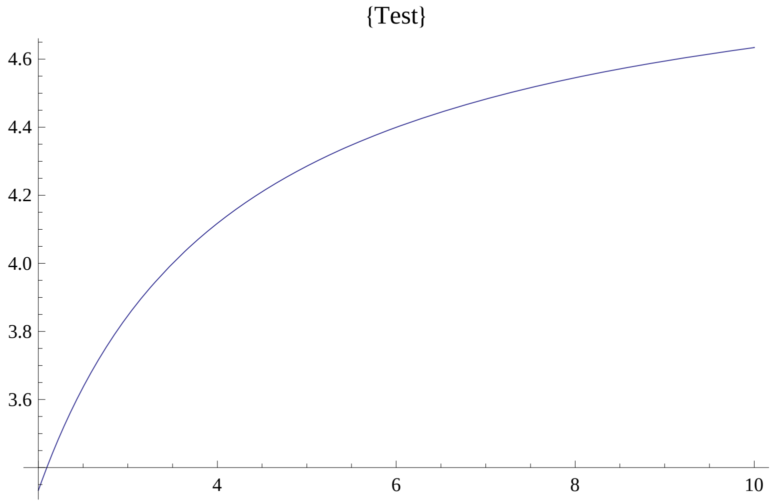
```
(*The following is required to have positive quantities, Test mus be positive*)
Test = FullSimplify[2 b - (n - 1) s /. {m → mparam, b → bparam, s → sparam}]
```

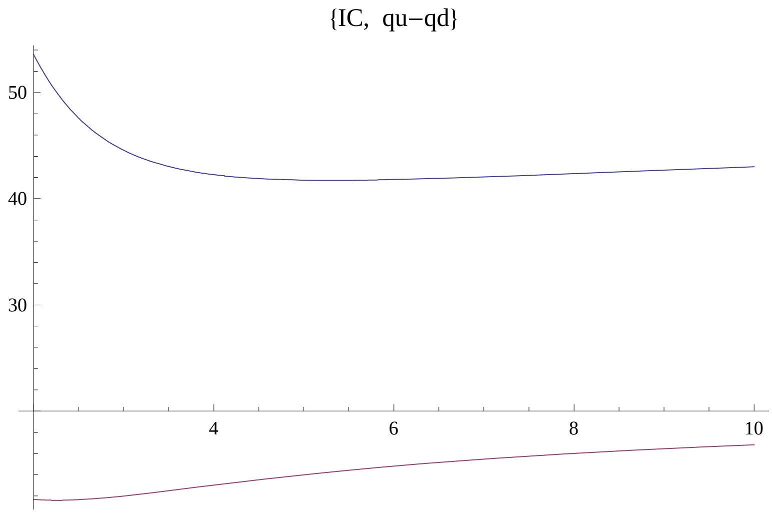
$$-\frac{2 + (-3 + n)\,\gamma}{(-1 + \gamma)\,(1 + (-1 + n)\,\gamma)}$$

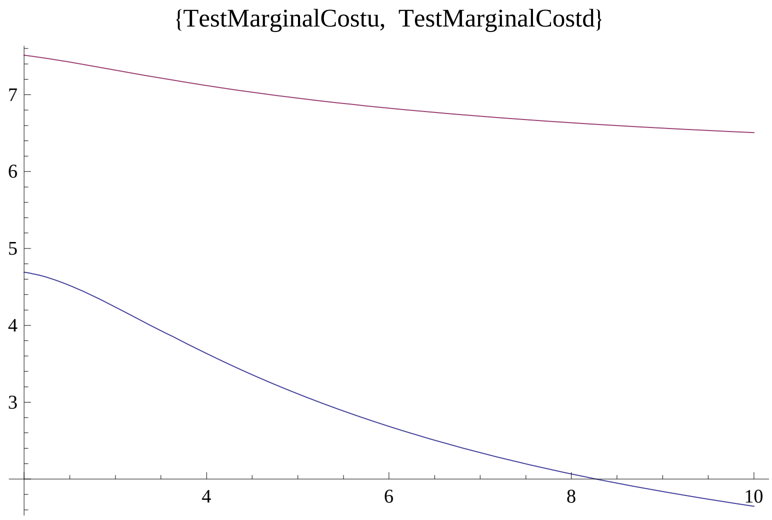
```
(*hence it is positive if the numerator is positive which is always the
 case because γ cannot be larger than 1 (see the conditions on parameters
   when defining the preferences on market u at the very beginning)*)

(*Thest to check that marginal costs are positive*)
TestMarginalCostu = c - θu qusol /. {m → mparam, b → bparam, s → sparam};
TestMarginalCostd = c - θd qdsol /. {m → mparam, b → bparam, s → sparam};

(*NOW WE ARE READY TO MAKE SOME NUMERICAL COMPARATIVE
 STATICS ANALYSIS ON MAIN VARIABLES AND WITH ALL TESTS
 for all main parameters we will have to define some variables to be plotted w.r.t
 to the main parameters, the labels in the graphs explain are self explanatory
 Recall that Mathematica plots in the following sequence of colors: Blue,
Red, Yellow, Green
 The lables in each picture indicate the plotted variables ordered with the above colors
*)
```

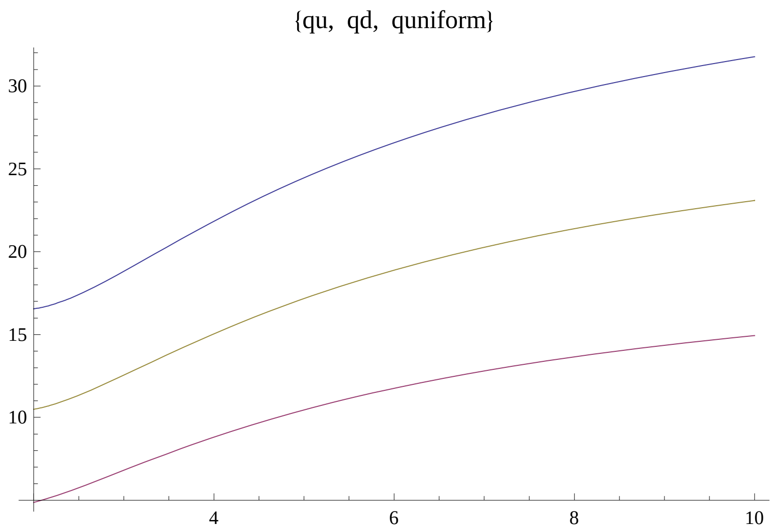
Analysis of n

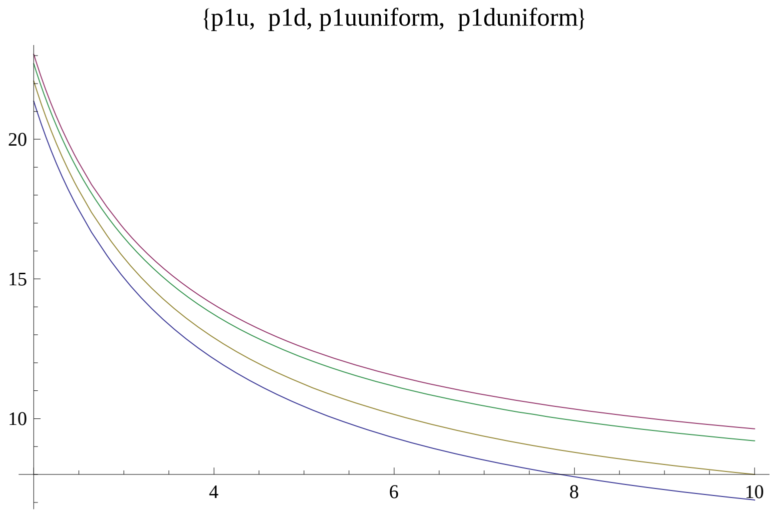
```
parameters = {μ → 100, γ → 0.8, mr → 10, c → 8, θu → 0.2, θd → 0.1, v → 0.5, a → 0.1};

nplotmin = 2;
nplotmax = 10;
ICsolplot = ICsol /. parameters;
TestMarginalCostuplot = TestMarginalCostu /. parameters;
TestMarginalCostdplot = TestMarginalCostd /. parameters;
qusolplot = qusol /. parameters;
qdsolplot = qdsol /. parameters;
p1uplot = p1u /. parameters;
p1dplot = p1d /. parameters;
puplot = pu /. parameters;
pdplot = pd /. parameters;
y1uplot = y1u /. parameters;
y1dplot = y1d /. parameters;
yuplot = yu /. parameters;
ydplot = yd /. parameters;
Yu = y1uplot + (n - 1) yuplot /. parameters;
Yd = y1dplot + (n - 1) ydplot /. parameters;

p1uuniformplot = p1uuniform /. parameters;
p1duniformplot = p1duniform /. parameters;
puuniformplot = puuniform /. parameters;
pduniformplot = pduniform /. parameters;

Rentplot = Rentsol /. parameters;

quniformplot = quniform /. parameters;
Rentunifsolplot = Rentunifsol /. parameters;

Testplot = Test /. parameters;

(*now we can plot...*)
Plot[Testplot /. parameters, {n, nplotmin, nplotmax}, PlotLabel → {"Test"}]

Plot[{ICsolplot, qusolplot - qdsolplot},
 {n, nplotmin, nplotmax}, PlotLabel → {"IC,  qu-qd"}]
Plot[{TestMarginalCostuplot, TestMarginalCostdplot}, {n, nplotmin, nplotmax},
 PlotLabel → {"TestMarginalCostu,  TestMarginalCostd"}]

Plot[{qusolplot, qdsolplot, quniformplot},
 {n, nplotmin, nplotmax}, PlotLabel → {"qu,  qd,  quniform"}]

Plot[{p1uplot, p1dplot, p1uuniformplot, p1duniformplot},
 {n, nplotmin, nplotmax}, PlotLabel → {"p1u,  p1d, p1uuniform,  p1duniform"}]

Plot[{puplot, pdplot, puuniformplot, pduniformplot},
 {n, nplotmin, nplotmax}, PlotLabel → {"pu,  pd,  puuniform,  pduniform"}]

Plot[{y1uplot, y1dplot}, {n, nplotmin, nplotmax}, PlotLabel → {"y1u,  y1d"}]
Plot[{yuplot, ydplot}, {n, nplotmin, nplotmax}, PlotLabel → {"yu,  yd"}]

Plot[{Yu, Yd}, {n, nplotmin, nplotmax}, PlotLabel → {"Yu,  Yd   (total quantities)"}]

Plot[{Rentplot, Rentunifsolplot},
 {n, nplotmin, nplotmax}, PlotLabel → {"Rent,  Rentuniform"}]
```

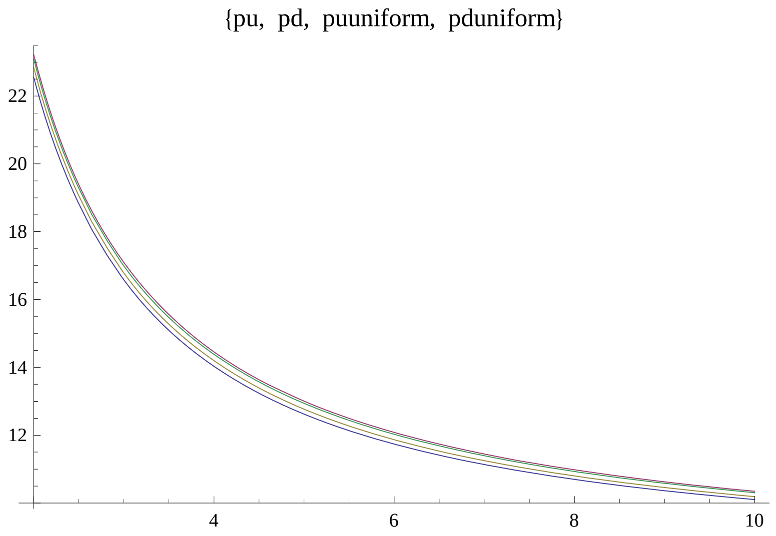

{Test}
4.6
4.4
4.2
4.0
3.8
3.6
4
6
8
10


{IC, qu−qd}
50
40
30
4
6
8
10

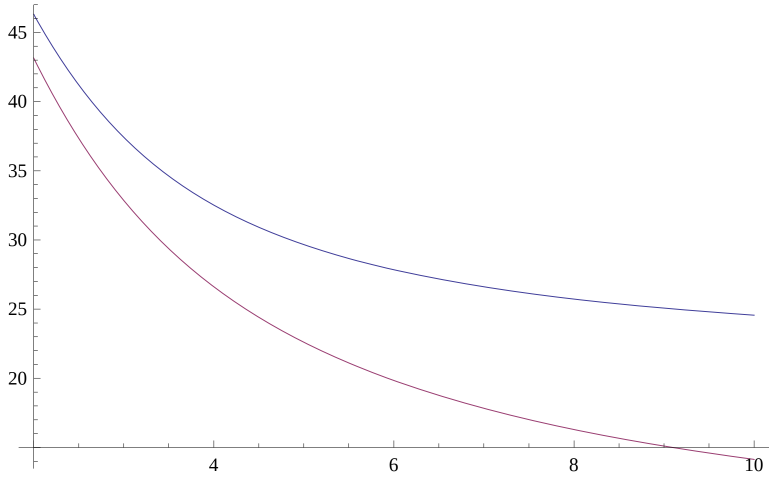

{TestMarginalCostu, TestMarginalCostd}
7
6
5
4
3
4
6
8
10


{qu, qd, quniform}
30
25
20
15
10
4
6
8
10

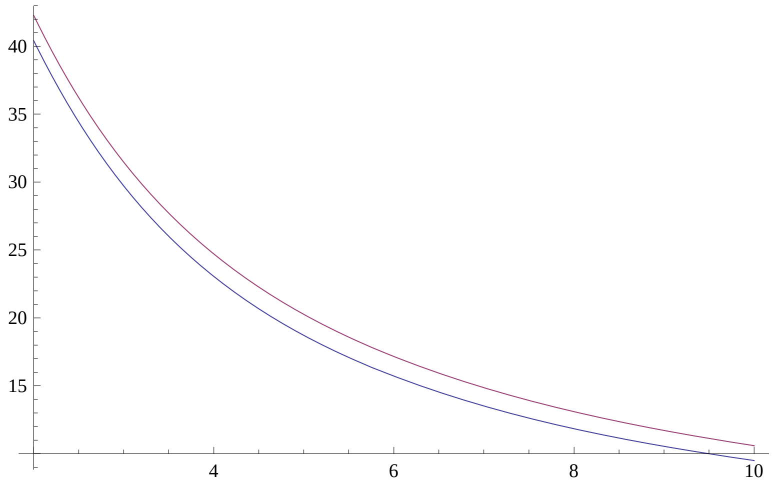

{p1u, p1d, p1uuniform, p1duniform}
20
15
10
4
6
8
10


{pu, pd, puuniform, pduniform}
22
20
18
16
14
12
4
6
8
10

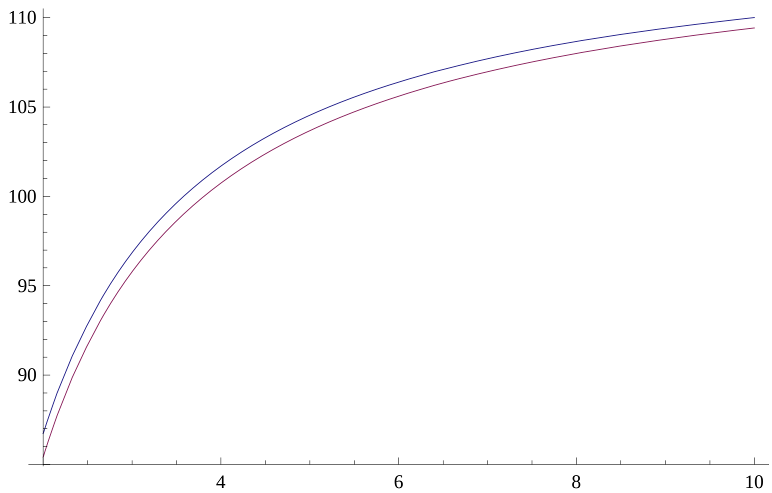
{y1u, y1d}

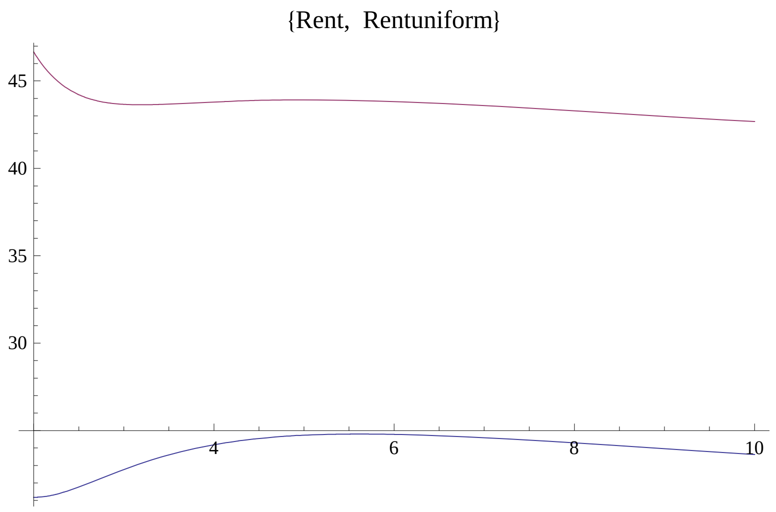


{yu, yd}



{Yu, Yd (total quantities)}



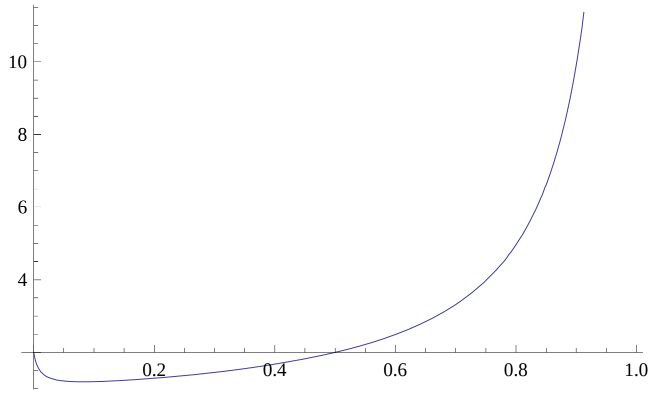

{Rent, Rentuniform}
45
40
35
30
4
6
8
10

```
(*With these paramters: all tests are OK, IC is not binding and,
what is more, the Rent is increasing in n for low n*)
```

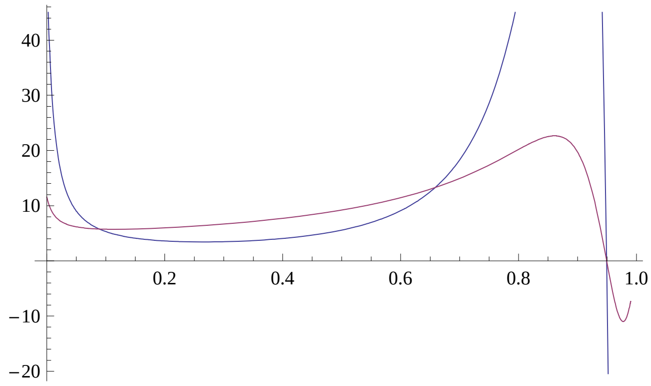
Analysis of γ

```
parameters = {n → 100, μ → 100, mr → 10, c → 8, θu → 0.2, θd → 0.1, v → 0.5, a → 0.1};

ICsolplot = ICsol /. parameters;
TestMarginalCostuplot = TestMarginalCostu /. parameters;
TestMarginalCostdplot = TestMarginalCostd /. parameters;
qusolplot = qusol /. parameters;
qdsolplot = qdsol /. parameters;
p1uplot = p1u /. parameters;
p1dplot = p1d /. parameters;
puplot = pu /. parameters;
pdplot = pd /. parameters;

y1uplot = y1u /. parameters;
y1dplot = y1d /. parameters;
yuplot = yu /. parameters;
ydplot = yd /. parameters;

Yu = y1uplot + (n - 1) yuplot /. parameters;
Yd = y1dplot + (n - 1) ydplot /. parameters;

p1uuniformplot = p1uuniform /. parameters;
p1duniformplot = p1duniform /. parameters;
puuniformplot = puuniform /. parameters;
pduniformplot = pduniform /. parameters;

Rentplot = Rentsol /. parameters;
quniformplot = quniform /. parameters;
Rentunifsolplot = Rentunifsol /. parameters;

Testplot = Test /. parameters;

Plot[Testplot /. parameters, {γ, 0, 0.99}, PlotLabel → {"Test"}]

Plot[{ICsolplot, qusolplot - qdsolplot}, {γ, 0, 0.99}, PlotLabel → {"IC,  qu-qd"}]
Plot[{TestMarginalCostuplot, TestMarginalCostdplot},
 {γ, 0, 0.99}, PlotLabel → {"TestMarginalCostu,  TestMarginalCostd"}]

Plot[{qusolplot, qdsolplot, quniformplot},
 {γ, 0, 0.99}, PlotLabel → {"qu,  qd,  quniform"}]

Plot[{p1uplot, p1dplot, p1uuniformplot, p1duniformplot},
 {γ, 0, 0.99}, PlotLabel → {"p1u,  p1d,  p1uuniform,  p1duniform"}]
Plot[{puplot, pdplot, puuniformplot, pduniformplot}, {γ, 0, 0.99},
 PlotLabel → {"pu,  pd,  plotpuuniform,  pduniform"}]

Plot[{Yu, Yd}, {γ, 0, 0.99}, PlotLabel → {"Yu,  Yd    (total demands)"}]

Plot[{Rentplot, Rentunifsolplot}, {γ, 0, 0.99}, PlotLabel → {"Rent,  Rentuniform"}]
```

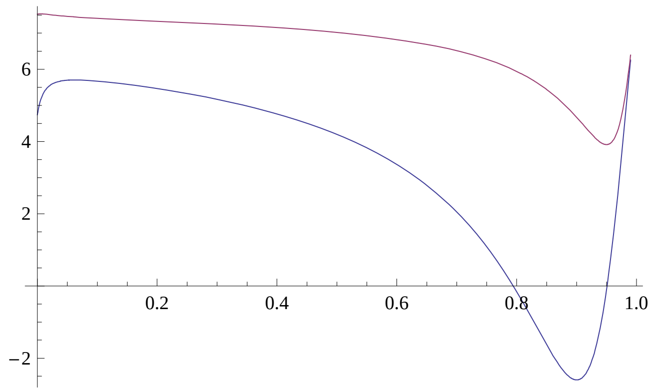
{Test}



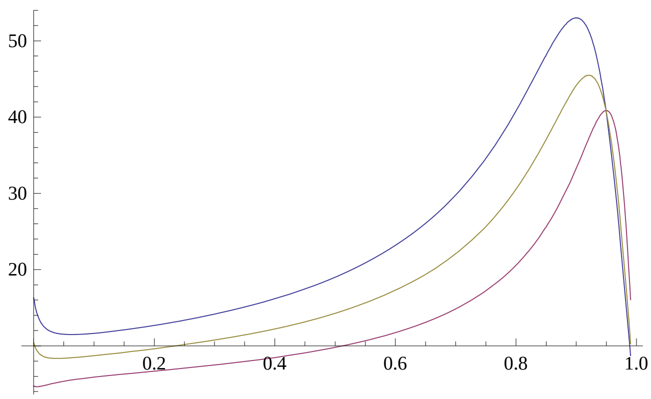
{IC, qu−qd}



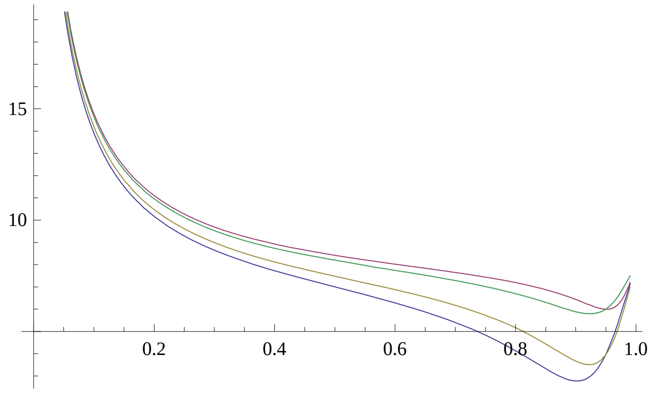
{TestMarginalCostu, TestMarginalCostd}



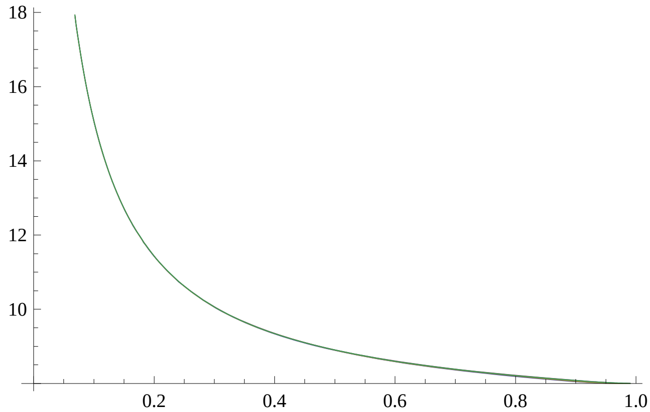
{qu, qd, quniform}



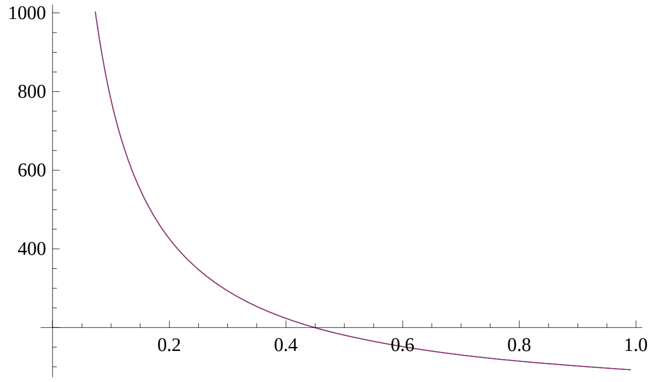
{p1u, p1d, p1uuniform, p1duniform}

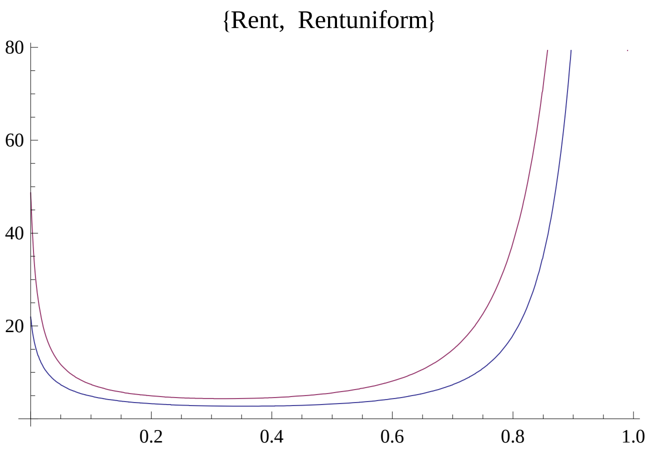


{pu, pd, plotpuuniform, pduniform}



{Yu, Yd (total demands)}

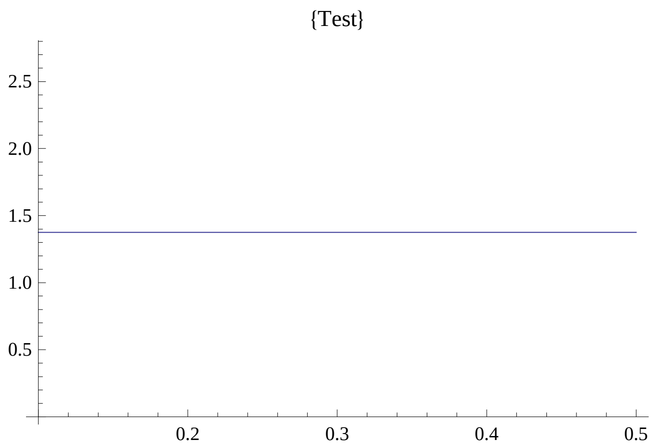


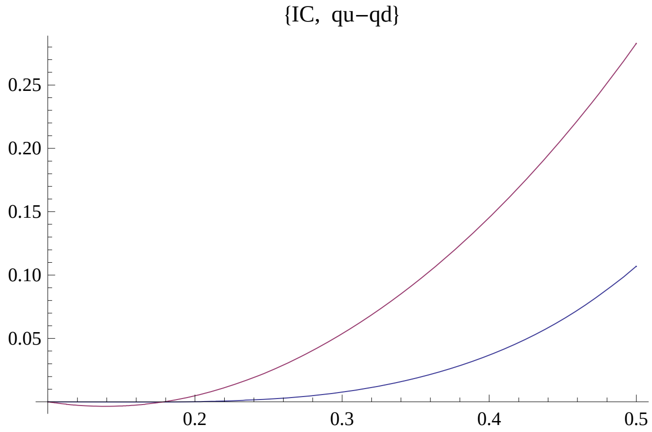


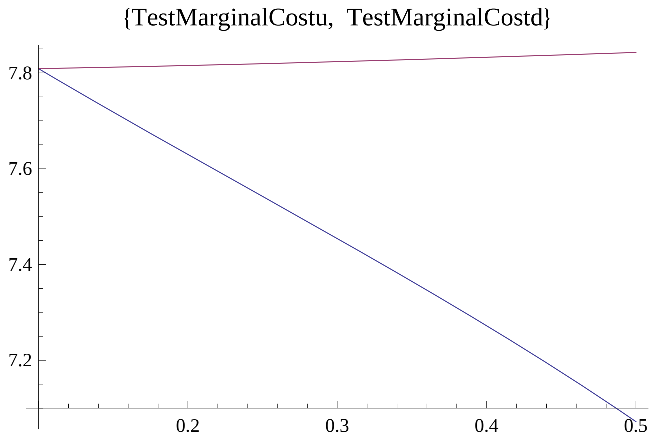
(*With these paramters: the max admissible value for γ is γ= 0.8 since for larger values the marginal cost becomes negative for θu; however the last plot again shows that Rent is non monotone in γ*)

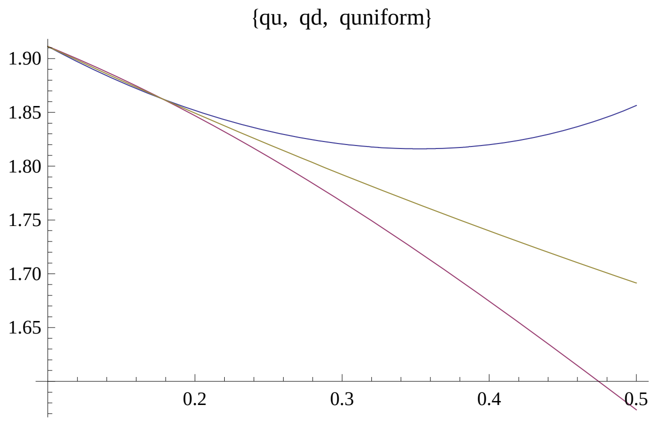
Analysis of θu

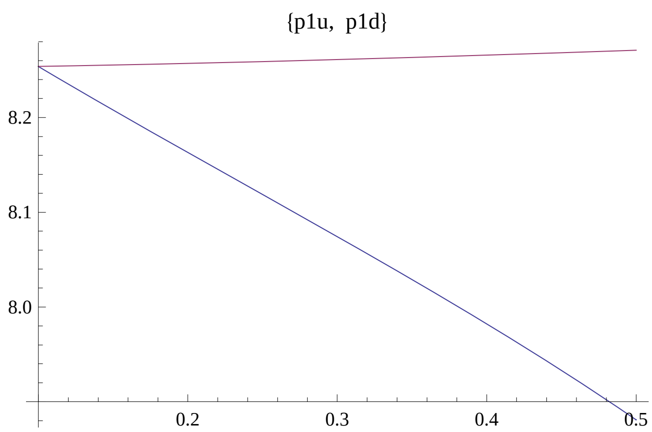
```
parameters = {n → 22, γ → 0.15, μ → 10, mr → 10, c → 8, θd → 0.1, v → 0.5, a → 0.1};

ICsolplot = ICsol /. parameters;
TestMarginalCostuplot = TestMarginalCostu /. parameters;
TestMarginalCostdplot = TestMarginalCostd /. parameters;
qusolplot = qusol /. parameters;
qdsolplot = qdsol /. parameters;
p1uplot = p1u /. parameters;
p1dplot = p1d /. parameters;
puplot = pu /. parameters;
pdplot = pd /. parameters;
y1uplot = y1u /. parameters;
y1dplot = y1d /. parameters;
yuplot = yu /. parameters;
ydplot = yd /. parameters;

Yu = y1uplot + (n - 1) yuplot /. parameters;
Yd = y1dplot + (n - 1) ydplot /. parameters;

Testplot = Test /. parameters;
Rentplot = Rentsol /. parameters;
quniformplot = quniform /. parameters;
Rentunifsolplot = Rentunifsol /. parameters;

Plot[Testplot /. parameters, {θu, 0.1, 0.5}, PlotLabel → {"Test"}]

Plot[{ICsolplot, qusolplot - qdsolplot}, {θu, 0.1, 0.5}, PlotLabel → {"IC,  qu-qd"}]
Plot[{TestMarginalCostuplot, TestMarginalCostdplot},
 {θu, 0.1, 0.5}, PlotLabel → {"TestMarginalCostu,  TestMarginalCostd"}]

Plot[{qusolplot, qdsolplot, quniformplot},
 {θu, 0.1, 0.5}, PlotLabel → {"qu,  qd,  quniform"}]
Plot[{p1uplot, p1dplot}, {θu, 0.1, 0.5}, PlotLabel → {"p1u,  p1d"}]
Plot[{puplot, pdplot}, {θu, 0.1, 0.5}, PlotLabel → {"pu,  pd"}]

Plot[{y1uplot, y1dplot}, {θu, 0.1, 0.5}, PlotLabel → {"y1u,  y1d"}]
Plot[{yuplot, ydplot}, {θu, 0.1, 0.5}, PlotLabel → {"yu,  yd"}]
Plot[{Yu, Yd}, {θu, 0.1, 0.5}, PlotLabel → {"Yu,  Yd  (total demands)"}]

Plot[{Rentplot, Rentunifsolplot}, {θu, 0.1, 0.5}, PlotLabel → {"Rent,  Rentuniform"}]
```

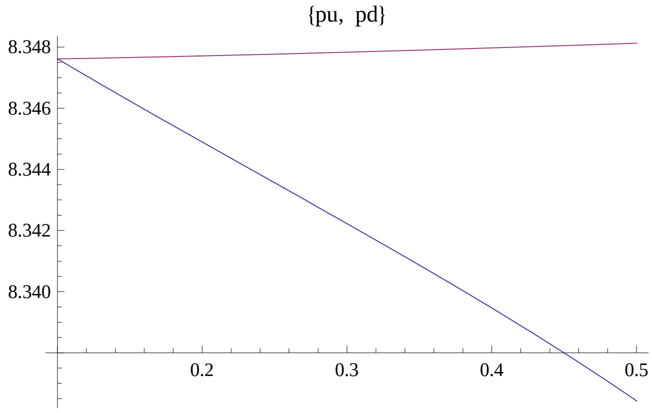


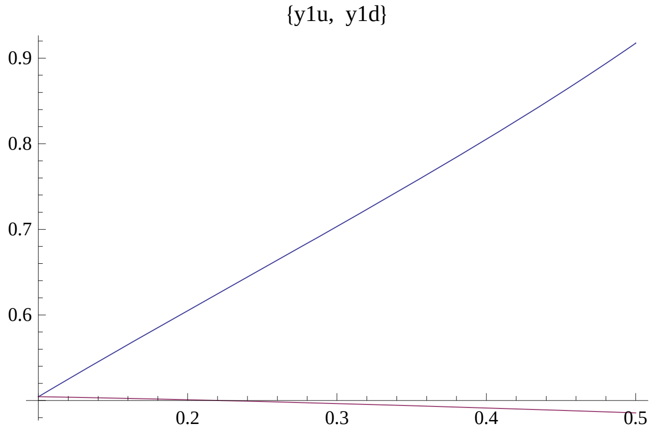

{IC, qu−qd}
0.25
0.20
0.15
0.10
0.05
0.2
0.3
0.4
0.5

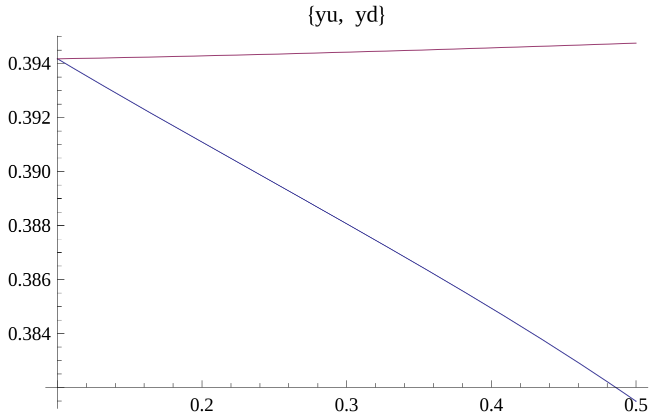

{TestMarginalCostu, TestMarginalCostd}
7.8
7.6
7.4
7.2
0.2
0.3
0.4
0.5

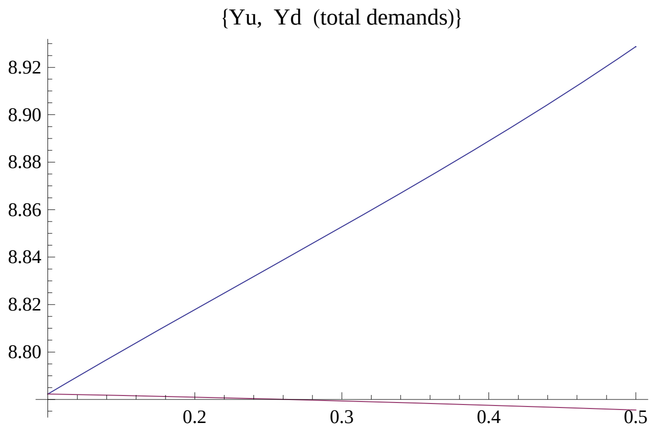

{qu, qd, quniform}
1.90
1.85
1.80
1.75
1.70
1.65
0.2
0.3
0.4
0.5

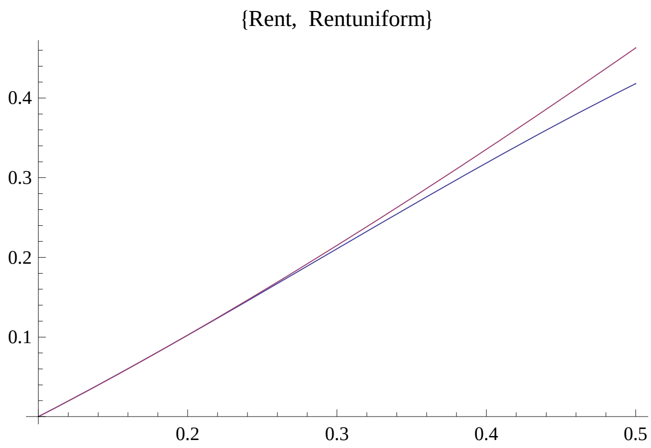

{p1u, p1d}
8.2
8.1
8.0
0.2
0.3
0.4
0.5


{pu, pd}
8.348
8.346
8.344
8.342
8.340
0.2
0.3
0.4
0.5


{y1u, y1d}
0.9
0.8
0.7
0.6
0.2
0.3
0.4
0.5


{yu, yd}
0.394
0.392
0.390
0.388
0.386
0.384
0.2
0.3
0.4
0.5


{Yu, Yd (total demands)}
8.92
8.90
8.88
8.86
8.84
8.82
8.80
0.2
0.3
0.4
0.5




```
(*The plot of IC and regulated quantities show that for low θu IC is not
  satisfied and also monotonicity of regulated quantities. With the next
  analysis we derive the precise values of θu such that IC is satisifed with =
 and the two regulated quantities are identical*)
```

```
Solve[ICsolplot == 0, θu]
FindRoot[ICsolplot, {θu, 0.2}]
```

```
{{θu → -0.802372 - 1.88485 ⅈ}, {θu → -0.802372 + 1.88485 ⅈ}, {θu → -0.415046},
 {θu → 0.1}, {θu → 0.1}, {θu → 0.193339}, {θu → 1.29084}, {θu → 2.87646}}
```

```
{θu → 0.193339}
```

```
Solve[qusolplot - qdsolplot == 0, θu]
FindRoot[qusolplot - qdsolplot, {θu, 0.2}]
```

```
{{θu → 0.1}, {θu → 0.178676}, {θu → 2.87246}}
```

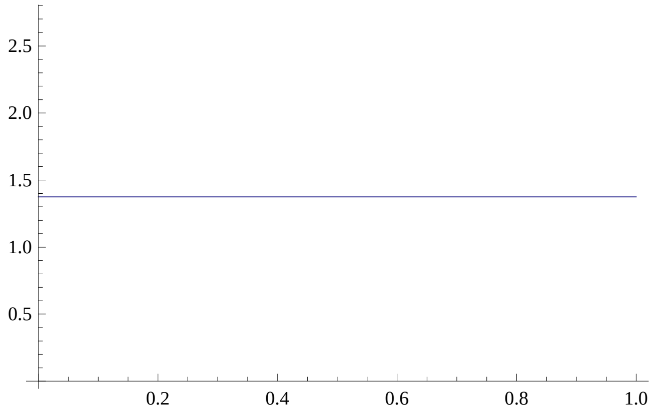
```
{θu → 0.178676}
```

```
(*Hence, this shows that monotinicity is not sufficient for IC since for θu in
 the interval [0.1786, 0.1933] monotonicity is verified but IC is violated*)
```

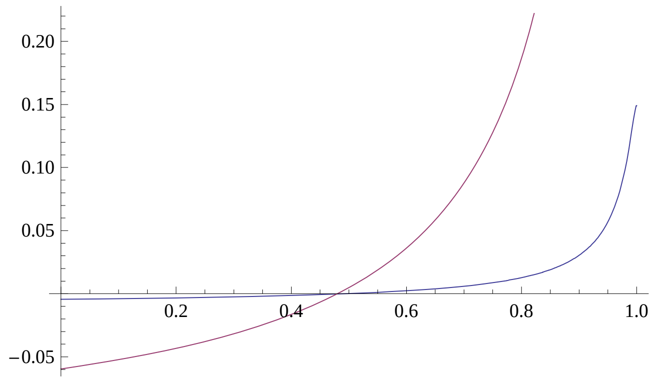
Analysis of v

```
parameters = {n → 22, γ → 0.15, μ → 10, mr → 10, c → 8, θu → 0.2, θd → 0.1, a → 0.1};

ICsolplot = ICsol /. parameters;
TestMarginalCostuplot = TestMarginalCostu /. parameters;
TestMarginalCostdplot = TestMarginalCostd /. parameters;
qusolplot = qusol /. parameters;
qdsolplot = qdsol /. parameters;
p1uplot = p1u /. parameters;
p1dplot = p1d /. parameters;
puplot = pu /. parameters;
pdplot = pd /. parameters;
y1uplot = y1u /. parameters;
y1dplot = y1d /. parameters;
yuplot = yu /. parameters;
ydplot = yd /. parameters;


p1uuniformplot = p1uuniform /. parameters;
p1duniformplot = p1duniform /. parameters;
puuniformplot = puuniform /. parameters;
pduniformplot = pduniform /. parameters;

Yu = y1uplot + (n - 1) yuplot /. parameters;
Yd = y1dplot + (n - 1) ydplot /. parameters;
ExpY = Yu v + (1 - v) Yd /. parameters;
Ysep = n * ysep /. parameters;
Psep = psep /. parameters;

Testplot = Test /. parameters;
Rentplot = Rentsol /. parameters;
quniformplot = quniform /. parameters;
Rentunifsolplot = Rentunifsol /. parameters;
Expectedp1 = p1uplot v + (1 - v) p1dplot /. parameters;
Expectedp = puplot v + (1 - v) pdplot /. parameters;


Plot[Testplot /. parameters, {v, 0, 1}, PlotLabel → {"Test"}]

Plot[{ICsolplot, qusolplot - qdsolplot}, {v, 0, 1}, PlotLabel → {"IC,  qu-qd"}]
Plot[{TestMarginalCostuplot, TestMarginalCostdplot},
 {γ, 0, 0.99}, PlotLabel → {"TestMarginalCostu,  TestMarginalCostd"}]

Plot[{qusolplot, qdsolplot, quniformplot}, {v, 0, 1}, PlotLabel → {"qu,  qd,  quniform"}]
Plot[{p1uplot, p1dplot, Expectedp1, Psep, p1uuniformplot, p1duniformplot}, {v, 0, 1},
 PlotLabel → {"p1u,  p1d, ExpectedPrice1,  Pseparation,  p1uuniform,  p1duniform"}]
Plot[{puplot, pdplot, Expectedp, Psep}, {v, 0, 1},
 PlotLabel → {"pu,  pd, Expectedprice,  Pseparation"}]

Plot[{y1uplot, y1dplot}, {v, 0, 1}, PlotLabel → {"y1u,  y1d"}]
Plot[{yuplot, ydplot}, {v, 0, 1}, PlotLabel → {"yu,  yd"}]
Plot[{Yu, Yd, ExpY, Ysep}, {v, 0, 1},
 PlotLabel → {"Yu,  Yd,  ExpectedY, Yseparation    (total demands)"}]

Plot[{Rentplot, Rentunifsolplot}, {v, 0, 1}, PlotLabel → {"Rent,  Rentuniform"}]
```

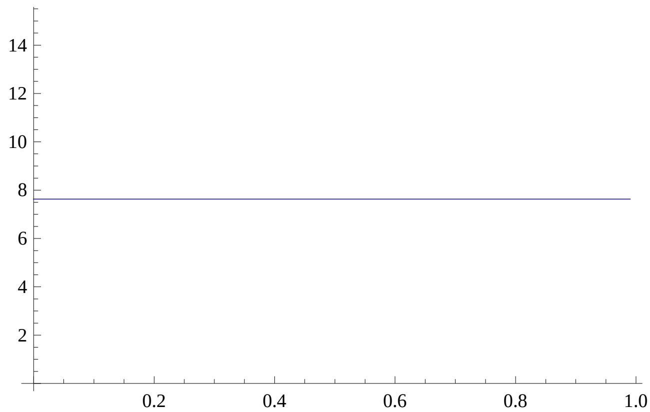
{Test}



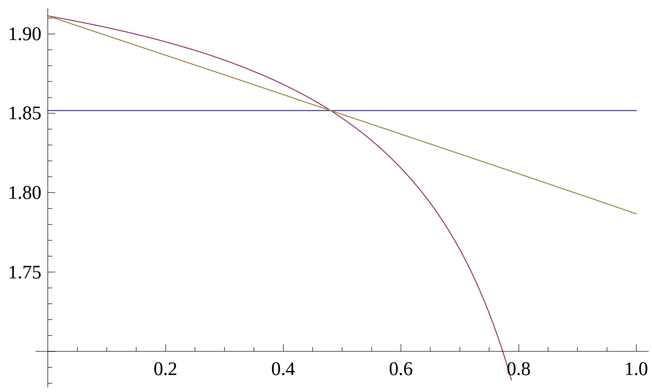
{IC, qu−qd}



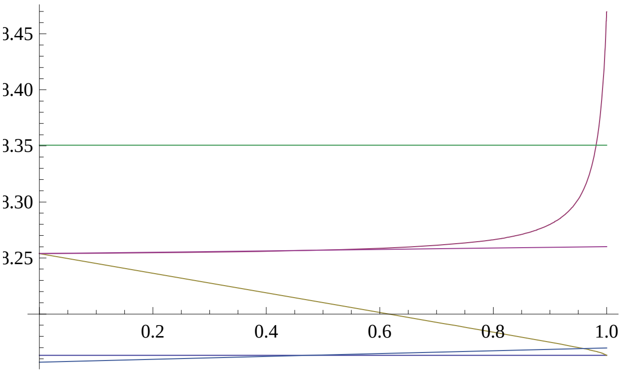
{TestMarginalCostu, TestMarginalCostd}



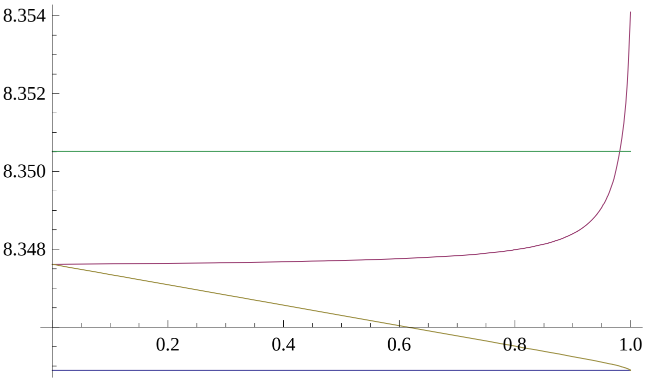
{qu, qd, quniform}



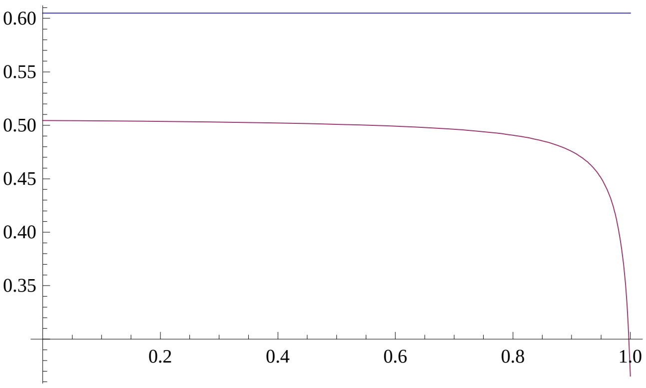
.u, p1d, ExpectedPrice1, Pseparation, p1uuniform, p1dunifor



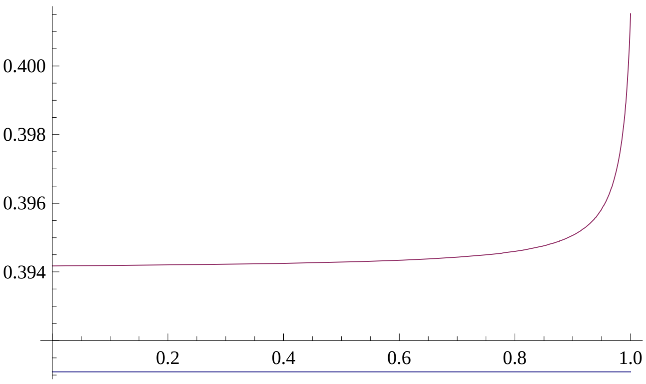
{pu, pd, Expectedprice, Pseparation}

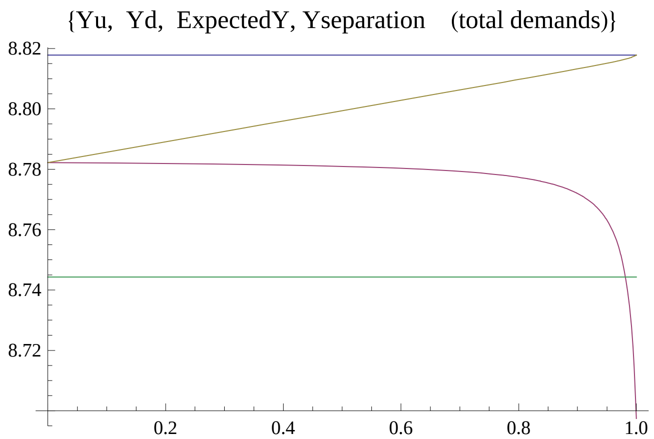


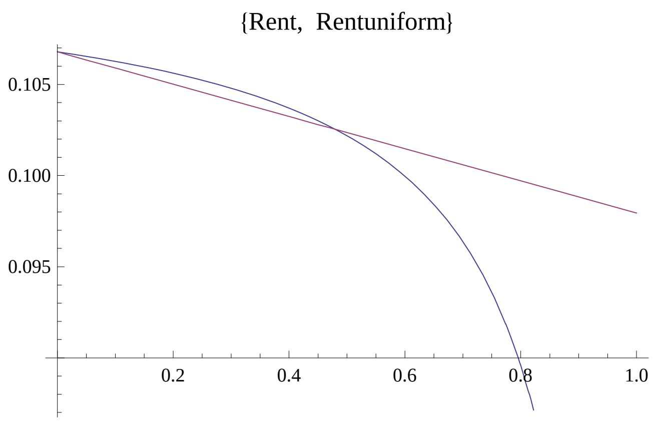
{y1u, y1d}



{yu, yd}








```
Solve[ICsolplot == 0, v]
FindRoot[ICsolplot, {v, 0.2}]
```

{{v → 0.493853}, {v → 1.00766}}

{v → 0.493853}

```
Solve[qusolplot - qdsolplot == 0, v]
FindRoot[qusolplot - qdsolplot, {v, 0.2}]
```

{{v → 0.48028}}

{v → 0.48028}

```
(*These paramters show the same observation about monotonicity
 of regulated quantities and IC as in the case above about θu *)
```